A system is a combination of devices and networks (subsystems) chosen to perform a desired function.

The analysis of most systems can be reduced to the study of the relationship between certain input excitation and the resulting outputs.

**Example:**

\[ f(t) \rightarrow M \rightarrow \text{Mech. system} \quad \text{input} \rightarrow \text{output (excitation, cause)} \]

**Example:**

\[ v(t) + \rightarrow + \rightarrow i(t) \rightarrow \text{Elect. system} \rightarrow v_o(t) \]
major categories of the system are as follows:
1. **Linear and Nonlinear Systems**

For a linear system, we should have:

\[
\begin{align*}
&x_1(t) \rightarrow y_1(t) \\
&x_2(t) \rightarrow y_2(t)
\end{align*}
\]

If

\[
\begin{align*}
&c_1 x_1 + c_2 x_2 \rightarrow c_1 y_1 + c_2 y_2
\end{align*}
\]

then

\[
\text{this is called superposition}
\]

If this is not true, then the system is nonlinear.

**Example.** Let \( y(t) = K + x(t) \)

\[
\begin{align*}
&x_1 \Rightarrow y_1 = K + x_1 \\
&x_2 \Rightarrow y_2 = K + x_2
\end{align*}
\]

\[
\begin{align*}
&c_1 x_1 + c_2 x_2 \Rightarrow y(t) = K + c_1 x_1(t) + c_2 x_2(t)
\end{align*}
\]

\[
\begin{align*}
&c_1 y_1 + c_2 y_2 = c_1 (K + x_1) + c_2 (K + x_2) \\
&= (c_1 + c_2) K + c_1 x_1 + c_2 x_2
\end{align*}
\]

\[\therefore\text{this system is nonlinear}\]
Remarks

1. Additivity Property:

\[ x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t) \]

2. Scaling or Homogeneity Property:

\[ \alpha x_1(t) \rightarrow \alpha y_1(t) \]

2. Note from the scaling property that for \( \alpha = 0 \), zero input results in zero output.

\[ \alpha = 0 \cdot x_1(t) \rightarrow 0 \cdot y_1(t) = 0. \]
\[ \Sigma x_a : \quad L + \quad y(t) = k x(t) \]
\[ x_1(t) \rightarrow \quad y_1(t) = k x_1(t) \]
\[ x_2(t) \rightarrow \quad y_2(t) = k x_2(t) \]
\[ a x_1(t) + b x_2(t) \rightarrow \quad y(t) = k (a x_1(t) + b x_2(t)) \]
\[ = a \underbrace{k x_1(t)}_{y_1(t)} + b \underbrace{k x_2(t)}_{y_2(t)} \]
\[ = a y_1(t) + b y_2(t). \]

Linear.

\[ \Sigma x_a : \quad y(t) = c x^2(t) \]
\[ x_1(t) \rightarrow \quad y_1(t) = c x_1^2(t) \]
\[ x_2(t) \rightarrow \quad y_2(t) = c x_2^2(t) \]
\[ a x_1(t) + b x_2(t) \rightarrow \quad y(t) = c (a x_1(t) + b x_2(t))^2 \neq a y_1(t) + b y_2(t) \]

Nonlinear.

\[ \Sigma x_a : \quad y(t) = \sin(x(t)) \]

Nonlinear.
Consider the $n^{th}$ order linear DE:

$$a_n(t) \frac{d^n y}{dt^n} + a_{n-1}(t) \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_1(t) \frac{d y}{dt} + a_0(t) y =$$

$$b_m(t) \frac{d^m x}{dt^m} + b_{m-1}(t) \frac{d^{m-1} x}{dt^{m-1}} + \cdots + b_1(t) \frac{d x}{dt} + b_0(t) x$$

$x(t)$ is said to be the input (cause, excitation)

$y(t)$ is called the output (effect, response)

It is shown that any system which can be characterized by the given DE is a linear system.

**Example 1:**

$$\frac{d^2 y}{dt^2} + y = \frac{d h}{dt}$$

**Linear**

**Example 2:**

$$\frac{d^3 y}{dt^3} + y \frac{dy}{dt} = 0$$

**Nonlinear**
Eq. (\star), as written, represents a time-varying system since the coeffs $a_i(t), b_i(t)$ are functions of time.

If $a_i(t)$ and $b_i(t)$ are constant, then this represents a fixed or time-invariant system. For fixed systems we have the following:

\[
\text{if } \begin{array}{c}
\rightarrow & y(t) & \rightarrow \\
X(t) & \rightarrow & y(t-t_0)
\end{array} \quad \text{then } \begin{array}{c}
X(t-t_0) & \rightarrow & \rightarrow \\
\rightarrow & \rightarrow & y(t-t_0)
\end{array}
\]

That is, the shape of the system response depends only upon the shape of the input and not on the time of application.
\[ y(t) = \min \{ x(t) \} \]

\[ y_1(t) = \min \{ x_1(t) \} \quad (\star) \]

Now, let

\[ x_2(t) = x_1(t - t_0) \]

\[ \therefore y_2(t) = \min \{ x_2(t) \} = \min \{ x_1(t - t_0) \} \quad (1) \]

Similarly, from (\star) = 0

\[ y_1(t - t_0) = \min \{ x_1(t - t_0) \} \quad (2) \]

(1), (2) = 1

\[ y_2(t) = y_1(t - t_0) \]

\[ \therefore \text{The system is T.I.} \]
\[
\begin{align*}
\Sigma h : & \quad \gamma(t) = e^{x(t)} \\
& \quad y(t) = e^{x_1(t)} \\
& \quad x_1(t) = x_1(t-t_0) \\
\therefore & \quad y_1(t) = e^{x_2(t)} = e^{x_1(t-t_0)} \quad (1) \\
\text{(1)} \Rightarrow & \quad y_1(t-t_0) = e^{x_1(t-t_0)} \quad (2) \\
\therefore & \quad y_2(t) = y_1(t-t_0) \\
\therefore & \quad T, I.
\end{align*}
\]
Example

\[ \chi_u(n) = \begin{cases} \chi(\frac{m}{L}) & m = 0, \pm L, \pm 2L, \ldots \\ 0 & \text{otherwise} \end{cases} \]

\[ \chi_u(m) = \chi(\frac{m}{L}) \quad (1) \]

\[ \chi_u(m) = \chi(\frac{m}{L} - m_0) \]

\[ \chi_{1, u}(m) = \begin{cases} \chi(\frac{m - Lm_0}{L}) & m = Lm_0, (L \pm 1)m_0, (L \pm 2)m_0, \ldots \\ 0 & \text{otherwise} \end{cases} \]

\[ \chi_{1, u}(m) = \begin{cases} \chi(\frac{m - m_0}{L}) & m = m_0, m_0 \pm L, \ldots \\ \neq \chi_{1, u}(m) \end{cases} \]

T.V.
Lumped and distributed-parameter systems

A lumped system is one that is composed of finite non-zero elements satisfying ordinary DE. Physical size of the system is of no concern since excitations propagate through the system instantaneously. This assumption is usually valid if the largest physical dimension of the system is small with the wavelength of the highest significant freq concerned.

A distributed-parameter system is represented by a partial DE and generally has dimensions that are not small compared with the shortest wavelength of interest. Transmission lines, waveguides, antennas, and microwave tubes are typical examples of distributed systems.
Causal and Noncausal Systems

A causal (or physical, or nonanticipatory) system is one whose present response does not depend on future values of the input.

A noncausal system is one for which this condition is not assumed.

Instantaneous (memoryless) and dynamic (memory) Systems

A system for which the output is a function of the input at the present time only is said to be instantaneous.

\[ y(t) = f(x(t), t) \]

A dynamic system is one whose output depends on past or future values of the input in addition to the present time. If the system is also causal, the output of a dynamic system depends only on present and past values of the input.
Causal and noncausal systems

A causal (or physical, or nonanticipatory) system is one whose present response does not depend on future values of the input.

Example:

A noncausal system is one for which this condition is not assumed.

Example:

\[ y(t) = kx(t + 10) \]
Instantaneous (memoryless) and dynamic systems

A system for which the output is a function of the input at the present time only is said to be instantaneous.

\[ y(t) = f(x(t), t) \]

**Example:** a resistive network

**Example:** \[ y(t) = x(t-1) \]

This is a memory (dynamic) system.

\[ y = x + 2 \]

A dynamic system is one whose output depends on past or future values of the input in addition to present time. If the system is also causal, the output of a dynamic system depends only on present and past values of the input. Systems described by diff eqns are dynamic.
Except where otherwise specifically stated, all systems discussed in later chapters will be assumed to be causal, linear, fixed, lumped and dynamic.

Ex.

Ex. next page
State whether the following systems are linear or nonlinear, causal or noncausal, fixed or time-varying, dynamic or instantaneous. As usual, \( x(t) \) denotes the input and \( y(t) \) the output. Fill in the table with your answers.

(a) \( \frac{dy(t)}{dt} + 10y(t) = x(t) \)

(b) \( \frac{dy(t)}{dt} + 10y(t) + 5 = x(t) \)

(c) \( \frac{dy(t)}{dt} + t^2y(t) = x(t) \)

(d) \( \frac{dy(t)}{dt} + y^2(t) = x(t) \)

(e) \( \frac{dy(t)}{dt} + y(t) = x(t + 10) \)

(f) \( y(t) = 10x^2(t) + x(t) \)

(g) \( y(t) = x(t + 10) + x^2(t) \)

If true, check the appropriate box.

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<th>b</th>
<th>c</th>
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If true, check the appropriate box.

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