Perfect Reconstruction Two-Channel FIR Filter Banks

- A perfect reconstruction two-channel FIR filter bank with linear-phase FIR filters can be designed if the power-complementary requirement

\[ |H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 = 1 \]

between the two analysis filters \( H_0(z) \) and \( H_1(z) \) is not imposed.
Perfect Reconstruction Two-Channel FIR Filter Banks

• To develop the pertinent design equations we observe that the input-output relation of the 2-channel QMF bank

\[ Y(z) = \frac{1}{2} \{ H_0(z)G_0(z) + H_1(z)G_1(z) \} X(z) \]

\[ + \frac{1}{2} \{ H_0(-z)G_0(z) + H_1(-z)G_1(z) \} X(-z) \]

can be expressed in matrix form as

\[ Y(z) = \frac{1}{2} \begin{bmatrix} G_0(z) & G_1(z) \end{bmatrix} \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix} \begin{bmatrix} X(z) \\ X(-z) \end{bmatrix} \]
Perfect Reconstruction Two-Channel FIR Filter Banks

• From the previous equation we obtain

\[
Y(\!-\!z) = \frac{1}{2} \begin{bmatrix} G_0(\!-\!z) & G_1(\!-\!z) \end{bmatrix} \begin{bmatrix} H_0(z) & H_0(\!-\!z) \\ H_1(z) & H_1(\!-\!z) \end{bmatrix} \begin{bmatrix} X(z) \\ X(\!-\!z) \end{bmatrix}
\]

• Combining the two matrix equations we get

\[
\begin{bmatrix} Y(z) \\ Y(\!-\!z) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} G_0(z) & G_1(z) \\ G_0(\!-\!z) & G_1(\!-\!z) \end{bmatrix} \begin{bmatrix} H_0(z) & H_0(\!-\!z) \\ H_1(z) & H_1(\!-\!z) \end{bmatrix} \begin{bmatrix} X(z) \\ X(\!-\!z) \end{bmatrix}
\]

\[
= \frac{1}{2} \begin{bmatrix} G^{(m)}(z) & H^{(m)}(z) \end{bmatrix}^T \begin{bmatrix} X(z) \\ X(\!-\!z) \end{bmatrix}
\]
Perfect Reconstruction Two-Channel FIR Filter Banks

where

\[
G^{(m)}(z) = \begin{bmatrix}
G_0(z) & G_1(z) \\
G_0(-z) & G_1(-z)
\end{bmatrix}
\]

\[
H^{(m)}(z) = \begin{bmatrix}
H_0(z) & H_1(z) \\
H_0(-z) & H_1(-z)
\end{bmatrix}
\]

are called the modulation matrices.
Perfect Reconstruction Two-Channel FIR Filter Banks

• Now for perfect reconstruction we must have
  \( Y(z) = z^{-\ell} X(z) \) and correspondingly
  \( Y(-z) = (-z)^{-\ell} X(-z) \)

• Substituting these relations in the equation

\[
\begin{bmatrix}
Y(z) \\
Y(-z)
\end{bmatrix} = \frac{1}{2} G^{(m)}(z)[H^{(m)}(z)]^T \begin{bmatrix}
X(z) \\
X(-z)
\end{bmatrix}
\]

we observe that the PR condition is satisfied

if

\[
\frac{1}{2} G^{(m)}(z)[H^{(m)}(z)]^T = \begin{bmatrix}
z^{-\ell} & 0 \\
0 & (-z)^{-\ell}
\end{bmatrix}
\]
Perfect Reconstruction Two-Channel FIR Filter Banks

• Thus, knowing the analysis filters $H_0(z)$ and $H_1(z)$, the synthesis filters $G_0(z)$ and $G_1(z)$ are determined from

$$G^{(m)}(z) = 2 \begin{bmatrix} z^{-\ell} & 0 \\ 0 & (-z)^{-\ell} \end{bmatrix} \left(\left[H^{(m)}(z)\right]^T\right)^{-1}$$

• After some algebra we arrive at
Perfect Reconstruction Two-Channel FIR Filter Banks

\[ G_0(z) = \frac{2z^{-\ell}}{\det[H^{(m)}(z)]]} \cdot H_1(-z) \]
\[ G_1(z) = -\frac{2z^{-\ell}}{\det[H^{(m)}(z)]]} \cdot H_0(-z) \]

where
\[ \det[H^{(m)}(z)] = H_0(z)H_1(-z) - H_0(-z)H_1(z) \]
and \( \ell \) is an odd positive integer
Perfect Reconstruction Two-Channel FIR Filter Banks

- For FIR analysis filters $H_0(z)$ and $H_1(z)$, the synthesis filters $G_0(z)$ and $G_1(z)$ will also be FIR filters if

$$\det[H^{(m)}(z)] = cz^{-k}$$

where $c$ is a real number and $k$ is a positive integer

- In this case

$$G_0(z) = \frac{2}{c} z^{-(\ell-k)}H_1(-z)$$

$$G_1(z) = -\frac{2}{c} z^{-(\ell-k)}H_0(-z)$$
Orthogonal Filter Banks

Let $H_0(z)$ be an FIR filter of odd order $N$ satisfying the power-symmetric condition
\[ H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = 1 \]

Choose $H_1(z) = z^{-N}H_0(-z^{-1})$

Then $\det[H^{(m)}(z)]$
\[ = -z^{-N} \left( H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) \right) = -z^{-N} \]
Orthogonal Filter Banks

• Comparing the last equation with
  \[ \text{det}[H^{(m)}(z)] = cz^{-k} \]
  we observe that \( c = -1 \) and \( k = N \)

• Using \( H_1(z) = z^{-N} H_0(-z^{-1}) \) in
  \[ G_0(z) = \frac{2}{c} z^{-(\ell-k)} H_1(-z) \]
  \[ G_1(z) = -\frac{2}{c} z^{-(\ell-k)} H_0(-z) \]

with \( \ell = k = N \) we get
  \[ G_0(z) = 2z^{-N} H_0(z^{-1}), \quad G_1(z) = 2z^{-N} H_1(z^{-1}) \]
Orthogonal Filter Banks

- **Note:** If $H_0(z)$ is a causal FIR filter, the other three filters are also causal FIR filters.
- **Moreover,** $|H_1(e^{j\omega})| = |H_0(-e^{j\omega})|$
- **Thus,** for a real coefficient transfer function if $H_0(z)$ is a lowpass filter, then $H_1(z)$ is a highpass filter.
- **In addition,** $|G_i(e^{j\omega})| = |H_i(e^{j\omega})|$, $i = 1, 2$
Orthogonal Filter Banks

• A perfect reconstruction power-symmetric filter bank is also called an orthogonal filter bank.

• The filter design problem reduces to the design of a power-symmetric lowpass filter $H_0(z)$.

• To this end, we can design a an even-order $F(z) = H_0(z)H_0(z^{-1})$ whose spectral factorization yields $H_0(z)$. 
Orthogonal Filter Banks

• Now, the power-symmetric condition

\[ H_0(z)H_0(z^{-1}) + H_1(-z)H_1(-z^{-1}) = 1 \]

implies that \( F(z) \) be a zero-phase half-band lowpass filter with a non-negative frequency response \( F(e^{j\omega}) \)

• Such a half-band filter can be obtained by adding a constant term \( K \) to a zero-phase half-band filter \( Q(z) \) such that

\[ F(e^{j\omega}) = Q(e^{j\omega}) + K \geq 0 \quad \text{for all } \omega \]
Orthogonal Filter Banks

• Summarizing, the steps for the design of a real-coefficient power-symmetric lowpass filter $H_0(z)$ are:

• (1) Design a zero-phase real-coefficient FIR half-band lowpass filter $Q(z)$ of order $2N$ with $N$ an odd positive integer:

$$Q(z) = \sum_{n=-N}^{N} q[n] z^{-n}$$
Orthogonal Filter Banks

• (2) Let $\delta$ denote the peak stopband ripple of $Q(e^{j\omega})$

• Define $F(z) = Q(z) + \delta$ which guarantees that $F(e^{j\omega}) \geq 0$ for all $\omega$

• Note: If $q[n]$ denotes the impulse response of $Q(z)$, then the impulse response $f[n]$ of $F(z)$ is given by

$$f[n] = \begin{cases} 
q[n] + \delta, & \text{for } n=0 \\
q[n], & \text{for } n \neq 0 
\end{cases}$$

• (3) Determine the spectral factor $H_0(z)$ of $F(z)$
Orthogonal Filter Banks

• Example - Consider the FIR filter

\[ F(z) = z^N (1 + z^{-1})^{N+1} R(z) \]

where \( R(z) \) is a polynomial in \( z^{-1} \) of degree \( N - 1 \) with \( N \) odd

• \( F(z) \) can be made a half-band filter by choosing \( R(z) \) appropriately

• This class of half-band filters has been called the binomial or maxflat filter
The filter $F(z)$ has a frequency response that is maximally flat at $\omega = 0$ and at $\omega = \pi$.

For $N = 3$, $R(z) = \frac{1}{16}(-1 + 4z^{-1} - z^{-2})$ resulting in

$$F(z) = \frac{1}{16}(-z^3 + 9z + 16 + 9z^{-1} - z^{-3})$$

which is seen to be a symmetric polynomial with 4 zeros located at $z = -1$, a zero at $z = 2 - \sqrt{3}$, and a zero at $z = 2 + \sqrt{3}$.
Orthogonal Filter Banks

• The minimum-phase spectral factor is therefore the lowpass analysis filter

\[ H_0(z) = -0.3415(1 + z^{-1})^2 [1 - (2 - \sqrt{3})z^{-1}] \]

\[ = -0.3415(1 + 1.732z^{-1} + 0.464z^{-2} - 0.268z^{-3}) \]

• The corresponding highpass filter is given by

\[ H_1(z) = z^{-3}H_0(-z^{-1}) \]

\[ = -0.3415(0.2679 + 0.4641z^{-1} - 1.732z^{-2} + z^{-3}) \]
Orthogonal Filter Banks

- The two synthesis filters are given by
  \[ G_0(z) = 2z^{-3}H_0(z^{-1}) \]
  \[ = -0.683(-0.2679 + 0.4641z^{-1} + 1.732z^{-2} + z^{-3}) \]
  \[ G_1(z) = 2z^{-3}H_1(z^{-1}) \]
  \[ = -0.683(1 - 1.732z^{-1} + 0.4641z^{-2} + 0.2679z^{-3}) \]

- Magnitude responses of the two analysis filters are shown on the next slide.
Orthogonal Filter Banks

- Comments: (1) The order of $F(z)$ is of the form $4K+2$, where $K$ is a positive integer.
- Order of $H_0(z)$ is $N = 2K+1$, which is odd as required.
Orthogonal Filter Banks

• (2) Zeros of $F(z)$ appear with mirror-image symmetry in the $z$-plane with the zeros on the unit circle being of even multiplicity.

• Any appropriate half of these zeros can be grouped to form the spectral factor $H_0(z)$.

• For example, a minimum-phase $H_0(z)$ can be formed by grouping all the zeros inside the unit circle along with half of the zeros on the unit circle.
Orthogonal Filter Banks

- Likewise, a maximum-phase $H_0(z)$ can be formed by grouping all the zeros outside the unit circle along with half of the zeros on the unit circle.
- However, it is not possible to form a spectral factor with linear phase.
- (3) The stopband edge frequency is the same for $F(z)$ and $H_0(z)$. 
Orthogonal Filter Banks

• (4) If the desired minimum stopband attenuation of $H_0(z)$ is $\alpha_s$ dB, then the minimum stopband attenuation of $F(z)$ is $2\alpha_s + 6.02$ dB

• Example - Design a lowpass real-coefficient power-symmetric filter $H_0(z)$ with the following specifications: $\omega_s = 0.63\pi$, and $\alpha_s = 12$ dB
Orthogonal Filter Banks

- The specifications of the corresponding zero-phase half-band filter $F(z)$ are therefore:
  $\omega_s = 0.63\pi$ and $\alpha_s = 40$ dB

- The desired stopband ripple is thus $\delta_s = 0.01$ which is also the passband ripple

- The passband edge is at $\omega_p = \pi - 0.63\pi = 0.37\pi$

- Using the function `remezord` we first estimate the order of $F(z)$ and then using the function `remez` design $Q(z)$
Orthogonal Filter Banks

- The order of $F(z)$ is found to be 14 implying that the order of $H_0(z)$ is 7 which is odd as desired.
- To determine the coefficients of $F(z)$ we add $err$ (the maximum stopband ripple) to the central coefficient $q[7]$
- Next, using the function roots we determine the roots of $F(z)$ which should theoretically exhibit mirror-image symmetry with double roots on the unit circle.
Orthogonal Filter Banks

• However, the algorithm’s numerically quite sensitive and it is found that a slightly larger value than $\text{err}$ should be added to ensure double zeros of $F(z)$ on the unit circle.

• Choosing the roots inside the unit circle along with one set of unit circle roots we get the minimum-phase spectral factor $H_0(z)$. 
Orthogonal Filter Banks

- The zero locations of $F(z)$ and $H_0(z)$ are shown below
Orthogonal Filter Banks

• The gain responses of the two analysis filters are shown below
Orthogonal Filter Banks

• Separate realizations of the two filters $H_0(z)$ and $H_1(z)$ would require $2(N+1)$ multipliers and $2N$ two-input adders

• However, a computationally efficient realization requiring $N+1$ multipliers and $2N$ two-input adders can be developed by exploiting the relation

\[ H_1(z) = z^{-N} H_0(-z^{-1}) \]
Paraunitary System

• A $p$-input, $q$-output LTI discrete-time system with a transfer matrix $T_{pq}(z)$ is called a paraunitary system if $T_{pq}(z)$ is a paraunitary matrix, i.e.,

$$\tilde{T}_{pq}(z)T_{pq}(z) = cI_p$$

• Note: $\tilde{T}_{pq}(z)$ is the paraconjugate of $T_{pq}(z)$ given by the transpose of $T_{pq}(z^{-1})$ with each coefficient replaced by its conjugate

• $I_p$ is an $p \times p$ identity matrix, $c$ is a real constant
Paraunitary Filter Banks

- A causal, stable paraunitary system is also a lossless system
- It can be shown that the modulation matrix

\[ H^{(m)}(z) = \begin{bmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{bmatrix} \]

of a power-symmetric filter bank is a paraunitary matrix
Paraunitary Filter Banks

• Hence, a power-symmetric filter bank has also been referred to as a paraunitary filter bank

• The cascade of two paraunitary systems with transfer matrices $T_{pq}^{(1)}(z)$ and $T_{qr}^{(2)}(z)$ is also paraunitary

• The above property can be utilized in designing a paraunitary filter bank without resorting to spectral factorization
Consider a real-coefficient FIR transfer function $H_N(z)$ of order $N$ satisfying the power-symmetric condition

$$H_N(z)H_N(z^{-1}) + H_N(-z)H_N(-z^{-1}) = K_N$$

We shall show now that $H_N(z)$ can be realized in the form of a cascaded lattice structure as shown on the next slide.
Power-Symmetric FIR
Cascaded Lattice Structure

- Define

\[ H_i(z) = \frac{X_i(z)}{X_0(z)} \quad \text{and} \quad G_i(z) = \frac{Y_i(z)}{X_0(z)} \]

\[ 1 \leq i \leq N \]
Power-Symmetric FIR Cascaded Lattice Structure

• From the figure we observe that
  \[ X_1(z) = X_0(z) + k_1z^{-1}X_0(z) \]
  \[ Y_1(z) = -k_1X_0(z) + z^{-1}X_0(z) \]

• Therefore,
  \[ H_1(z) = 1 + k_1z^{-1}, \quad G_1(z) = -k_1 + z^{-1} \]

• It can be easily shown that
  \[ G_1(z) = z^{-1}H_1(-z^{-1}) \]
Power-Symmetric FIR
Cascaded Lattice Structure

• Next from the figure it follows that

\[ H_i(z) = H_{i-2}(z) + k_i z^{-2} G_{i-2}(z) \]
\[ G_i(z) = -k_i H_{i-2}(z) + z^{-2} G_{i-2}(z) \]

• It can easily be shown that

\[ G_i(z) = z^{-i} H_i(-z^{-1}) \]
provided

\[ G_{i-2}(z) = z^{-(i-2)} H_{i-2}(-z^{-1}) \]
We have shown that $G_i(z) = z^{-i}H_i(-z^{-1})$ holds for $i = 1$.

Hence the above relation holds for all odd integer values of $i$.

$N$ must be an odd integer.

It is a simple exercise to show that both $H_i(z)$ and $G_i(z)$ satisfy the power-symmetry condition $H_i(z)H_i(z^{-1}) + H_i(-z)H_i(-z^{-1}) = K_i$. 
Power-Symmetric FIR Cascaded Lattice Structure

• In addition, $H_i(z)$ and $G_i(z)$ are power-complementary, i.e.,

$$(1 + k_i^2)z^{-2}G_{i-2}(z) = k_iH_i(z) + G_i(z)$$

• To develop the synthesis equation we express $H_{i-2}(z)$ and $G_{i-2}(z)$ in terms of $H_i(z)$ and $G_i(z)$:

$$(1 + k_i^2)H_{i-2}(z) = H_i(z) - k_iG_i(z)$$

$$(1 + k_i^2)z^{-2}G_{i-2}(z) = k_iH_i(z) + G_i(z)$$
Power-Symmetric FIR Cascaded Lattice Structure

- Note: At the $i$-th step, the coefficient $k_i$ is chosen to eliminate the coefficient of $z^{-i}$, the highest power of $z^{-1}$ in $H_i(z) - k_iG_i(z)$.
- For this choice of $k_i$ the coefficient of also vanishes making $H_{i-2}(z)$ a polynomial of degree $i - 2$.
- The synthesis process begins with $i = N$ and compute $G_N(z)$ using $G_N(z) = z^{-N}H_N(-z^{-1})$. 
Power-Symmetric FIR Cascaded Lattice Structure

• Next, the transfer functions $H_{N-2}(z)$ and $G_{N-2}(z)$ are determined using the synthesis equations

\[(1 + k_i^2)H_{i-2}(z) = H_i(z) - k_iG_i(z)\]
\[(1 + k_i^2)z^{-2}G_{i-2}(z) = k_iH_i(z) + G_i(z)\]

• This process is repeated until all coefficients of the lattice have been determined
Power-Symmetric FIR
Cascaded Lattice Structure

• **Example** - Consider

\[ H_5(z) = 1 + 0.3z^{-1} + 0.2z^{-2} - 0.376z^{-3} \]
\[ - 0.06z^{-4} + 0.2z^{-5} \]

• It can be easily verified that \( H_5(z) \) satisfies the power-symmetric condition

• Next we form

\[ G_5(z) = z^{-5} H_5(-z^{-1}) = -0.2 - 0.06z^{-1} \]
\[ + 0.376z^{-2} + 0.2z^{-3} - 0.3z^{-4} + z^{-5} \]
Power-Symmetric FIR
Cascaded Lattice Structure

• To determine $H_5(z)$ we first form

$$H_5(z) - k_5 G_5(z) = (1 + 0.2k_5) + (0.3 + 0.06k_5)z^{-1}$$

$$+ (0.2 - 0.376k_5)z^{-2} + (-0.376 - 0.2k_5)z^{-3}$$

$$+ (-0.06 + 0.3k_5)z^{-4} + (0.2 - k_5)z^{-5}$$

• To cancel the coefficient of $z^{-5}$ in the above we choose

$$k_5 = 0.2$$
Power-Symmetric FIR Cascaded Lattice Structure

- Then \[ H_3(z) = \frac{1}{1-k_5^2} [H_5(z) - k_5G_5(z)] \]
  \[ = \frac{1}{1.04} (1.04 + 0.312z^{-1} + 0.1248z^{-2} - 0.416z^{-3}) \]

- We next form \[ G_3(z) = z^{-3}H_3(-z^{-1}) = 0.4 + 0.12z^{-1} - 0.3z^{-2} + z^{-3} \]

- Continuing the above process we get \[ k_3 = -0.4, \quad k_1 = 0.3 \]
Power-Symmetric FIR Banks

• Using the method outlined for the realization of a power-symmetric transfer function, we can develop a cascaded lattice realization of the 2-channel paraunitary QMF bank
• Three important properties of the QMF lattice structure are structurally induced
Power-Symmetric FIR Banks

• (1) The QMF lattice guarantees perfect reconstruction independent of the lattice parameters
• (2) It exhibits very small coefficient sensitivity to lattice parameters as each stage remains lossless under coefficient quantization
• (3) Computational complexity is about one-half that of any other realization as it requires \((N+1)/2\) total number of multipliers for an order-\(N\) filter
Example - Consider the analysis filter of the previous example:

\[ H_7(z) = 0.3231 + 0.51935z^{-1} + 0.30134z^{-2} \]
\[ - 0.0781z^{-3} - 0.13767z^{-4} + 0.321z^{-5} \]
\[ + 0.079z^{-6} - 0.049z^{-7} \]

We place a multiplier \( h[0] = 0.3231 \) at the input and synthesize a cascade lattice structure for the normalized transfer function \( H_7(z) / h[0] \)
Power-Symmetric FIR Banks

• The lattice coefficients obtained for the normalized analysis transfer function are:

\[ k_7 = -0.15165, \quad k_5 = 0.2354, \]
\[ k_3 = -0.48393, \quad k_1 = 1.61 \]

• Note: Because of the numerical problem, the coefficients of the spectral factor obtained in the previous example are not very accurate.
• As a result, the coefficients of $z^{-(i-1)}$ of the transfer function $H_{i-2}(z)$ generated from the transfer function $H_i(z)$ using the relation

$$H_{i-2}(z) = \frac{1}{1+k_i^2}[H_i(z) - k_i G_i(z)]$$

is not exactly zero, and has been set to zero at each iteration.
Power-Symmetric FIR Banks

- Two interesting properties of the cascaded lattice QMF bank can be seen by examining its multiplier coefficient values:
  - (1) Signs of coefficients alternate between stages
  - (2) The values of the coefficients \( \{k_i\} \) decrease with increasing \( i \)
Power-Symmetric FIR Banks

• The QMF lattice structure can be used directly to design the power-symmetric analysis filter $H_0(z)$ using an iterative computer-aided optimization technique

• Goal: Determine the lattice parameters $k_i$ by minimizing the energy in the stopband of $H_0(z)$
Power-Symmetric FIR Banks

• The pertinent objective function is given by

\[ \phi = \int_{\omega_s}^{\pi} \left| H_0(e^{j\omega}) \right|^2 d\omega \]

• Note: The power-symmetric property ensures good passband response
• In the design of an orthogonal 2-channel filter bank, the analysis filter $H_0(z)$ is chosen as a spectral factor of the zero-phase even-order half-band filter

$$F(z) = H_0(z)H_0(z^{-1})$$

• Note: The two spectral factors $H_0(z)$ and $H_0(z^{-1})$ of $F(z)$ have the same magnitude response
Biorthogonal FIR Banks

• As a result, it is not possible to design perfect reconstruction filter banks with linear-phase analysis and synthesis filters

• However, it is possible to maintain the perfect reconstruction condition with linear-phase filters by choosing a different factorization scheme
Biorthogonal FIR Banks

• To this end, the causal half-band filter $z^{-N}F(z)$ of order $2N$ is factorized in the form

$$z^{-N}F(z) = H_0(z)H_1(-z)$$

where $H_0(z)$ and $H_1(z)$ are linear-phase filters

• The determinant of the modulation matrix $H^{(m)}(z)$ is now given by

$$\det[H^{(m)}(z)] = H_0(z)H_1(-z) - H_0(-z)H_1(z) = z^{-N}$$
Biorthogonal FIR Banks

• Note: The determinant of the modulation matrix satisfies the perfect reconstruction condition

• The filter bank designed using the factorization scheme \( z^{-N} F(z) = H_0(z) H_1(-z) \) is called a biorthogonal filter bank

• The two synthesis filters are given by
  \[
  G_0(z) = H_1(-z), \quad G_1(z) = -H_0(-z)
  \]
Biorthogonal FIR Banks

• **Example** - The half-band filter

\[ F(z) = \frac{1}{16} z^3 (1 + z^{-1})^4 (-1 + 4 z^{-1} - z^{-2}) \]

• can be factored several different ways to yield linear-phase analysis filters \( H_0(z) \) and \( H_1(z) \)

• **For example, one choice yields**

\[ H_0(z) = \frac{1}{8} (-1 + 2 z^{-1} + 6 z^{-2} + 2 z^{-3} - z^{-4}) \]

\[ H_1(z) = \frac{1}{2} (1 - 2 z^{-1} + z^{-2}) \]
Biorthogonal FIR Banks

• Since the length of $H_0(z)$ is 5 and the length of $H_1(z)$ is 3, the above set of analysis filters is known as the 5/3 filter-pair of Daubechies

• A plot of the gain responses of the 5/3 filter-pair is shown below
Biorthogonal FIR Banks

• Another choice yields the 4/4 filter-pair of Daubechies

\[
H_0(z) = \frac{1}{8} (1 + 3z^{-1} + 3z^{-2} + z^{-3})
\]

\[
H_1(z) = \frac{1}{2} (-1 - 3z^{-1} + 3z^{-2} + z^{-3})
\]

• A plot of the gain responses of the 4/4 filter-pair is shown below