Perfect Reconstruction Two-Channel FIR Filter Banks

- A perfect reconstruction two-channel FIR filter bank with linear-phase FIR filters can be designed if the power-complementary requirement

$$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 = 1$$

between the two analysis filters $H_0(z)$ and $H_1(z)$ is not imposed

Perfect Reconstruction Two-Channel FIR Filter Banks

- To develop the pertinent design equations we observe that the input-output relation of the 2-channel QMF bank

$$Y(z) = \frac{1}{2}\{H_0(z)G_0(z) + H_1(z)G_1(z)\}X(z)$$
$$+\frac{1}{2}\{H_0(-z)G_0(z) + H_1(-z)G_1(z)\}X(-z)$$

can be expressed in matrix form as

$$Y(z) = \frac{1}{2}\begin{bmatrix} G_0(z) & G_1(z) \end{bmatrix} \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix} \begin{bmatrix} X(z) \\ X(-z) \end{bmatrix}$$

Perfect Reconstruction Two-Channel FIR Filter Banks

- From the previous equation we obtain

$$Y(-z) = \frac{1}{2}[G_0(-z) \;\; G_1(-z)] \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix} \begin{bmatrix} X(z) \\ X(-z) \end{bmatrix}$$

- Combining the two matrix equations we get

$$\begin{bmatrix} Y(z) \\ Y(-z) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} G_0(z) & G_1(z) \\ G_0(-z) & G_1(-z) \end{bmatrix} \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix} \begin{bmatrix} X(z) \\ X(-z) \end{bmatrix}$$

$$= \frac{1}{2} \mathbf{G}^{(m)}(z) [\mathbf{H}^{(m)}(z)]^T \begin{bmatrix} X(z) \\ X(-z) \end{bmatrix}$$

Perfect Reconstruction Two-Channel FIR Filter Banks

where

$$\mathbf{G}^{(m)}(z) = \begin{bmatrix} G_0(z) & G_1(z) \\ G_0(-z) & G_1(-z) \end{bmatrix}$$

$$\mathbf{H}^{(m)}(z) = \begin{bmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{bmatrix}$$

are called the modulation matrices

Perfect Reconstruction Two-Channel FIR Filter Banks

- Now for perfect reconstruction we must have $Y(z) = z^{-\ell} X(z)$ and correspondingly $Y(-z) = (-z)^{-\ell} X(-z)$
- Substituting these relations in the equation

$$\begin{bmatrix} Y(z) \\ Y(-z) \end{bmatrix} = \frac{1}{2}\mathbf{G}^{(m)}(z)[\mathbf{H}^{(m)}(z)]^T \begin{bmatrix} X(z) \\ X(-z) \end{bmatrix}$$

we observe that the PR condition is satisfied if $\frac{1}{2}\mathbf{G}^{(m)}(z)[\mathbf{H}^{(m)}(z)]^T = \begin{bmatrix} z^{-\ell} & 0 \\ 0 & (-z)^{-\ell} \end{bmatrix}$

Perfect Reconstruction Two-Channel FIR Filter Banks

- Thus, knowing the analysis filters $H_0(z)$ and $H_1(z)$, the synthesis filters $G_0(z)$ and $G_1(z)$ are determined from

$$\mathbf{G}^{(m)}(z) = 2\begin{bmatrix} z^{-\ell} & 0 \\ 0 & (-z)^{-\ell} \end{bmatrix}\left([\mathbf{H}^{(m)}(z)]^T\right)^{-1}$$

- After some algebra we arrive at

Perfect Reconstruction Two-Channel FIR Filter Banks

$$G_0(z) = \frac{2z^{-\ell}}{\det[\mathbf{H}^{(m)}(z)]} \cdot H_1(-z)$$

$$G_1(z) = -\frac{2z^{-\ell}}{\det[\mathbf{H}^{(m)}(z)]} \cdot H_0(-z)$$

where

$$\det[\mathbf{H}^{(m)}(z)] = H_0(z)H_1(-z) - H_0(-z)H_1(z)$$

and ℓ is an odd positive integer

Perfect Reconstruction Two-Channel FIR Filter Banks

- For FIR analysis filters $H_0(z)$ and $H_1(z)$, the synthesis filters $G_0(z)$ and $G_1(z)$ will also be FIR filters if

$$\det[\mathbf{H}^{(m)}(z)] = cz^{-k}$$

where c is a real number and k is a positive integer

- In this case $G_0(z) = \frac{2}{c} z^{-(\ell-k)} H_1(-z)$

$$G_1(z) = -\frac{2}{c} z^{-(\ell-k)} H_0(-z)$$

Orthogonal Filter Banks

- Let $H_0(z)$ be an FIR filter of odd order N satisfying the power-symmetric condition

$$H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = 1$$

- Choose $H_1(z) = z^{-N}H_0(-z^{-1})$
- Then $\det[\mathbf{H}^{(m)}(z)]$

$$= -z^{-N}\left(H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1})\right) = -z^{-N}$$

Orthogonal Filter Banks

- Comparing the last equation with

$$\det[\mathbf{H}^{(m)}(z)] = cz^{-k}$$

we observe that $c = -1$ and $k = N$

- Using $H_1(z) = z^{-N} H_0(-z^{-1})$ in

$$G_0(z) = \frac{2}{c} z^{-(\ell-k)} H_1(-z)$$

$$G_1(z) = -\frac{2}{c} z^{-(\ell-k)} H_0(-z)$$

with $\ell = k = N$ we get

$$G_0(z) = 2z^{-N} H_0(z^{-1}), \quad G_1(z) = 2z^{-N} H_1(z^{-1})$$

Orthogonal Filter Banks

- Note: If $H_0(z)$ is a causal FIR filter, the other three filters are also causal FIR filters
- Moreover, $|H_1(e^{j\omega})| = |H_0(-e^{j\omega})|$
- Thus, for a real coefficient transfer function if $H_0(z)$ is a lowpass filter, then $H_1(z)$ is a highpass filter
- In addition, $|G_i(e^{j\omega})| = |H_i(e^{j\omega})|, \quad i = 1,2$

Orthogonal Filter Banks

- A perfect reconstruction power-symmetric filter bank is also called an orthogonal filter bank
- The filter design problem reduces to the design of a power-symmetric lowpass filter $H_0(z)$
- To this end, we can design a an even-order $F(z) = H_0(z)H_0(z^{-1})$ whose spectral factorization yields $H_0(z)$

Orthogonal Filter Banks

- Now, the power-symmetric condition

$$H_0(z)H_0(z^{-1}) + H_1(-z)H_1(-z^{-1}) = 1$$

implies that $F(z)$ be a zero-phase half-band lowpass filter with a non-negative frequency response $F(e^{j\omega})$

- Such a half-band filter can be obtained by adding a constant term K to a zero-phase half-band filter $Q(z)$ such that

$$F(e^{j\omega}) = Q(e^{j\omega}) + K \geq 0 \quad \text{for all } \omega$$

Orthogonal Filter Banks

- Summarizing, the steps for the design of a real-coefficient power-symmetric lowpass filter $H_0(z)$ are:
- (1) Design a zero-phase real-coefficient FIR half-band lowpass filter $Q(z)$ of order $2N$ with N an odd positive integer:

$$Q(z) = \sum_{n=-N}^{N} q[n] z^{-n}$$

Orthogonal Filter Banks

- (2) Let δ denote the peak stopband ripple of $Q(e^{j\omega})$
- Define $F(z) = Q(z) + \delta$ which guarantees that $F(e^{j\omega}) \geq 0$ for all ω
- Note: If $q[n]$ denotes the impulse response of $Q(z)$, then the impulse response $f[n]$ of $F(z)$ is given by

$$f[n] = \begin{cases} q[n] + \delta, & \text{for } n = 0 \\ q[n], & \text{for } n \neq 0 \end{cases}$$

- (3) Determine the spectral factor $H_0(z)$ of $F(z)$

Orthogonal Filter Banks

- Example - Consider the FIR filter

$$F(z) = z^{N}(1 + z^{-1})^{N+1}R(z)$$

where $R(z)$ is a polynomial in z^{-1} of degree $N-1$ with N odd

- $F(z)$ can be made a half-band filter by choosing $R(z)$ appropriately
- This class of half-band filters has been called the binomial or maxflat filter

Orthogonal Filter Banks

- The filter $F(z)$ has a frequency response that is maximally flat at $\omega = 0$ and at $\omega = \pi$
- For $N = 3$, $R(z) = \frac{1}{16}(-1 + 4z^{-1} - z^{-2})$ resulting in

$$F(z) = \frac{1}{16}(-z^3 + 9z + 16 + 9z^{-1} - z^{-3})$$

which is seen to be a symmetric polynomial with 4 zeros located at $z = -1$, a zero at $z = 2 - \sqrt{3}$, and a zero at $z = 2 + \sqrt{3}$

Orthogonal Filter Banks

- The minimum-phase spectral factor is therefore the lowpass analysis filter

$$H_0(z) = -0.3415(1 + z^{-1})^2[1 - (2 - \sqrt{3})z^{-1}]$$
$$= -0.3415(1 + 1.732z^{-1} + 0.464z^{-2} - 0.268z^{-3})$$

- The corresponding highpass filter is given by

$$H_1(z) = z^{-3}H_0(-z^{-1})$$
$$= -0.3415(0.2679 + 0.4641z^{-1} - 1.732z^{-2} + z^{-3})$$

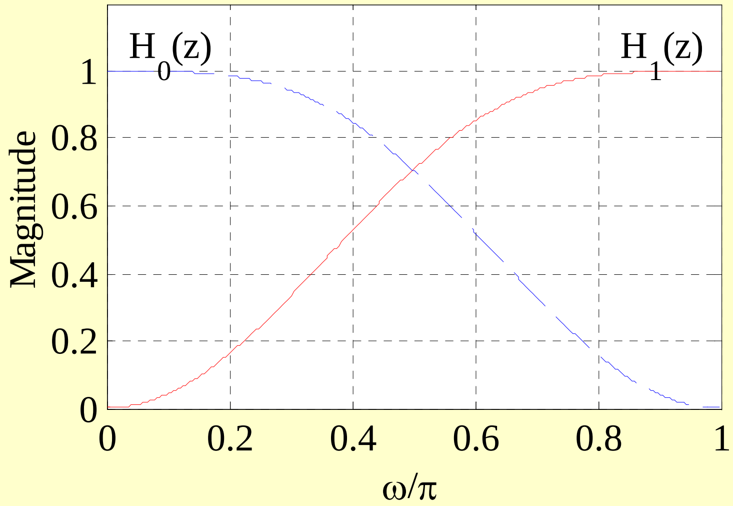
Orthogonal Filter Banks

- The two synthesis filters are given by

$$G_0(z) = 2z^{-3}H_0(z^{-1})$$
$$= -0.683(-0.2679 + 0.4641z^{-1} + 1.732z^{-2} + z^{-3})$$
$$G_1(z) = 2z^{-3}H_1(z^{-1})$$
$$= -0.683(1 - 1.732z^{-1} + 0.4641z^{-2} + 0.2679z^{-3})$$

- Magnitude responses of the two analysis filters are shown on the next slide

Orthogonal Filter Banks



- Comments: (1) The order of $F(z)$ is of the form $4K+2$, where K is a positive integer
- ⇒ Order of $H_0(z)$ is $N = 2K+1$, which is odd as required

Orthogonal Filter Banks

- (2) Zeros of $F(z)$ appear with mirror-image symmetry in the z-plane with the zeros on the unit circle being of even multiplicity
- Any appropriate half of these zeros can be grouped to form the spectral factor $H_0(z)$
- For example, a minimum-phase $H_0(z)$ can be formed by grouping all the zeros inside the unit circle along with half of the zeros on the unit circle

Orthogonal Filter Banks

- Likewise, a maximum-phase $H_0(z)$ can be formed by grouping all the zeros outside the unit circle along with half of the zeros on the unit circle
- However, it is not possible to form a spectral factor with linear phase
- (3) The stopband edge frequency is the same for $F(z)$ and $H_0(z)$

Orthogonal Filter Banks

- (4) If the desired minimum stopband attenuation of $H_0(z)$ is α_s dB, then the minimum stopband attenuation of $F(z)$ is $2\alpha_s + 6.02$ dB
- Example - Design a lowpass real-coefficient power-symmetric filter $H_0(z)$ with the following specifications: $\omega_s = 0.63\pi$, and $\alpha_s = 12$ dB

Orthogonal Filter Banks

- The specifications of the corresponding zero-phase half-band filter $F(z)$ are therefore:
  $\omega_s = 0.63\pi$ and $\alpha_s = 40$ dB
- The desired stopband ripple is thus $\delta_s = 0.01$ which is also the passband ripple
- The passband edge is at $\omega_p = \pi - 0.63\pi = 0.37\pi$
- Using the function `remezord` we first estimate the order of $F(z)$ and then using the function `remez` design $Q(z)$

Orthogonal Filter Banks

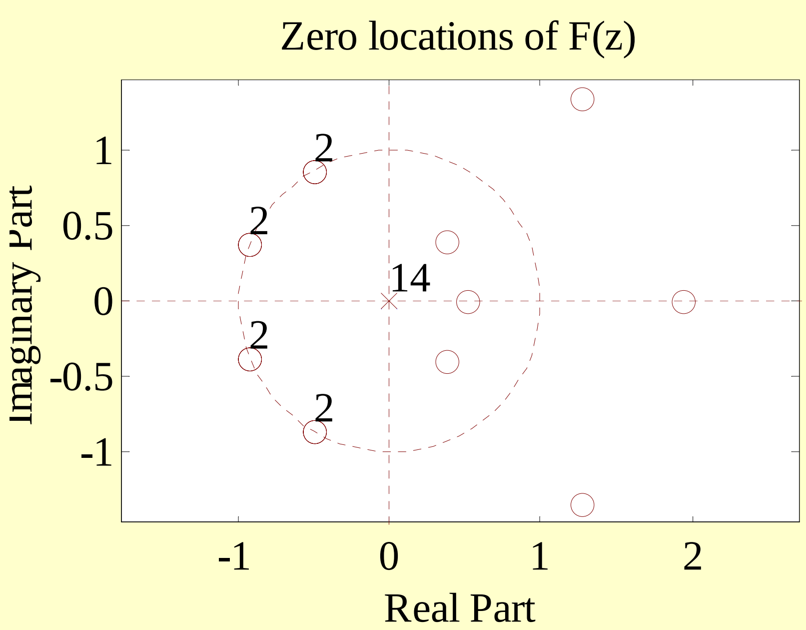
- The order of $F(z)$ is found to be 14 implying that the order of $H_0(z)$ is 7 which is odd as desired
- To determine the coefficients of $F(z)$ we add `err` (the maximum stopband ripple) to the central coefficient $q[7]$
- Next, using the function `roots` we determine the roots of $F(z)$ which should theretically exhibit mirror-image symmetry with double roots on the unit circle

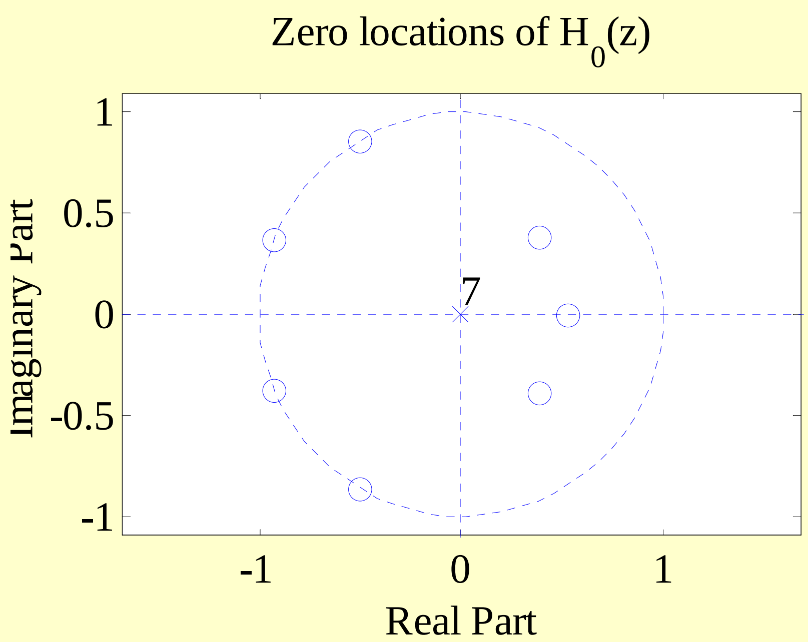
Orthogonal Filter Banks

- However, the algorithm s numerically quite sensitive and it is found that a slightly larger value than `err` should be added to ensure double zeros of $F(z)$ on the unit circle
- Choosing the roots inside the unit circle along with one set of unit circle roots we get the minimum-phase spectral factor $H_0(z)$

Orthogonal Filter Banks

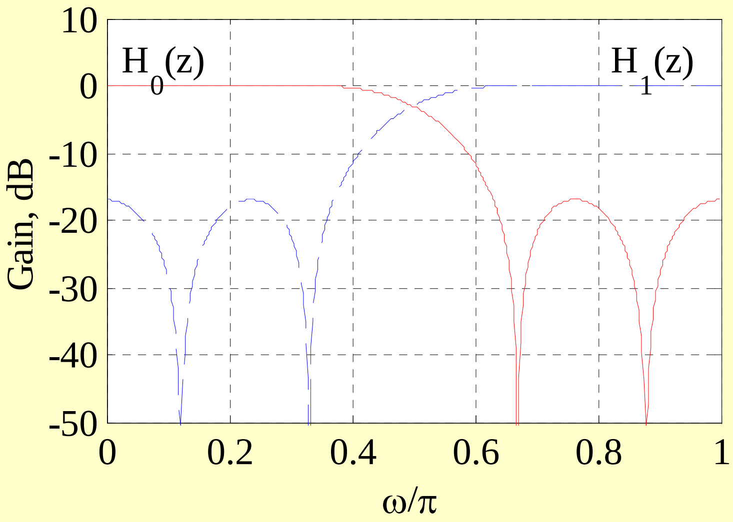
- The zero locations of $F(z)$ and $H_0(z)$ are shown below





Orthogonal Filter Banks

- The gain responses of the two analysis filters are shown below



Orthogonal Filter Banks

- Separate realizations of the two filters $H_0(z)$ and $H_1(z)$ would require $2(N+1)$ multipliers and $2N$ two-input adders
- However, a computationally efficient realization requiring $N+1$ multipliers and $2N$ two-input adders can be developed by exploiting the relation

$$H_1(z) = z^{-N} H_0(-z^{-1})$$

Paraunitary System

- A p-input, q-output LTI discrete-time system with a transfer matrix $\mathbf{T}_{pq}(z)$ is called a paraunitary system if $\mathbf{T}_{pq}(z)$ is a paraunitary matrix, i.e.,

$$\tilde{\mathbf{T}}_{pq}(z)\mathbf{T}_{pq}(z) = c\mathbf{I}_p$$

- Note: $\tilde{\mathbf{T}}_{pq}(z)$ is the paraconjugate of $\mathbf{T}_{pq}(z)$ given by the transpose of $\mathbf{T}_{pq}(z^{-1})$ with each coefficient replaced by its conjugate
- $\mathbf{I}_p$ is an $p \times p$ identity matrix, c is a real constant

Paraunitary Filter Banks

- A causal, stable paraunitary system is also a lossless system
- It can be shown that the modulation matrix

$$\mathbf{H}^{(m)}(z) = \begin{bmatrix} H_0(z) & H_1(z) \\ H_0(-z) & H_1(-z) \end{bmatrix}$$

of a power-symmetric filter bank is a paraunitary matrix

Paraunitary Filter Banks

- Hence, a power-symmetric filter bank has also been referred to as a paraunitary filter bank
- The cascade of two paraunitary systems with transfer matrices $\mathbf{T}_{pq}^{(1)}(z)$ and $\mathbf{T}_{qr}^{(2)}(z)$ is also paraunitary
- The above property can be utilized in designing a paraunitary filter bank without resorting to spectral factorization

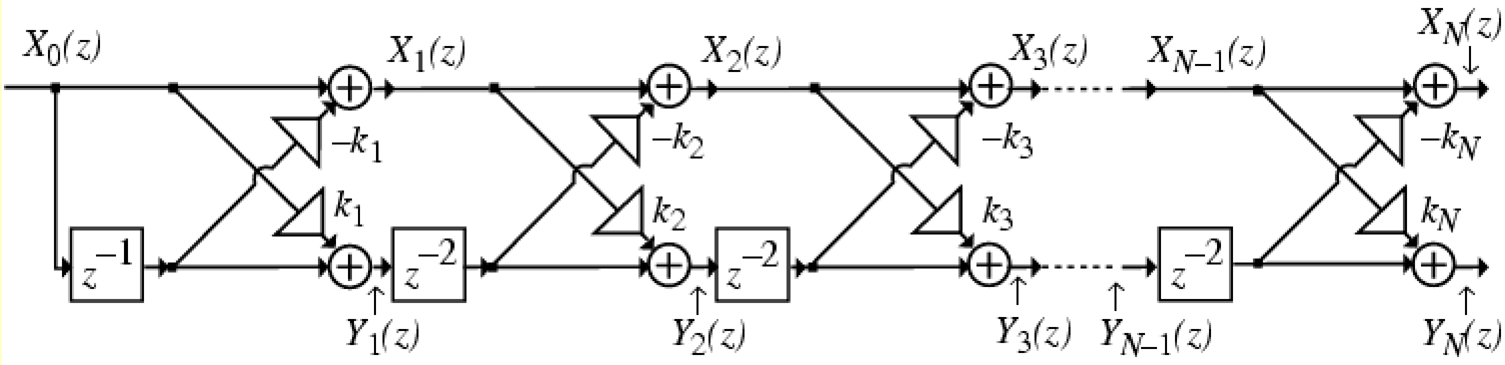
Power-Symmetric FIR Cascaded Lattice Structure

- Consider a real-coefficient FIR transfer function $H_N(z)$ of order N satisfying the power-symmetric condition

$$H_N(z)H_N(z^{-1}) + H_N(-z)H_N(-z^{-1}) = K_N$$

- We shall show now that $H_N(z)$ can be realized in the form of a cascaded lattice structure as shown on the next slide

Power-Symmetric FIR Cascaded Lattice Structure



- Define

$$H_i(z) = \frac{X_i(z)}{X_0(z)}, \quad G_i(z) = \frac{Y_i(z)}{X_0(z)}$$

$$1 \le i \le N$$

Power-Symmetric FIR Cascaded Lattice Structure

- From the figure we observe that

$$X_1(z) = X_0(z) + k_1 z^{-1} X_0(z)$$

$$Y_1(z) = -k_1 X_0(z) + z^{-1} X_0(z)$$

- Therefore,

$$H_1(z) = 1 + k_1 z^{-1}, \quad G_1(z) = -k_1 + z^{-1}$$

- It can be easily shown that

$$G_1(z) = z^{-1} H_1(-z^{-1})$$

Power-Symmetric FIR Cascaded Lattice Structure

- Next from the figure it follows that

$$H_i(z) = H_{i-2}(z) + k_i z^{-2} G_{i-2}(z)$$

$$G_i(z) = -k_i H_{i-2}(z) + z^{-2} G_{i-2}(z)$$

- It can easily be shown that

$$G_i(z) = z^{-i} H_i(-z^{-1})$$

provided

$$G_{i-2}(z) = z^{-(i-2)} H_{i-2}(-z^{-1})$$

Power-Symmetric FIR Cascaded Lattice Structure

- We have shown that $G_i(z) = z^{-i} H_i(-z^{-1})$ holds for $i = 1$
- Hence the above relation holds for all odd integer values of i
- $\Longrightarrow$ N must be an odd integer
- It is a simple exercise to show that both $H_i(z)$ and $G_i(z)$ satisfy the power-symmetry condition $H_i(z)H_i(z^{-1}) + H_i(-z)H_i(-z^{-1}) = K_i$

Power-Symmetric FIR Cascaded Lattice Structure

- In addition, $H_i(z)$ and $G_i(z)$ are power-complementary, i.e.,

$$(1+k_i^2)z^{-2}G_{i-2}(z) = k_i H_i(z) + G_i(z)$$

- To develop the synthesis equation we express $H_{i-2}(z)$ and $G_{i-2}(z)$ in terms of $H_i(z)$ and $G_i(z)$:

$$(1+k_i^2)H_{i-2}(z) = H_i(z) - k_i G_i(z)$$
$$(1+k_i^2)z^{-2}G_{i-2}(z) = k_i H_i(z) + G_i(z)$$

Power-Symmetric FIR Cascaded Lattice Structure

- Note: At the i-th step, the coefficient k_i is chosen to eliminate the coefficient of z^{-i}, the highest power of z^{-1} in $H_i(z) - k_i G_i(z)$
- For this choice of k_i the coefficient of also vanishes making $H_{i-2}(z)$ a polynomial of degree $i-2$
- The synthesis process begins with $i = N$ and compute $G_N(z)$ using $G_N(z) = z^{-N} H_N(-z^{-1})$

Power-Symmetric FIR Cascaded Lattice Structure

- Next, the transfer functions $H_{N-2}(z)$ and $G_{N-2}(z)$ are determined using the synthesis equations

$$(1+k_i^2)H_{i-2}(z) = H_i(z) - k_i G_i(z)$$
$$(1+k_i^2)z^{-2}G_{i-2}(z) = k_i H_i(z) + G_i(z)$$

- This process is repeated until all coefficients of the lattice have been determined

Power-Symmetric FIR Cascaded Lattice Structure

- Example - Consider

$$H_5(z) = 1 + 0.3z^{-1} + 0.2z^{-2} - 0.376z^{-3} - 0.06z^{-4} + 0.2z^{-5}$$

- It can be easily verified that $H_5(z)$ satisfies the power-symmetric condition
- Next we form

$$G_5(z) = z^{-5}H_5(-z^{-1}) = -0.2 - 0.06z^{-1} + 0.376z^{-2} + 0.2z^{-3} - 0.3z^{-4} + z^{-5}$$

Power-Symmetric FIR Cascaded Lattice Structure

- To determine $H_5(z)$ we first form

$$H_5(z) - k_5 G_5(z) = (1 + 0.2k_5) + (0.3 + 0.06k_5)z^{-1}$$

$$+ (0.2 - 0.376k_5)z^{-2} + (-0.376 - 0.2k_5)z^{-3}$$

$$+ (-0.06 + 0.3k_5)z^{-4} + (0.2 - k_5)z^{-5}$$

- To cancel the coefficient of z^{-5} in the above we choose

$$k_5 = 0.2$$

Power-Symmetric FIR Cascaded Lattice Structure

- Then $H_3(z) = \frac{1}{1-k_5^2}\left[H_5(z) - k_5 G_5(z)\right]$

$$= \frac{1}{1.04}(1.04 + 0.312z^{-1} + 0.1248z^{-2} - 0.416z^{-3})$$

- We next form

$$G_3(z) = z^{-3}H_3(-z^{-1}) = 0.4 + 0.12z^{-1} - 0.3z^{-2} + z^{-3}$$

- Continuing the above process we get

$$k_3 = -0.4, \quad k_1 = 0.3$$

Power-Symmetric FIR Banks

- Using the method outlined for the realization of a power-symmetric transfer function, we can develop a cascaded lattice realization of the 2-channel paraunitary QMF bank
- Three important properties of the QMF lattice structure are structurally induced

Power-Symmetric FIR Banks

- (1) The QMF lattice guarantees perfect reconstruction independent of the lattice parameters
- (2) It exhibits very small coefficient sensitivity to lattice parameters as each stage remains lossless under coefficient quantization
- (3) Computational complexity is about one-half that of any other realization as it requires $(N+1)/2$ total number of multipliers for an order-N filter

Power-Symmetric FIR Banks

- Example - Consider the analysis filter of the previous example:

$$H_7(z) = 0.3231 + 0.51935z^{-1} + 0.30134z^{-2} - 0.0781z^{-3} - 0.13767z^{-4} + 0.321z^{-5} + 0.079z^{-6} - 0.049z^{-7}$$

- We place a multiplier $h[0] = 0.3231$ at the input and synthesize a cascade lattice structure for the normalized transfer function $H_7(z)/h[0]$

Power-Symmetric FIR Banks

- The lattice coefficients obtained for the normalized analysis transfer function are:

$$k_7 = -0.15165, \quad k_5 = 0.2354,$$
$$k_3 = -0.48393, \quad k_1 = 1.61$$

- Note: Because of the numerical problem, the coefficients of the spectral factor obtained in the previous example are not very accurate

Power-Symmetric FIR Banks

- As a result, the coefficients of $z^{-(i-1)}$ of the transfer function $H_{i-2}(z)$ generated from the transfer function $H_i(z)$ using the relation
$$H_{i-2}(z) = \frac{1}{1+k_i^2}[H_i(z) - k_i G_i(z)]$$
is not exactly zero, and has been set to zero at each iteration

Power-Symmetric FIR Banks

- Two interesting properties of the cascaded lattice QMF bank can be seen by examining its multiplier coefficient values
- (1) Signs of coefficients alternate between stages
- (2) The values of the coefficients $\{k_i\}$ decrease with increasing i

Power-Symmetric FIR Banks

- The QMF lattice structure can be used directly to design the power-symmetric analysis filter $H_0(z)$ using an iterative computer-aided optimization technique
- Goal: Determine the lattice parameters k_i by minimizing the energy in the stopband of $H_0(z)$

Power-Symmetric FIR Banks

- The pertinent objective function is given by

$$\phi = \int_{\omega_s}^{\pi} \left| H_0(e^{j\omega}) \right|^2 d\omega$$

- Note: The power-symmetric property ensures good passband response

Biorthogonal FIR Banks

- In the design of an orthogonal 2-channel filter bank, the analysis filter $H_0(z)$ is chosen as a spectral factor of the zero-phase even-order half-band filter

$$F(z) = H_0(z)H_0(z^{-1})$$

- Note: The two spectral factors $H_0(z)$ and $H_0(z^{-1})$ of $F(z)$ have the same magnitude response

Biorthogonal FIR Banks

- As a result, it is not possible to design perfect reconstruction filter banks with linear-phase analysis and synthesis filters
- However, it is possible to maintain the perfect reconstruction condition with linear-phase filters by choosing a different factorization scheme

Biorthogonal FIR Banks

- To this end, the causal half-band filter $z^{-N}F(z)$ of order $2N$ is factorized in the form

$$z^{-N}F(z) = H_0(z)H_1(-z)$$

where $H_0(z)$ and $H_1(z)$ are linear-phase filters

- The determinant of the modulation matrix $\mathbf{H}^{(m)}(z)$ is now given by

$$\det[\mathbf{H}^{(m)}(z)] = H_0(z)H_1(-z) - H_0(-z)H_1(z) = z^{-N}$$

Biorthogonal FIR Banks

- Note: The determinant of the modulation matrix satisfies the perfect reconstruction condition
- The filter bank designed using the factorization scheme $z^{-N}F(z) = H_0(z)H_1(-z)$ is called a biorthogonal filter bank
- The two synthesis filters are given by

$$G_0(z) = H_1(-z), \quad G_1(z) = -H_0(-z)$$

Biorthogonal FIR Banks

- Example - The half-band filter

$$F(z) = \frac{1}{16} z^3 (1 + z^{-1})^4 (-1 + 4z^{-1} - z^{-2})$$

- can be factored several different ways to yield linear-phase analysis filters $H_0(z)$ and $H_1(z)$

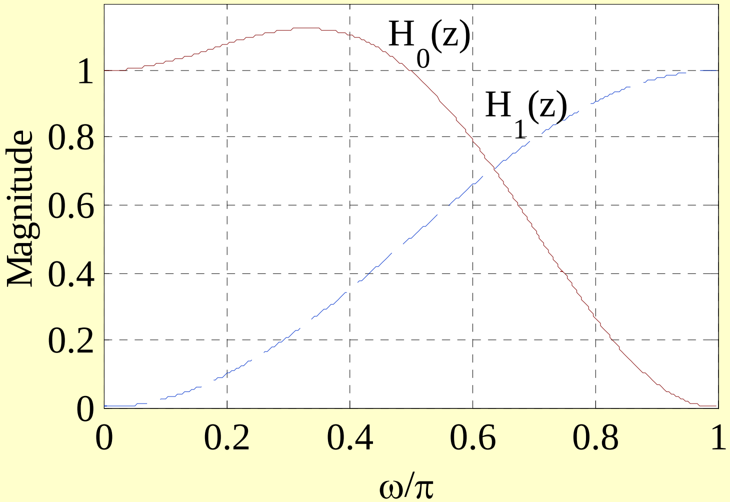
- For example, one choice yields

$$H_0(z) = \frac{1}{8}(-1 + 2z^{-1} + 6z^{-2} + 2z^{-3} - z^{-4})$$

$$H_1(z) = \frac{1}{2}(1 - 2z^{-1} + z^{-2})$$

Biorthogonal FIR Banks

- Since the length of $H_0(z)$ is 5 and the length of $H_1(z)$ is 3, the above set of analysis filters is known as the 5/3 filter-pair of Daubechies
- A plot of the gain responses of the 5/3 filter-pair is shown below



Biorthogonal FIR Banks

- Another choice yields the 4/4 filter-pair of Daubechies

$$H_0(z) = \frac{1}{8}(1 + 3z^{-1} + 3z^{-2} + z^{-3})$$

$$H_1(z) = \frac{1}{2}(-1 - 3z^{-1} + 3z^{-2} + z^{-3})$$

- A plot of the gain responses of the 4/4 filter-pair is shown below