thus far in our study of communication systems, we have been primarily concerned with the performance of communication systems in terms of BW efficiency and energy efficiency (i.e., detected $S/N$ or $P_E$) wrt natural noise. However, in some applications, we are also concerned with:

- antijam capability,
- interference rejection,
- multiple-access capability,
- covert operation or low prob of intercept (LPI) capability.

These performance objectives can be optimized by using spread-spectrum (SS) techniques.

There are many types of SS systems. To be considered an SS system, two criteria need to be satisfied:

1. the BW of the transmitted signal, $S(t)$, needs to be much greater than that of the message, $m(t)$. 

the relatively wide BW of $s(t)$ must be caused by an indep modulating waveform, $c(t)$, called the spreading signal, and this signal must be known by the receiver in order for the message signal, $m(t)$, to be detected.

There are many types of SS systems. Some of the most common types are

1. Direct Sequence (DS),
2. Freq hopping (FH),
3. Hybrid techniques that include both DS and FH.
Direct - Sequence spread spectrum (DS-55).

One method of spreading the spectrum of a data-modulated signal is to modulate the signal a second time using a very wideband spreading signal. The spreading signal is chosen to have properties which facilitates demodulation of the transmitted signal by the intended receiver, and which make demodulation difficult as possible by an unintended receiver. If the BW of the spreading signal is large relative to the data BW, the 55 transmission BW is dominated by the spreading signal and is nearly independent of the data signal.

BPSK Direct - Sequence Spread Spectrum (BPSK-DS-55):

Consider

\[ S(t) = \sqrt{2P} \cos \left[ \omega_d t + \Theta_s(t) \right] \]

\[ S_s(t) = \sqrt{2P} C(t) \cos \left[ \omega_d t + \Theta_s(t) \right] \]

\[ C(t) \in \{ +1, -1 \} \]

Note that the 'data modulation' and 'DS spread' are not necessarily the same type of modulation.
Note that

\[ S(t) = \sqrt{2P} \cos (\omega_0 t + \theta_d(t)) \]

\[ t \]

\[ S(t) = \sqrt{2P} C(t) \cos [\omega_0 t + \theta_d(t)] \]

The BPSK-DS-SS receiver can be implemented as follows:

DS despread

\[ r(t) = \sqrt{2P} c(t-T_d) \cos [\omega_0 t + \theta_d(t-T_d) + \phi] \]

\[ + \text{Interference} \]

\[ x = \sqrt{2P} c(t-T_d) c(t-T_d) \cos [\omega_0 t + \theta_d(t-T_d) + \phi] \]

since \( c(t) = 1 \Rightarrow c(t-T_d) c(t-T_d) = 1 \) if \( T_d = T_d \)

\[ x = \sqrt{2P} c(t-T_d) \cos [\omega_0 t + \theta_d(t-T_d) + \phi] \]
Remark

The data modulation above does not have to be BPSK; no restrictions have been placed on the form of \( \Theta(t) \). However, it is common to use the same type of digital phase modulation for the data and the spreading code. When BPSK is used for both modulators, one mixer can be eliminated. The double-modulation process is replaced by a single modulation as shown:

\[
\begin{align*}
d(t) & \rightarrow \times \rightarrow \times \rightarrow \text{modulation} \rightarrow \\
\sqrt{2p} \cos \omega t & \text{binary sequence} \\
\end{align*}
\]

\[
\begin{align*}
d(t) & \rightarrow + \rightarrow \times \rightarrow \\
\sqrt{2p} \cos \omega t & \\
\end{align*}
\]

If \( d(t) = C(t) \rightarrow 1 \)

\( d(t) \neq C(t) \rightarrow -1 \)

The following figure illustrates the DS spreading and despreading operation, where the data modulation and the spreading modulation are BPSK.
BPSK direct-sequence spreading and despreading.

Fig 8-5
Power Considerations: (See Previous Page)

Recall that for BPSK signal $S(t)$, the power spectrum is:

$$S_d(f) = \frac{1}{2} PT \left\{ \text{sinc}^2 \left[ (f - f_0) T \right] + \text{sinc}^2 \left[ (f + f_0) T \right] \right\}$$

Note that $S_t(t) = \sqrt{2P} C(t) C_0 \left[ \omega_0 t + \Theta_0(t) \right]$ is also a BPSK carrier and therefore has a PSD similar to above with $T \rightarrow T_c$. 

$VTc \ll T$ \quad \frac{PTE}{\frac{1}{Tc}} \gg \frac{1}{\frac{1}{T}}$

$\left\uparrow \right\downarrow \frac{PTE}{\frac{1}{Tc}} \gg \frac{1}{\frac{1}{T}}$

$\left\uparrow \right\downarrow \frac{PTE}{\frac{1}{Tc}} \gg \frac{1}{\frac{1}{T}}$

$T_c$ is often referred to as a spreading code chip.
Remark 1

The effect of the modulation by the spreading code \( c(t) \) is to spread the BW. For example, if

\[
T_c = \frac{T}{k}, \quad \text{then} \quad BW_{s(t)} = k \cdot BW_{s_d(t)}
\]

and the height of PSD of \( s(t) \) is \( \frac{1}{k} \) times the height of PSD of \( s_d(t) \).

Remark 2

\( s_d(t) \) is an ergodic random process and the spreading code, \( c(t) \), is both deterministic and periodic.

\[
\therefore s(t) = c(t) \cdot s_d(t) \quad \text{is ergodic.}
\]

Also, \( c(t) \) and \( s_d(t) \) are independent.

\[
\therefore R_s(t) = R_c(t) \cdot R_d(t) = 0 \quad S_s(f) = S_c(f) \cdot S_d(f)
\]
the effect of Jamming

Suppose that BPSK is used for both data and spreading modulation and that the interference is a single tone having power T. The Jammer's best strategy is to place the Jamming tone directly in the center of the modem's Transmission BW.

Recall that

\[ s(t) = \sqrt{2P} c(t-T_d) \cos[w_0 t + \phi(t-T_d) + \phi] \]

\[ r(t) = s(t) + \text{Jamming (interference)} \]

\[ S_r(f) = \frac{1}{2} P T_c \left\{ \sin^2 [(f-f_0) T_c] + \sin^2 [(f+f_0) T_c] \right\} \]

\[ + \frac{1}{2} J \left\{ \delta(f-f_0) + \delta(f+f_0) \right\} \]

If the receiver despreading code is correctly phased, then the output of the despreading mixer is

\[ r(t) c(t-T_d), \quad \hat{T}_d = \hat{T}_d \]
\[ y(t) = \sqrt{2P} \ d(t-T_d) \ \cos\ (\omega_0 t + \phi) + \sqrt{2J} \ c(t-T_d) \ \cos\ (\omega_0 t + \phi') \]

and

\[ S_y(f) = \frac{1}{2} PT \left\{ \sin^2 [(f-f_0) T] + \sin^2 [(f+f_0) T] \right\} \]

\[ + \frac{1}{2} JT_c \left\{ \sin^2 [(f-f_0) T_c] + \sin^2 [(f+f_0) T_c] \right\} \]

Observe that the data signal has been spread to the data BW, while the single-tone jammer has been spread over the full transmission BW of the SS - system. This is illustrated in the following figure.
that the output

\[ (7-18) \]

width, while the

\[ (7-19) \]

spectra after the

desubstitution in spread-

width at

The power

Figure 7-10c and

represents the

IF filter whose

power is passed

the other hand,

passed by the IF

\[ (7-20) \]

spreading mixer. If

then

\[ (7-21) \]

is, \( T_c \ll T \), the

and

\[ (7-22) \]

has been reduced by

The processing

the inverse of this

\[ (7-23) \]

\underline{FIGURE 8.10. Receiver power spectral densities with tone jamming.}

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7.3 DIRECT-SEQUENCE SPREAD SPECTRUM
Note that nearly all of signal power is passed by the IF filter. A large fraction of the spread jammer power, on the other hand, is rejected by this filter. The magnitude of the jammer power passed by the IF filter is

$$J_0 = \int_{-f_o}^{+f_o} |S_J(f)|^2 |H(f)|^2 \, df$$

where

$$J_0 = \int_{-f_o - \frac{1}{2T}}^{-f_o + \frac{1}{2T}} S_J(f) \, df + \int_{f_o - \frac{1}{2T}}^{f_o + \frac{1}{2T}} S_J(f) \, df$$

$$= \frac{1}{2} JT_c \int_{-f_o - \frac{1}{2T}}^{-f_o + \frac{1}{2T}} \text{pinc}^2 \left( (f + f_o) T_c \right) \, df + \frac{1}{2} JT_c \int_{f_o - \frac{1}{2T}}^{f_o + \frac{1}{2T}} \text{pinc}^2 \left( (f - f_o) T_c \right) \, df$$

For $T_c \ll T \Rightarrow \text{pinc}^2 \left( (f \pm f_o) T_c \right) \approx \text{constant over the range of}$

$$J_0 \approx J \frac{T_c}{T} \left( \frac{1}{2} \right)^{\text{small} \#}$$

$$G_p \triangleq \frac{J}{J_0} = \frac{T}{T_c} = \text{processing gain}.$$
QPSK Direct Sequence Spread Spectrum:

Recall that the main reason for using quadrature multiplexing is the conservation of the BW. In SS systems, however, BW is not usually of primary importance, but quadrature modulation is still important. The reason is that the quadrature modulations are:

1. Less sensitive to some types of jamming,
2. Difficult to detect in low prob of detection applications.

QPSK - DS-SS modulator can be structured as shown:

![Diagram of QPSK - DS-SS modulator]
\[ S(t) = \sqrt{P} C_1(t) \cos \left[ \omega_0 t + \theta_1(t) \right] + \sqrt{P} C_2(t) \sin \left[ \omega_0 t + \theta_2(t) \right]. \]

\[ = a(t) + b(t). \]

The assumption here is that \( C_1(t) \) and \( C_2(t) \) are chip synchronous and independent of one another. The independence condition implies that the power spectrum of \( S(t) \) is the sum of that of \( a(t) \) and \( b(t) \).

The receiver for this case can be implemented as shown:

\[ S(t - T_d) \]

\[ \text{Power divider} \]

\[ X \]

\[ \times 2 \cos \left[ (\omega_0 + \omega_{IF}) t + \phi \right]. \]

\[ X \]

\[ \times (t) \]

\[ + \]

\[ \Sigma \]

\[ Z(t) \]

\[ \text{BPF} \]

\[ \hat{d}(t) \]

\[ \text{Phase demodulator} \]

\[ X \]

\[ X \]

\[ \times \]

\[ \times 2 \sin \left[ (\omega_0 + \omega_{IF}) t + \phi \right]. \]

\[ C_1(t - T_d) \]

\[ C_2(t - T_d) \]
\[ x(t) = \frac{S(t-T_d)}{P} C_1(t-T_d) e^{j \phi} \left[ (w_0 + w_{IF}) t + \theta(t) \right] \]

\[ = \sqrt{\frac{P}{2}} \left( C_1(t-T_d) e^{j \phi} \left[ (w_0 + w_{IF}) t + \theta(t) \right] \right) \left( C_2(t-T_d) e^{j \phi} \left[ (w_0 + w_{IF}) t + \theta(t) \right] \right) \]

\[ = \sqrt{\frac{P}{2}} \left[ \cos \left[ (w_0 + w_{IF}) t + \theta(t) \right] \right] \left[ \cos \left[ (w_0 + w_{IF}) t + \theta(t) \right] \right] \]

\[ = \sqrt{\frac{P}{2}} \left[ C_0 \left[ \frac{w_0}{w_{IF}} t - \theta_d(t) + \phi \right] + C_0 \left[ \frac{w_0}{w_{IF}} t - \theta_d(t) + \phi \right] \right] \]

\[ y(t) = \sqrt{\frac{P}{2}} C_0 \left[ \frac{w_0}{w_{IF}} t - \theta_d(t) + \phi \right] \]

\[ z(t) = BPF \left\{ x(t) + y(t) \right\} = 2 \sqrt{\frac{P}{2}} C_0 \left( \frac{w_0}{w_{IF}} t - \theta_d(t) \right) \]

\[ = \sqrt{2P} C_0 \left[ \frac{w_0}{w_{IF}} t - \theta_d(t) \right] \]

Note that the spreading terms are completely removed and \( z(t) \) after demodulation renders \( \hat{d}(t) \).
Frequency-Hop Spread spectrum (FH-SS)

A second method for widening the spectrum of a data-modulated carrier is to change the frequency of the carrier periodically. The frequency hopping is accomplished by using a mixer circuit where the LO (local oscillator) signal is provided by the output of a frequency synthesizer that is hopped by the spreading code. The serial-to-parallel converter feeds $k$ serial chips of the spreading code and outputs a $k$ chip parallel word to the programmable dividers in the frequency synthesizer. The $k$ chips and specifies one of the possible $M = 2^k$ hop freqs.

![Diagram showing the FH spreader](image-url)

Note that we have $k$ chips in parallel. Each chip can be $+1$ or $-1$, thus there are $2^k$ different words and therefore $2^k$ different freqs can be put in the synthesizer.
Although in most cases FH is done non-coherently, a fully coherent FH system is theoretically possible. Consider, for example, the FH system shown in the previous page:

$$h(t) = \sum_{n=-\infty}^{\infty} 2P(t-nT_c) \cos(\omega_n t + \phi_n)$$

L.O. Signal

where $P(t)$ is a unit amp pulse of duration $T_c$.

Notice that $h(t)$ is a pure random process. It is simply a cosine function with random frequency controlled by $C(t)$. 
\[
S_t(t) = S_d(t) \frac{h(t)}{T} / \text{HPF (sum freq comp)}
\]

\[
= S_d(t) \sum_{n=-\infty}^{\infty} P(t-nT_c) \cos(\omega_n t + \phi_n)
\]

\[
\sqrt{2P} \cos[\omega_n t + \phi_d(t)]
\]

Notice that \( S_d(t) \) and \( h(t) \) are independent.

\[
S_t(f) = S_d(f) \ast S_h(f) / \text{sum freq comp.}
\]

For the coherent FH, the same \( P_m \) is used each time \( h_T(t) \) returns to \( w_m \). With these assumptions, \( S_h(f) \) is given by (p. 350):

\[
S_h(f) = \frac{1}{T_c^2} \sum_{n=-\infty}^{\infty} \left| \sum_{m=1}^{K} P_m G_m(n/T_c) \right|^2 \delta(f - \frac{m}{T_c})
\]

\[
+ \frac{1}{T_c} \sum_{m=1}^{K} P_m (1 - P_m) |G_m(f)|^2
\]

\[
- \frac{2}{T_c} \sum_{m=1}^{K} \sum_{m' \neq m} P_m P_{m'} \text{Re} \left\{ G_m(f) G_{m'}^*(f) \right\}
\]
Where $P_m$ is the part that $w_m$ is selected, and

$$G_m(f) = \mathcal{F} \left\{ g(t) \right\}, \text{ where}$$

$$g(t) = \begin{cases} 2P(t) \cos(w_m t + \Phi_m) & 0 \leq t \leq T_c \\ 0 & \text{elsewhere} \end{cases}$$

Note that this PSD has discrete components due to the assumption that the same phase is used each time $h_T(t)$ returns to $w_m$. Also

$$G_m(f) = T_c \exp \left\{ -j \left[ \pi (f - f_m) T_c - \Phi_m \right] \right\} \sin \left[ (f - f_m) T_c \right]$$

$$+ T_c \exp \left\{ -j \left[ \pi (f + f_m) T_c + \Phi_m \right] \right\} \sin \left[ (f + f_m) T_c \right].$$

$S_n(f)$ can be simplified. See Text Page 350 for details!
Returning now to the original plan, the received signal \( s(t-T_d) + n(t) \) can be processed as shown:

\[
\begin{align*}
S(t-T_d) &= \sqrt{2P} \sum_{n=-\infty}^{\infty} P(t-T_d-nT_c) \cos \left[ (w_0 + w_n) t + \phi_n + \theta(t-T_d) \right] \\
&\quad - (w_0 + w_n) T_d \\
\end{align*}
\]

\[
h_R(t) = \sum_{n=-\infty}^{\infty} 2P(t-T_d-nT_c) \cos \left[ w_n (t-T_d) + \phi_n \right]
\]

\[
y(t) = S(t-T_d) \frac{h_R(t)}{\text{BPF about diff freq.}} \]
\[
\begin{align*}
&= \sqrt{2p} \sum_{n=-\infty}^{\infty} P(t-T_d-nT_c) C_n \left[ (\omega_n + w_n) t + \phi_n + \Theta(t-T_d) - (\omega_n + w_n) \hat{T_d} \right] \\
&= \sum_{n=-\infty}^{\infty} P(t-T_d-nT_c) C_n \left[ w_n(t-T_d) + \phi_n \right] / \text{BPF diff comp.} \\
&= \sqrt{2p} \sum_{n} \sum_{m} P(t-T_d-nT_c) P(t-T_d-mT_c) \frac{1}{T_c} \left[ C_n (\alpha + \beta) \right. \\
&\quad \left. + C_n (\alpha - \beta) \right] / \text{BPF diff comp.} \\
&= \sqrt{2p} \sum_{n} \sum_{m} P(t-T_d-nT_c) P(t-T_d-mT_c) C_n (\alpha - \beta) \\
&\quad \text{has value if } n=m \text{ and zero elsewhere! } (T_d = \hat{T_d}), \quad P^2 = P. \\
&= \sqrt{2p} \sum_{n} P(t-T_d-nT_c) C_n \left[ w_n t - \omega_n T_d + \Theta(t-T_d) \right] \\
&= \sqrt{2p} C_n \left[ w_n t - \omega_n T_d + \Theta(t-T_d) \right] \sum_{m} P(t-T_d-mT_c)
\end{align*}
\]
Non Coherent Slow-Freq-Hop Spread Spectrum.

Because of the difficulty of implementing truly (in the Trans and Receiver) coherent freq synthesizers as well as the code tracking requirements, most Freq-Hop SS systems use either noncoherent or differentially coherent data modulation schemes.

The block diagram of the modem (Trans.) is the same as the coherent one (shown below) but in the receiver, no effort is made to precisely recover the phase of the "data modulated carrier".

A common data modulation for FH systems is M-ary FSK (MFSK).
Binary data → M-ary FSK modulator → Mixer → BPF → FH/MFSK signal

Freq synthesizer

PN code generator

Received signal → Mixer → BPF → M-ary FSK detector → Estimate of binary data
Suppose that the data modulator outputs one of \( L \) tones each \( LT \) seconds, where \( T \) is the duration of one information bit.

\[
M = 2^L
\]

One symbol is \( L \) bits, and takes \( LT \) seconds.

\[
\begin{array}{c}
\text{Possible ways} \\
\text{True bit!} \\
\end{array}
\]

If each bit takes \( T \) seconds, then each symbol takes \( LT \) seconds. Note that the MFSK sends a symbol for each \( M \) and usually these signals are orthogonal. A symbol spacing of at least \( \frac{1}{LT} \) guarantees orthogonality. Thus, the data modulated carrier has the spectral width, approximately,

\[
\gtrsim M \left( \frac{1}{LT} \right) = 2^{L/LT}.
\]
Each $T_c$ seconds, the data modulator output is translated to a new freq by the freq-hop modulator.

When

$$T_s = \text{symbol time}$$

$$T_c \geq L T_a = 0 \quad \text{slow-FH-ss system}.$$ 

Recall that the FH modulator consists of

$$2^k \text{ freqs}$$

Thus, each of data mod and freq is randomly shifted to each of $2^k$ FH output. For the sake of illustration,

Let $L = 2$ and $k = 3$. Thus, each of $2^k = 4$ data modulated symbol is shifted to one of $2^3 = 8$ FH freqs. Note that, this is the slow-FH-ss and thus the action should be for

$$T_c \geq L T_a = 2T_a \text{ sec}.$$
That \( T_c = 4T = 2T_s \)

Every \( 2T_s \) sec, any of the 4 data mod
signal \( f(t) \) is shifted into one of 8

8-hop bands by the FH mod.

**FIGURE 7-19.** Pictorial representation of (a) transmitted signal for an M-ary FSK slow-frequency hop spread-spectrum system; (b) receiver downconverter output.
In the receiver, the transmitted signal is down-converted using a local oscillator and then demodulated using the conventional methods for noncoherent MFSK.

Note that in the absence of the FH-ss, the jammer chooses a BW $W_d$ centered on the carrier freq $f_0$ and forces the receiver operating $S/N$ ratio to

$$\frac{E_b}{N_J} = \frac{E_b}{J/W_d} = \frac{E_b W_d}{J}$$

This is the same as $N_0$ which is the PSD $\frac{Watts}{Hz}$.

When FH is added, the jammer must place noise in all $2^k$ freqs to cause the receiver to have the same performance as before.
\[
\frac{E_b}{N_J} = \frac{E_b}{J/W_s} = \frac{E_b}{J/2 K W_d} = 2^K \frac{E_b W_d}{J}
\]

Thus, the jammer requires a total power $2^K$ times as large as before and the processing gain is:

\[
P.G = 2^K = \frac{W_s}{W_d}
\]
A fast FH/MFSK system differs from a slow FH/MFSK system in that there are multiple hops per M-ary symbol. In this case

\[ T_c \leq \frac{T_s}{L} \]

Now, let \( K = 2 \), \( L = 2 \), and \( T_c = T = \frac{T_s}{2} \).

\[ W_d = \frac{2^L}{T_c} = K \frac{2^L}{LT} \]

\[ W_i = 2^k W_d \]

**FIGURE 7-20.** Pictorial representation of (a) transmitted signal for an M-ary FSK fast frequency hop spread-spectrum system; (b) receiver downconverter output.
One reason for using hybrid techniques is that some of the advantages of both types of systems are combined in a single system. These systems are widely used in military and are currently the only practical way of achieving extremely wide spectrum spreading.

FIGURE 7.21. Hybrid direct-sequence/frequency-hop spread-spectrum modem.