composite hypotheses:

So far, we assumed that the signal was known and it was the addition of noise which caused uncertainty as to which hypothesis to choose. There are many cases, however, when the signal is not known precisely. For example, phase of a sin wave or freq.... For these cases we could choose to generate a hypothesis for each possible value of the unknown parameter.

Example 1.

\[ H_0 : \ r \sim (0, \sigma^2) \]
\[ H_1 : \ r \sim (m, \sigma^2) \]

\[ \uparrow \in [M_0, M_1] \]

\( m \) is a rv and \( H_1 \) is a composite hypothesis.
In general we may have

\[ H_0 : \theta_0 \text{ unknown or a r.v} \]

\[ H_1 : \theta_1 \text{ } \]
Case 1: \( \theta \) is a r.v. with known pdf:

We assume that \( P(\theta | H_0), P(\theta | H_1) \)
are known. Then

\[
\Lambda(R) = \frac{P_{R|H_1}(R | H_1)}{P_{R|H_0}(R | H_0)} = \frac{\int_{\theta} P(R, \theta | H_1) d\theta}{\int_{\theta} P(R, \theta | H_0) d\theta}
\]

\[
= \frac{\int_{\theta} \frac{P_{\theta,H_1}(R, \theta | H_1)}{P(H_1)} d\theta}{\int_{\theta} \frac{P_{\theta,H_0}(R, \theta | H_0)}{P(H_0)} d\theta} = \frac{\int_{\theta} \frac{P(R|\theta, H_1) P(\theta | H_1)}{P(H_1)} d\theta}{\int_{\theta} \frac{P(R|\theta, H_0) P(\theta | H_0)}{P(H_0)} d\theta}
\]

\[
\geq n^{(*)}
\]

\[
\int_{R_{10}} P(R|\theta, H_1) P(\theta | H_1) d\theta \geq n^{(*)}
\]

\[
\int_{R_{10}} P(R|\theta, H_0) P(\theta | H_0) d\theta
\]
Example 1 cont. (single measurement)

Let \( P_{m_{1}H_{1}} = \frac{1}{\sqrt{2\pi \sigma_{m}^{2}}} \exp \left( -\frac{M^{2}}{2\sigma_{m}^{2}} \right) \); \(-\infty < M < \infty\)

then

\[
P_{r_{1}M,H_{1}} = \frac{1}{\sqrt{2\pi \sigma_{r_{1}m}^{2}}} \exp \left( -\frac{(R-M)^{2}}{2\sigma_{r_{1}m}^{2}} \right)
\]

\[
P_{m_{1}H_{1}} = \frac{1}{\sqrt{2\pi \sigma_{m}^{2}}} \exp \left( -\frac{M^{2}}{2\sigma_{m}^{2}} \right)
\]

\[
P_{r_{1}M,H_{0}} = \frac{1}{\sqrt{2\pi \sigma_{r_{1}m}^{2}}} \exp \left( -\frac{R^{2}}{2\sigma_{r_{1}m}^{2}} \right)
\]

\[
P_{m_{1}H_{0}} = S(M)
\]

\[
\therefore \text{(*)} \Rightarrow \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \sigma_{r_{1}m}^{2}}} \exp \left( -\frac{(R-M)^{2}}{2\sigma_{r_{1}m}^{2}} \right) \cdot \frac{1}{\sqrt{2\pi \sigma_{m}^{2}}} \exp \left( -\frac{M^{2}}{2\sigma_{m}^{2}} \right) dM
\]

\[
\Lambda(R) = \frac{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \sigma_{r_{1}m}^{2}}} \exp \left( -\frac{R^{2}}{2\sigma_{r_{1}m}^{2}} \right) S(M) dM}{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \sigma_{r_{1}m}^{2}}} \exp \left( -\frac{R^{2}}{2\sigma_{r_{1}m}^{2}} \right) S(M) dM}
\]
Integrating and taking the log of both sides, we obtain

\[
R^2 \geq \frac{20^2 (0^2 + 0_m^2)}{\sigma_m^2} \left[ \ln \eta + \frac{1}{2} \ln \left( 1 + \frac{\sigma_m^2}{\sigma^2} \right) \right] = \gamma
\]

Case 2: \( \Theta \) is a r.v with unknown pdf:

If \( \Theta \) is a r.v with unknown pdf the role is not clearly specified. A possibility is to treat \( \Theta \) as an unknown. (Suggest Neyman-Pearson test).
Case 3: \( \theta \) is an unknown nonrandom parameter.

A first observation in this case is that, because \( \theta \) has no PDF over which to average, a Bayes test is not meaningful. Thus, we devote our time to Neyman-Pearson tests.

**Example**

\begin{align*}
H_1: & \quad r \sim (\mu, \sigma^2) \\
H_0: & \quad r \sim (0, \sigma^2)
\end{align*}

\[
\therefore \quad H_1: P_{\text{RM}} (R|\mu) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left( -\frac{(r-\mu)^2}{2\sigma^2} \right)
\]

\[
H_0: P_{\text{RM}} (R|\mu) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left( -\frac{r^2}{2\sigma^2} \right)
\]

where \( \mu \) is an unknown nonrandom parameter.

\[
L(R) = \frac{\exp \left( -\frac{(R-\mu)^2}{2\sigma^2} \right)}{\exp \left( -\frac{R^2}{2\sigma^2} \right)} \geq \eta.
\]
The largest $P_D$ for the value of $M$