THE COMPLEX NUMBER SYSTEM

There is no real number x which satisfies the polynomial

$$x^2 + 1 = 0$$

To permit solutions of this and similar equations, the set of complex numbers is introduced.

We can consider a complex number as having the form

$$z = x + j\,y$$

where x and y are real numbers and

$$j^2 = -1$$

x and y are referred to as the real and imaginary parts of z:

$$x = Re(z)$$

and

$$y = \mathcal{I}m\,(z)$$

The symbol z is called a complex variable.

<u>Remarks</u>

1. Let $z_1 = x_1 + j\,y_1$ and $z_2 = x_2 + j\,y_2$, then

a. $z_1 = z_2$ iff $x_1 = x_2$ and $y_1 = y_2$.

b. $z_1 + z_2 = (x_1 + x_2) + j\,(y_1 + j\,y_2)$.

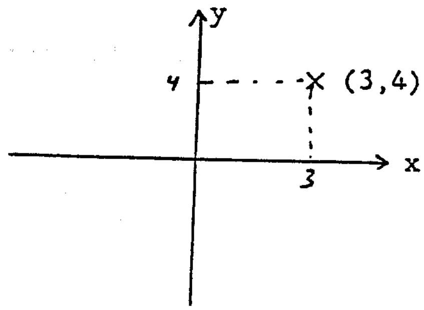
c. $z_1 - z_2 = (x_1 + j\,y_1) - (x_2 + j\,y_2) = (x_1 - x_2) + j\,(y_1 - y_2)$.

d. $z_1 z_2 = (x_1 + j\,y_1)(x_2 + j\,y_2) = x_1 x_2 + j\,x_1 y_2 + j\,y_1 x_2 + \overset{-1}{\cancel{j^2}}\,y_1 y_2$
$= (x_1 x_2 - y_1 y_2) + j\,(x_1 y_2 + y_1 x_2)$

2. Let $z_1 = x_1 + j\,y_1$ then $z_1^* = x_1 - j\,y_1$ is called the conjugate of z_1.

GRAPHICAL REPRESENTATION OF COMPLEX NUMBERS

The complex number $z = 3 + j\,4$ can be shown as



POLAR FORM OF COMPLEX NUMBERS

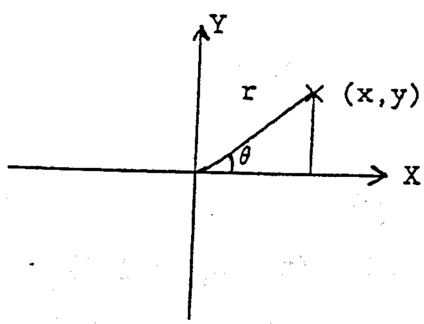
If P is a point in the complex plane corresponding to the complex number

$$z = x + j\,y$$

then we see from the figure that

$$x = r \cos \theta$$

$$y = r \sin \theta$$



where $r = (x^2 + y^2)^{1/2} = |x + j\,y|$ is called the absolute value of z and θ is called the argument of z. Note that

$$z = x + j\,y = r(\cos \theta + j \sin \theta) = r e^{j\theta} = r\angle\theta$$

which is called the polar form of z and (r, θ) are called the polar coordinates of z.