

CHAPTER I

Voltage and Current

The concept of electric charge is the basis for describing all electrical phenomena. First, the charge is bipolar, that is, electrical effects are described in terms of positive and negative charges. Second, electric charge exists in discrete quantities. Specifically, all quantities are integral multiples of electron charge. The electron charge is 1.6022×10^{-19} C. Third, electrical effects are attributed to both the separation of charge and charge in motion. In circuit theory, we regard the separation of charge as creating an *electric force (voltage)* and the motion of charge as creating an *electric fluid (current)*.

Def. Voltage is the energy per unit charge that is created by the separation:

$$v = dw/dq$$

where

- v = the voltage in volts,
- w = the energy in joules, and
- q = the charge in coulombs,

Example

The potential energy of an electron is found to change by 1.2×10^{-16} J when it moves between two points. Calculate the voltage between the two points.

solution

Here

$$dw = 1.2 \times 10^{-16} \text{ J}$$

and we know that

$$dq = 1 \times (1.6022 \times 10^{-19}) \text{ C.}$$

Thus, the voltage between two points is:

$$v = dw/dq = 1.2 \times 10^{-16} / 1.6022 \times 10^{-19} = 750 \text{ V}$$

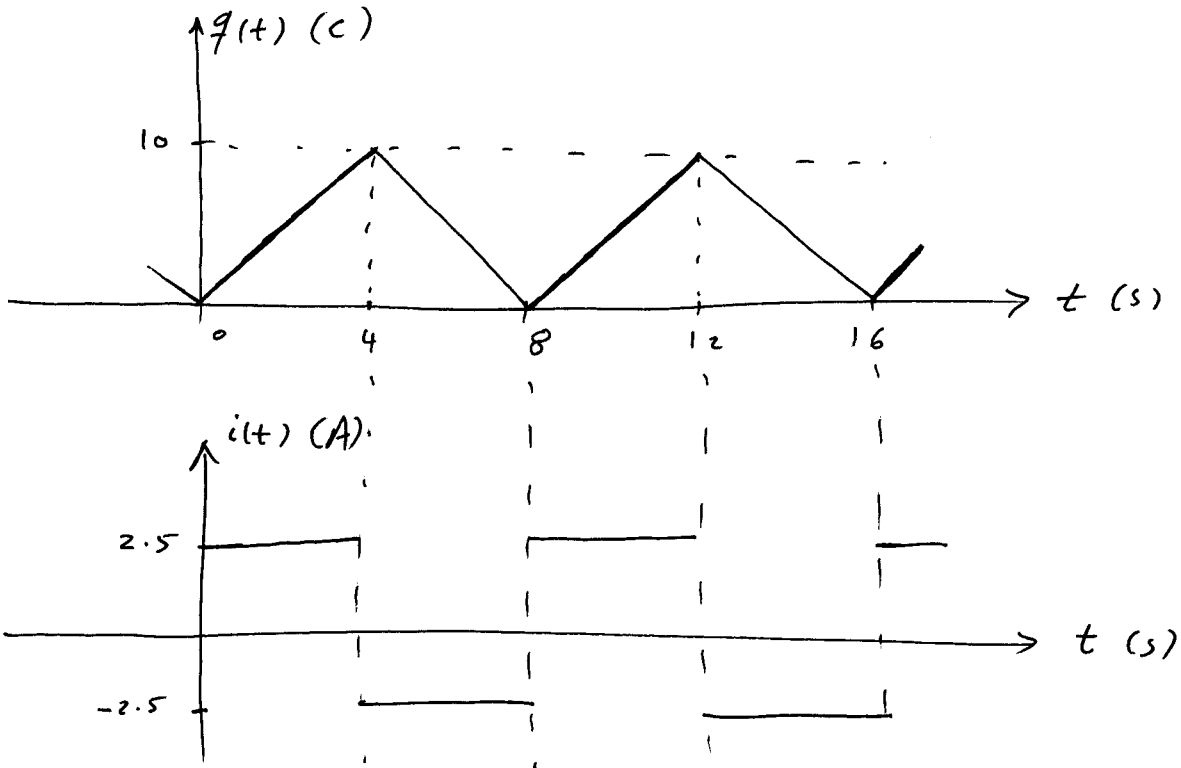
Def. The electrical effects caused by charges in motion depend on the rate of charge flow:

$$i = dq/dt$$

where i is the current in amperes.

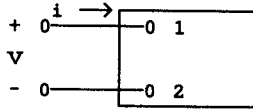
Example

The electric charge in a cross section of a material is found to vary with time as shown. Determine and sketch $i(t)$.



Prob 1.5 (Drill)

Let $i = 10 e^{-2000t} u(t)$. Calculate the total charge (in microcoulombs) entering the element at its upper terminal.



Solⁿ:

$$i = \frac{dq}{dt}$$

$$q = \int_0^{\infty} i(t) dt$$

$$= \int_0^{\infty} 10 e^{-2000t} dt$$

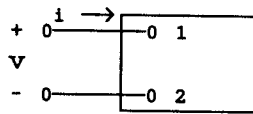
$$= \frac{10}{-2000} e^{-2000t} \Big|_0^{\infty}$$

$$= -\frac{1}{200} [0 - 1] = \frac{1}{200} \text{ C}$$

$$= 5000 \text{ } \mu\text{C.}$$

Remarks

Consider the basic element shown:



The interpretation of the element can be summarized as follows.

If:

- a) $v > 0$, then there is a drop in voltage in going from terminal 1 to 2.
- b) $v < 0$, then there is a rise in voltage in going from terminal 1 to 2.
- c) $i > 0$, then positive charge carriers flow from terminal 1 to 2 or negative charge carriers flow from terminal 2 to 1.
- d) $i < 0$, then the positive charge carriers flow from terminal 2 to terminal 1 or the negative charge carriers flow from terminal 1 to 2.

Passive Sign Convention

Whenever the reference direction for the current in an element is in the direction of the voltage drop across the element (as shown above), use a positive sign in the expression that relates the voltage to the current. Otherwise, use a negative sign.

Power and Energy

All practical devices have limitations on the amount of power they can handle. In the design process, therefore, voltage and current calculations by themselves are not sufficient. Here we define power and energy.

We recall from basic physics that power is the time rate of expending or absorbing energy. (A water pump rated 100 hp can deliver more gallons per minute than one rated 10 hp). Thus,

$$p = dw/dt$$

where

p = the power in watts

w = the energy in joules

Thus, 1 watt is equivalent to 1 joules per second. It follows that

$$p = dw/dt = (dw/dq)(dq/dt) = vi$$

As a result, power is a quantity associated with a pair of terminals and we have to be able ^{to see} from our calculation whether power is being delivered or extracted from the pair of terminals. This information comes from the correct application of the passive sign convention.

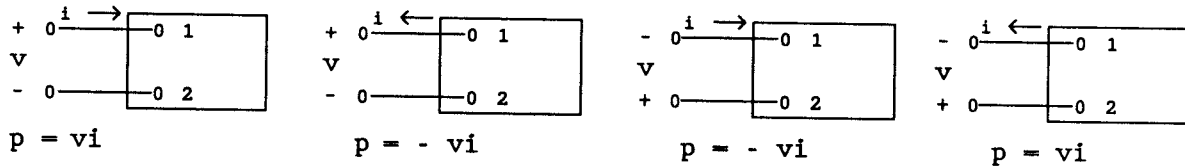
If we use the passive sign convention, then

$$p = vi$$

Otherwise

$$p = -vi$$

The relationship between the polarity references are shown below.



We can now state the rule for interpreting the algebraic sign of power.

a) If $p > 0$, then the power is being **delivered** to the circuit inside the box.

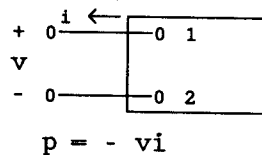
b) If $p < 0$, then power is being **extracted** from the circuit inside the box.

Example

Consider the polarity reference shown with

$$i = 4 \text{ A}$$

$$v = -10 \text{ V}$$



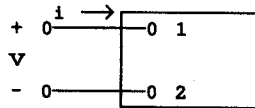
Then

$$p = -(-10)(4) = 40$$

Thus the circuit inside the box is absorbing 40 W.

Prob 1.7 (Drill)

Let $v = 50 e^{-2000t} u(t)$. Calculate the total energy (in millijoules) delivered to the circuit element.



Sol: know $i = 10 e^{-2000t} u(t)$.

$$\therefore P = v i = 500 e^{-4000t} u(t)$$

$$P = \frac{dw}{dt} \Rightarrow W(t) = \int_0^{\infty} P(t) dt$$

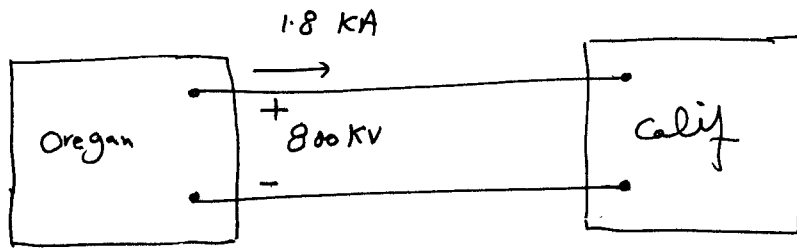
$$= \int_0^{\infty} 500 e^{-4000t} dt$$

$$= \frac{500}{-4000} e^{-4000t} \Big|_0^{\infty} = \frac{5}{40} \text{ joules}$$

$$= 125 \text{ mJ.}$$

1.8 (Drill) :

11



Calculate the power at the Oregon end of the line and state the direction of power flow.

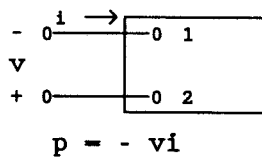
to Calif

$$P = vi = (800 \text{ k})(1.8 \text{ k}) = 1440 \text{ MW}$$

since $P > 0$ for Calif \Rightarrow power is drawn from Oregon and delivered to Calif.

Prob 1.6 (p. 13)

Let $v = 15$ V and $i = -8$ A.



a) Calculate p .

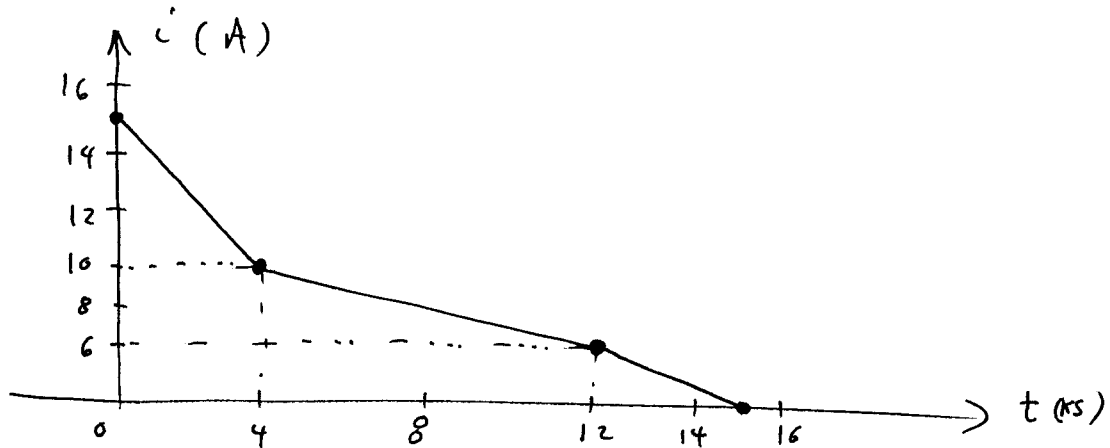
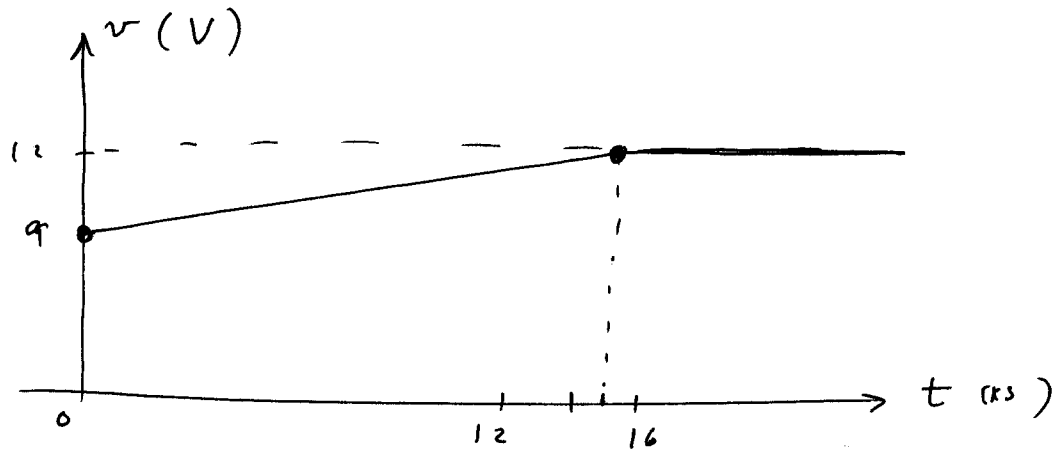
$$p = -vi = -(15)(-8) = 120 \text{ W}$$

Since $p > 0$, the power is being *absorbed* in the circuit element shown.

b) Since $i = -8$ A, it implies that the positive charge carriers (protons) flow from terminal 2 to terminal 1 or negative charge carriers (electrons) flow in the opposite direction, that is, from terminal 1 to terminal 2.

c) Part (a) implies that the circuit element is absorbing power, therefore, the electrons are *losing* energy as they pass through the element in the box.

1.8
13 :



$$\begin{aligned}
 a) \quad q_{\text{total}} &= \int_0^{\infty} i(t) dt = (15+10)\frac{4}{2} + (10+6)\frac{8}{2} + 3 \cdot \frac{6}{2} \\
 &= 50 + 64 + 9 = 123 \text{ Kc.}
 \end{aligned}$$

$$b) \quad P(t) = v(t) i(t) = \frac{dW}{dt}$$

$$\therefore W = \int_0^{\infty} P(t) dt = \int_0^{\infty} v(t) i(t) dt$$

$$[d] p = -vi = -4800 \text{ W} \quad \text{B to A}$$

$$\mathbf{P1.8} \quad [a] q = \text{area under } i \text{ vs. } t \text{ plot}$$

$$= 12.5 \times 4 \times 10^3 + 8 \times 8 \times 10^3 + 3 \times 3 \times 10^3$$

$$= 123,000 \text{ Coulombs}$$

$$[b] v = 9 + 0.2 \times 10^{-3}t \text{ V}, \quad 0 \leq t \leq 15 \text{ ks}$$

$$i = 15 - 1.25 \times 10^{-3}t \text{ A}, \quad 0 \leq t \leq 4 \text{ ks}$$

$$= 12 - 0.5 \times 10^{-3}t \text{ A}, \quad 4 \leq t \leq 12 \text{ ks}$$

$$= 30 - 2 \times 10^{-3}t \text{ A}, \quad 12 \leq t \leq 15 \text{ ks}$$

$$= 0, \quad t \geq 15 \text{ ks}$$

$$0 \leq t \leq 4000 \text{ s:}$$

$$p = vi = 135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2 \text{ W}$$

$$w = \int_0^{4000} p dt = 468.67 \text{ kJ}$$

$$4000 \leq t \leq 12,000 \text{ s:}$$

$$p = vi = 108 - 2.1 \times 10^{-3}t - 0.1 \times 10^{-6}t^2 \text{ W}$$

$$w = \int_{4,000}^{12,000} p dt = 674.53 \text{ kJ}$$

$$12,000 \leq t \leq 15,000 \text{ s:}$$

$$p = vi = 270 - 12 \times 10^{-3}t - 0.4 \times 10^{-6}t^2 \text{ W}$$

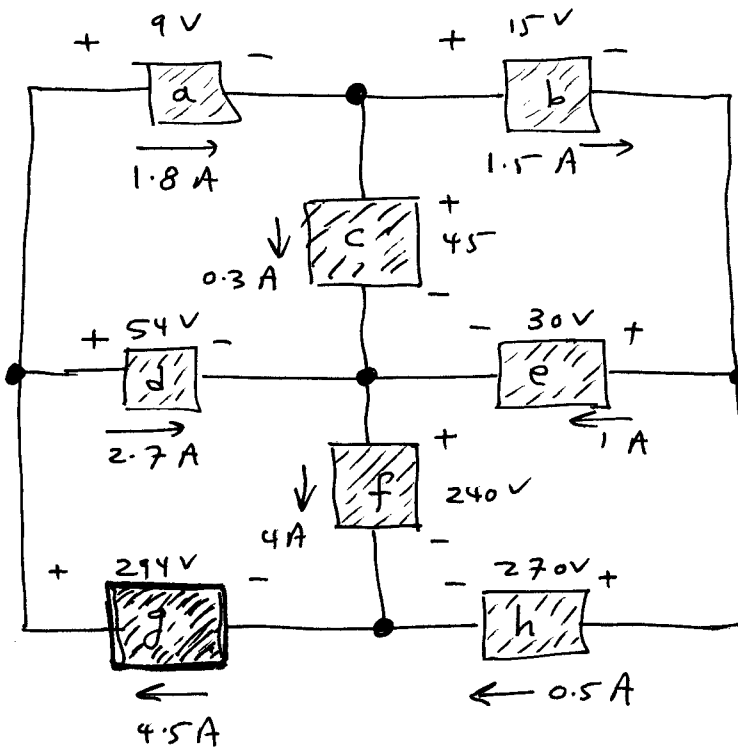
$$w = \int_{12,000}^{15,000} p dt = 104.40 \text{ kJ}$$

$$w_{\text{TOT}} = 468.67 + 674.53 + 104.40 = 1247.60 \text{ kJ}$$

$$\mathbf{P1.9} \quad [a] v = 40(4e^{-16t} - e^{-4t}) \text{ V}$$

$$i = 5(4e^{-16t} - e^{-4t}) \text{ A}$$

$$\frac{1.13}{14} :$$



$$P_a = (9)(1.8) = 16.2 \quad \text{abs.}$$

$$P_b = (15)(1.5) = 22.5 \quad \text{abs.}$$

$$P_c = (45)(0.3) = 13.5 \quad \text{abs.}$$

$$P_d = (54)(2.7) = 145.8 \quad \text{abs.}$$

$$P_e = (30)(1) = 30 \quad \text{abs.}$$

$$P_f = (240)(4) = 960 \quad \text{abs.}$$

$$P_g = -(294)(4.5) = -1323 \quad \text{Del}$$

$$P_h = (270)(0.5) = 135 \quad \text{abs.}$$

$$\sum P_{\text{abs}} = 1323 = \sum P_{\text{Del}} \quad \checkmark$$