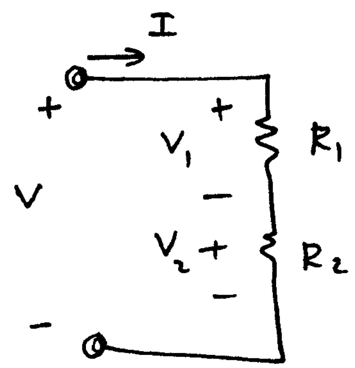
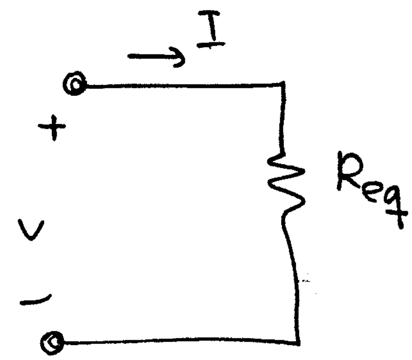
Chapter 3

Simple Resistive Circuits

1.



$\Longrightarrow$

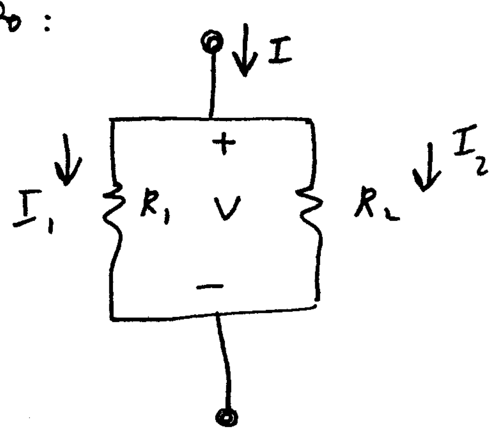


KVL : $V = V_1 + V_2$

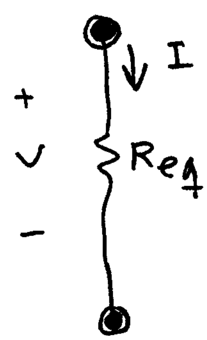
$$\therefore \quad \cancel{I} R_{eq} = \cancel{I} R_1 + \cancel{I} R_2$$

$$\therefore \quad \boxed{R_{eq} = R_1 + R_2}$$

Also :



$\Longrightarrow$



KCL :

$$I = I_1 + I_2$$

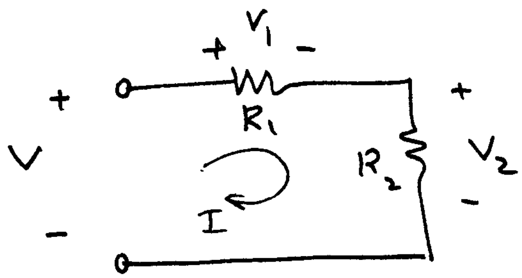
$$\frac{\cancel{V}}{R_{eq}} = \frac{\cancel{V}}{R_1} + \frac{\cancel{V}}{R_2}$$

$$\therefore \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

or $$G_{eq} = G_1 + G_2$$

2. Voltage divider and Current divider :

a. Voltage divider :



Pf : $V = (R_1 + R_2) I$

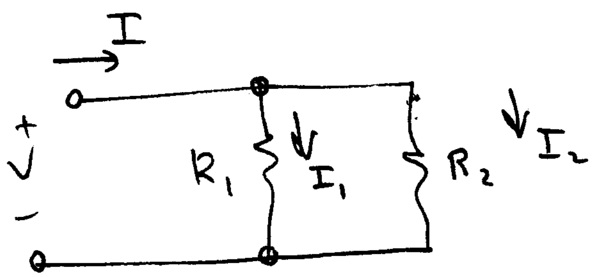
$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = R_1 I = \frac{R_1}{R_1 + R_2} V.$$

$$V_1 = \frac{R_1}{R_1 + R_2} V$$

$$V_2 = \frac{R_2}{R_1 + R_2} V$$

b. Current divider



Pf : $I_1 = \frac{V}{R_1}$

$$V = I (R_1 /\!/ R_2)$$

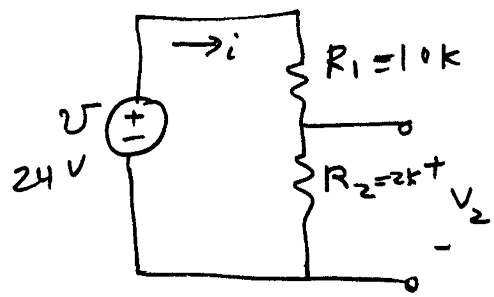
$$= I \frac{R_1 R_2}{R_1 + R_2}$$

$$\therefore \quad I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I_1 = \frac{R_2}{R_1 + R_2} I$$

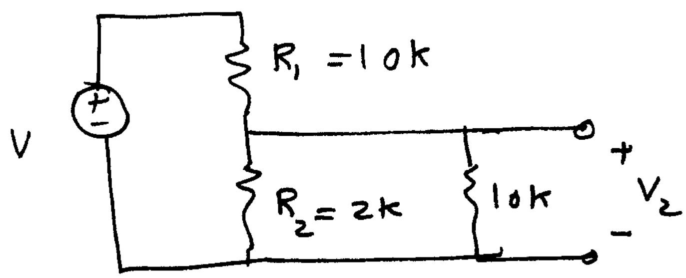
$$I_2 = \frac{R_1}{R_1 + R_2} I$$

$\frac{E20}{55}$:



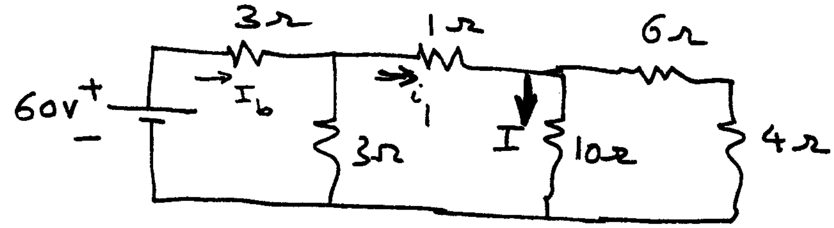
(a) $V_2 = \frac{R_2}{R_1 + R_2} V = \frac{2}{10+2} 24 = \underline{4\ V}$

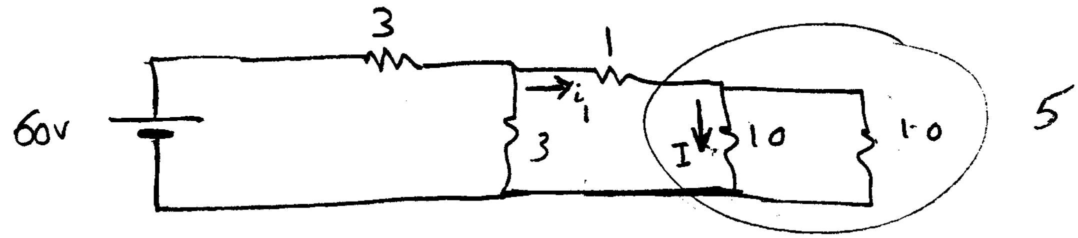
(b)

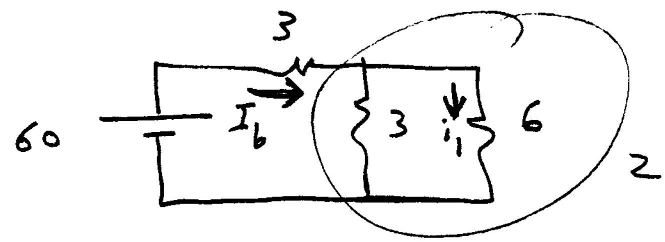


$V_2 = \frac{10 /\!/ 2}{10 + 10/\!/2} 24 = \underline{3.43}\ V$

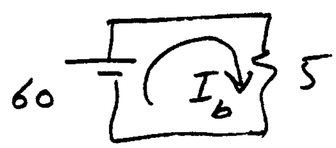
$\frac{E16}{55}$: $I_b = ?$, $i_1 = ?$, $I = ?$







$$\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

∴ 

$$-60 + 5 I_b = 0$$

$$I_b = \underline{12 \text{ A}}.$$

$$i_1 = \frac{3}{3+6} I_b = \frac{3}{9}(12) = \underline{4 \text{ A}}.$$

$$I = \frac{10}{10+10} i_1 = \frac{1}{2} i_1 = \underline{2 \text{ A}}.$$

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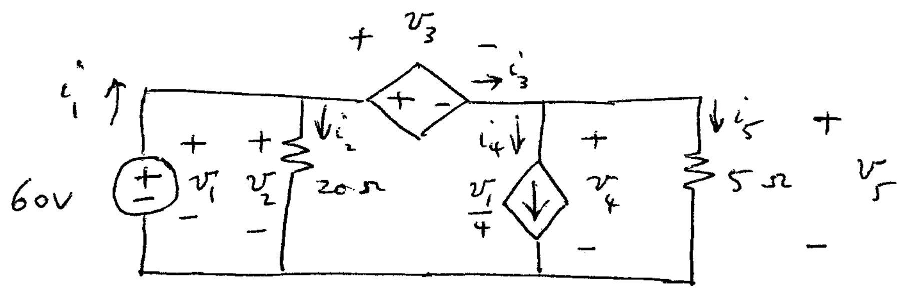
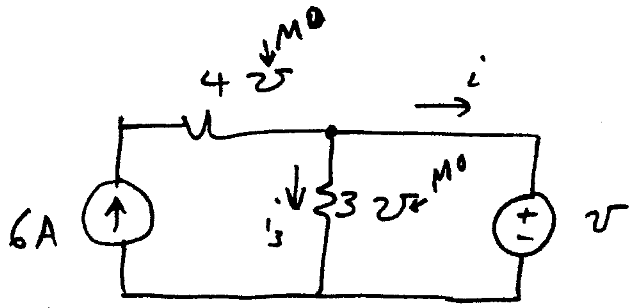
(a) Use KCL and KVL to evaluate all the currents and voltages in the following circuit.

(b) Calculate the power absorbed by each of the five circuit elements and show that the sum is zero.



$\frac{1.16}{41}$ (Bobrow):



$i = ?$

a) $v = 1$ V.

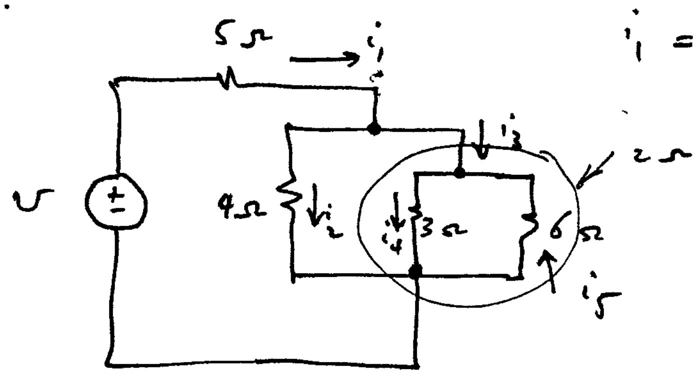
KCL: $6 = i_3 + i$

$$i_3 = \frac{v}{R_3} = v\,G_3 = (1)(3) = 3 \text{ A}.$$

$$\therefore \quad i = 6 - i_3 = 6 - 3 = \underline{3 \text{ A}}.$$

$\frac{1.18}{42}$:

$i_1 = 5$, $i_2, i_3, i_4, i_5 = ?$

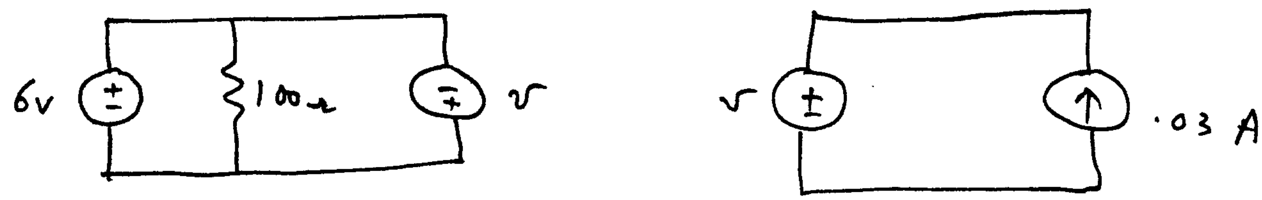


Current division:

$$i_2 = \frac{2}{2+4}\,\cancel{i_1}^{\,5} = \frac{5}{3}\text{ A}, \qquad i_3 = \frac{4}{2+4}\,\cancel{i_1}^{\,5} = \frac{10}{3}\text{ A}$$

$$i_4 = \frac{6}{6+3}\,i_3 = \frac{6}{9}\cdot\frac{10}{3} = \frac{20}{9}\text{ A}, \quad -i_5 = \frac{3}{6+3}\,i_3 \Rightarrow i_5 = -\frac{10}{9}\text{ A}.$$

#1.25 (Bobrow):
43



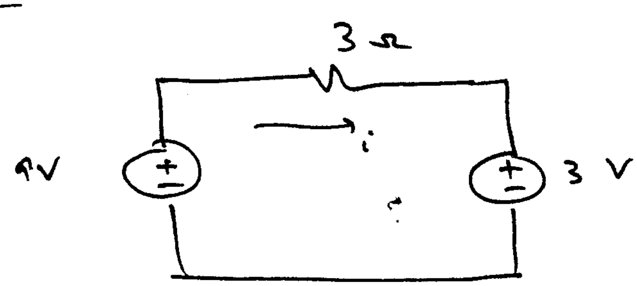
(a) (b)

$v = -6$ V. for (a)

Any value for v is permissible for (b).

(b) is both in series and in parallel.

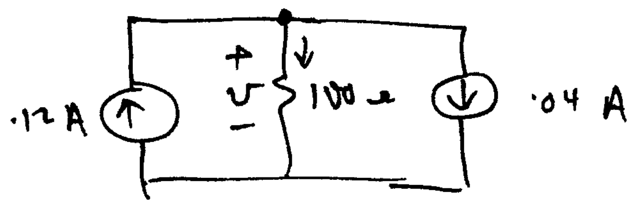
#1.27 (a):
43



KVL:

$-9 + 3i + 3 = 0 \Rightarrow \underline{i = 2 \text{ A}}$

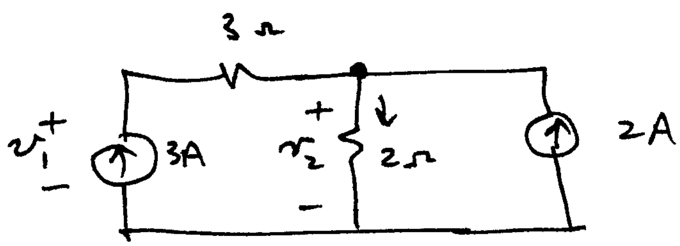
(b) :



KCL :

$$.12 = \frac{v}{100} + .04 \implies v = \underline{8\,V.}$$

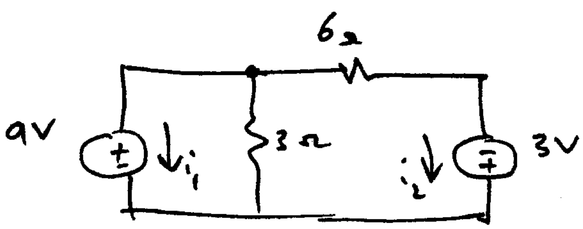
(c)



KCL: $$3 + 2 = \frac{v_2}{2} \implies v_2 = \underline{10\ V}$$

KVL :
(Loop 1) $$-v_1 + 3(3) + \overset{10}{\cancel{v_2}} = 0 \implies v_1 = \underline{19\ V}.$$

(d)

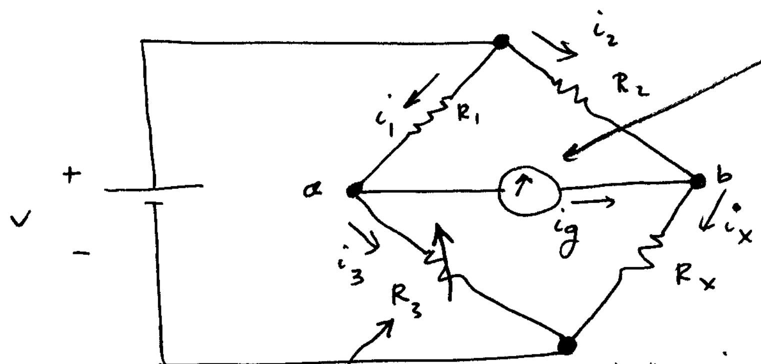


KVL
(outer Loop) : $$-9 + 6 i_2 - 3 = 0 \implies i_2 = \frac{12}{6} = \underline{2\ A.}$$

KCL: $$i_1 + \overset{9/3}{\cancel{i/3}} + \overset{2}{\cancel{i_2}} = 0. \implies i_1 = \underline{-5\ A}$$

The Wheatstone Bridge

The Wheatstone bridge circuit is a method used for the precision measurement of resistances of medium values, that is, resistances in the range of 1 Ω to 1 MΩ.



microammeter which shows zero when the bridge is in balance.

$\therefore$ $i_g = 0$

adjust R_3 until there is no current in the galvanometer

when $i_g = 0 \Rightarrow i_1 = i_3$

$i_2 = i_x$

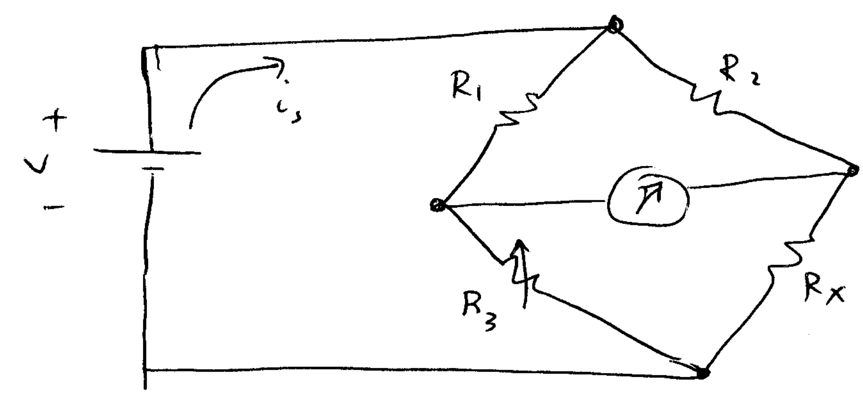
since $i_g = 0 \Rightarrow$ pnts a and b have the same potential;

Thus,

$$i_1 R_1 = i_2 R_2 \qquad\qquad \Rightarrow \qquad \frac{\cancel{i_1} R_1}{\cancel{i_3} R_3} = \frac{\cancel{i_2} R_2}{\cancel{i_x} R_x}$$
$$i_3 R_3 = i_x R_x$$

$$\therefore \quad \boxed{R_1 R_x = R_2 R_3}$$

DE $\frac{3.5}{77}$:



$R_1 = 100\ \Omega$

$R_2 = 1000\ \Omega$

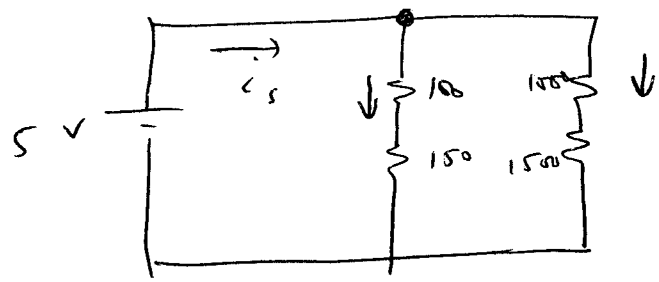
$R_3 = 150\ \Omega$

$V = 5\ V$

(a) $\overset{100}{\cancel{R_1}}\, R_x = \overset{1000}{\cancel{R_2}}\, \overset{150}{\cancel{R_3}} \Rightarrow R_x = 1.5\ K.$

(b) suppose each bridge resistor is capable of dissipating 250 mW. Can the bridge be left in the balanced state without exceeding the power-dissipating capacity of the resistors, thereby damaging the bridge?

KCL.



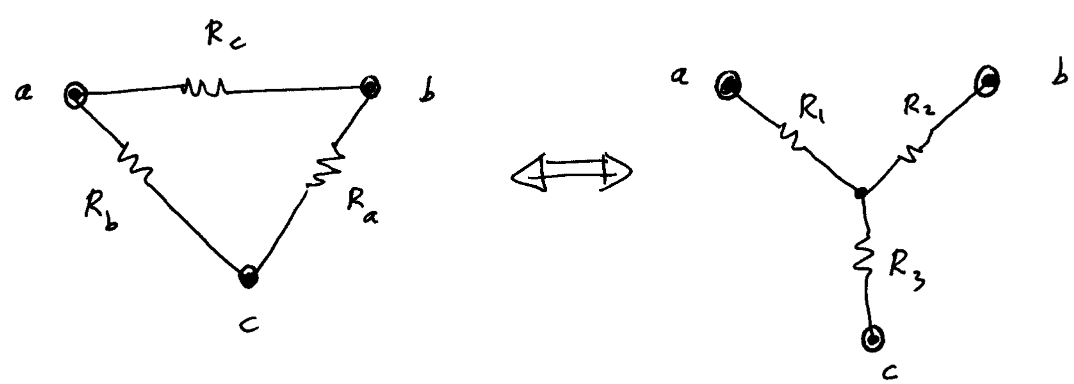
$$i_s = \frac{5}{250} + \frac{5}{2500} = 0.022\ A = 22\ mA.$$

$\therefore\ P_{5V}\ (\text{delivered}) = (5)(22\ mA) = 110\ mW < 250\ mW$

$\therefore$ Bridge will not be damaged.

Δ to Y Equivalent Circuits

The bridge configuration introduced an interconnection of R's that suggests further discussion. Consider



Note that, by definition, the two circuits are equivalent wrt their terminal behavior.

$$\therefore \quad R_{ab} = R_1 + R_2 = (R_b + R_a) // R_c$$

$$R_{bc} = R_2 + R_3 = (R_b + R_c) // R_a$$

$$R_{ca} = R_1 + R_3 = (R_c + R_a) // R_b$$

$$\therefore \quad R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

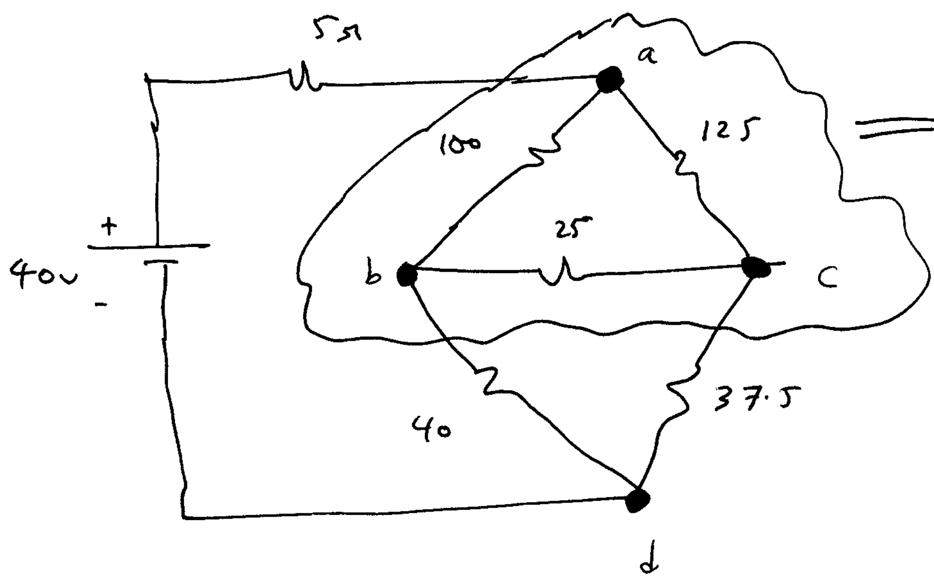
or

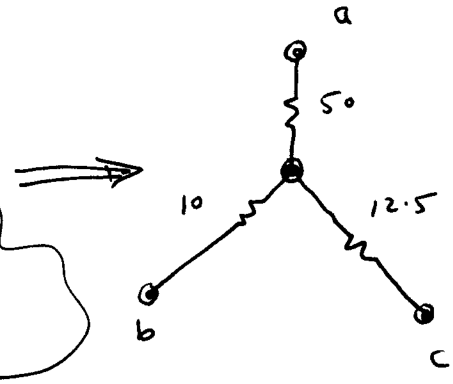
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

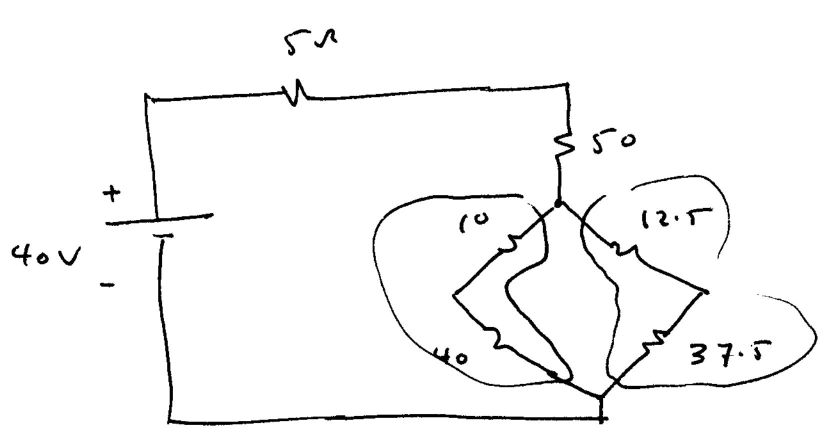
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

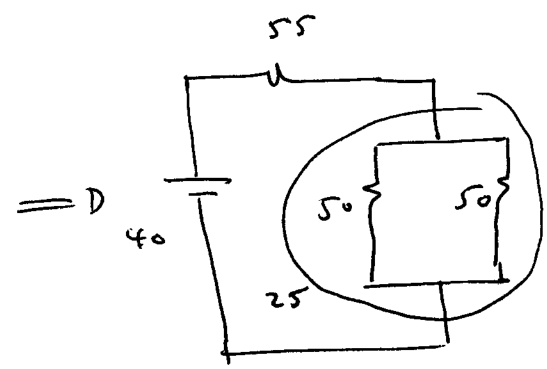
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} .$$

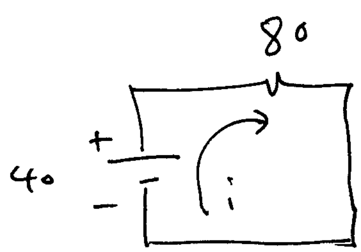
Exa : $\frac{3.6}{79}$











$\Rightarrow \quad i = 1/2 \text{ A}$

$P_{Del.} = (40)(\frac{1}{2}) = 20 \text{ W}$