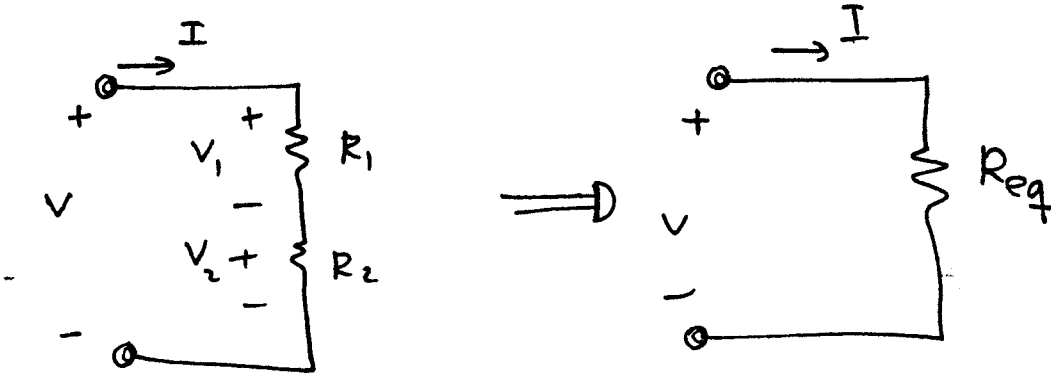


Chapter 3

Simple Resistive Circuits

1.

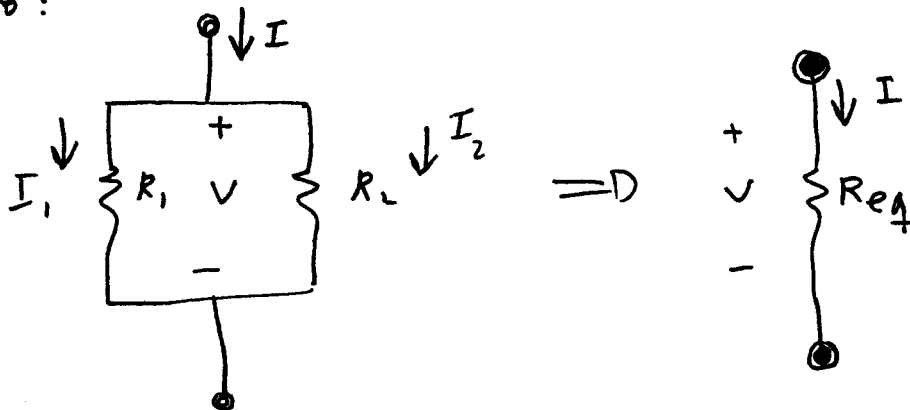


KVL : $V = V_1 + V_2$

$$\therefore I R_{eq} = I R_1 + I R_2$$

$$\therefore R_{eq} = R_1 + R_2$$

Also :



KCL :

$$I = I_1 + I_2$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2}$$

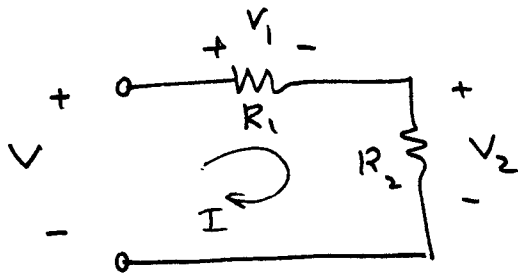
$$\therefore \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

or

$$G_{eq} = G_1 + G_2$$

2. Voltage divider and Current divider :

a. Voltage divider :



$$V_1 = \frac{R_1}{R_1 + R_2} V$$

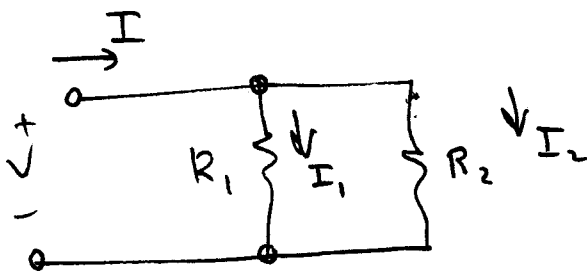
$$V_2 = \frac{R_2}{R_1 + R_2} V$$

pf: $V = (R_1 + R_2) I$

$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = R_1 I = \frac{R_1}{R_1 + R_2} V$$

b. Current divider



$$I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{R_1}{R_1 + R_2} I$$

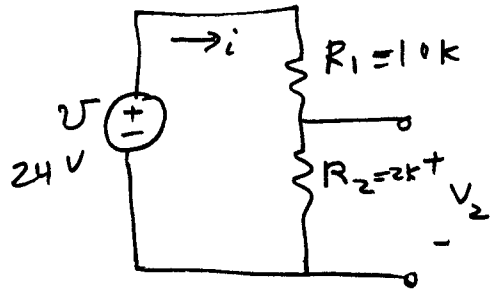
pf: $I_1 = \frac{V}{R_1}$

$$V = I (R_1 \parallel R_2)$$

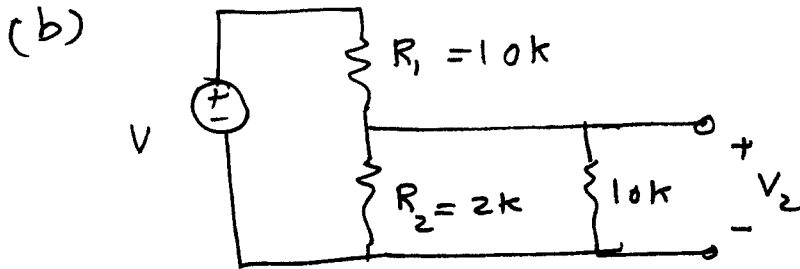
$$= I \frac{R_1 R_2}{R_1 + R_2}$$

$$\therefore I_1 = \frac{R_2}{R_1 + R_2} I$$

$\frac{E_{20}}{55}$:

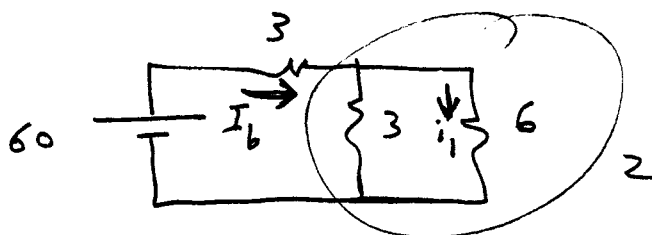
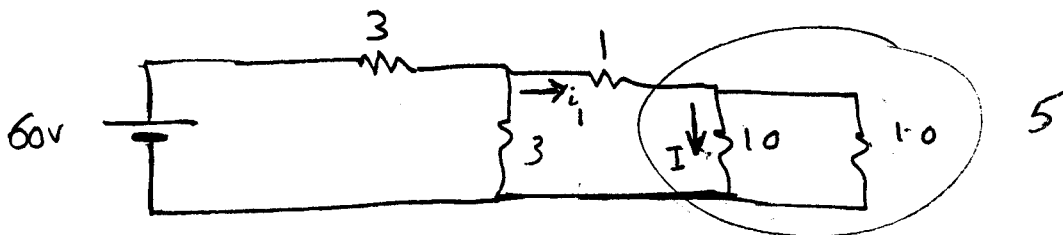
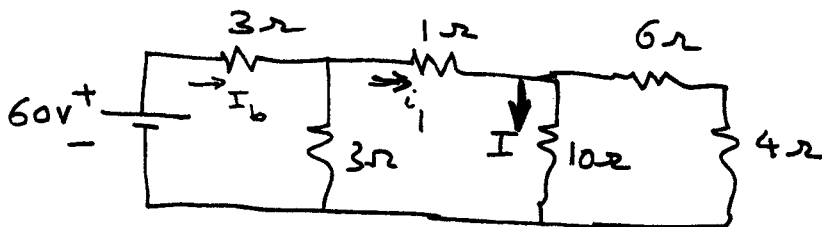


$$(a) \quad V_2 = \frac{R_2}{R_1 + R_2} \cdot 24 = \frac{2}{10 + 2} \cdot 24 = \underline{4 \text{ V}}$$

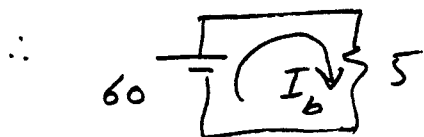


$$V_2 = \frac{10 // 2}{10 + 10 // 2} \cdot 24 = \underline{3.43 \text{ V}}$$

$$\frac{E16}{55} : \quad \bar{I}_b = ? , \quad i_1 = ? , \quad \bar{I} = ?$$



$$\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$



$$-60 + 5 I_b = 0$$

$$I_b = \underline{12 \text{ A}}$$

$$i_1 = \frac{3}{3+6} I_b = \frac{3}{9} (12) = \underline{4 \text{ A}}$$

$$I = \frac{10}{10+10} i_1 = \frac{1}{2} i_1 = \underline{2 \text{ A}}$$

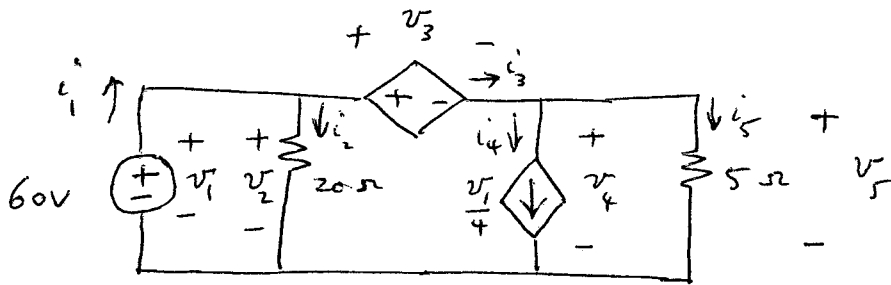
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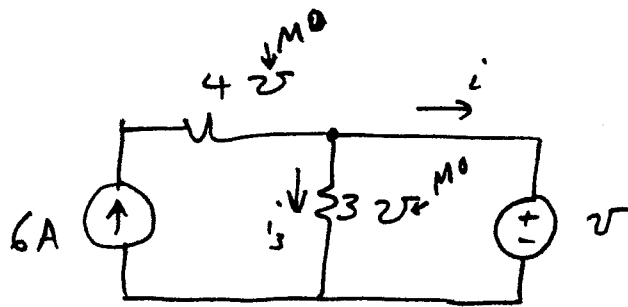
Quiz I

Spring 2002

- (a) Use KCL and KVL to evaluate all the currents and voltages in the following circuit.
- (b) Calculate the power absorbed by each of the five circuit elements and show that the sum is zero.



1.16 (Bobrow):
41



$$i = ?$$

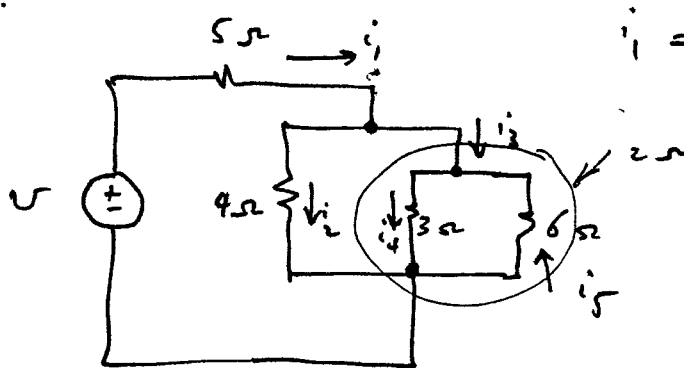
a) $v = 1 \text{ V}$.

KCL: $6 = i_3 + i$

$$i_3 = \frac{v}{R_3} = v G_3 = (1)(3) = 3 \text{ A}.$$

$$\therefore i = 6 - i_3 = 6 - 3 = \underline{3 \text{ A}}.$$

1.18:
42



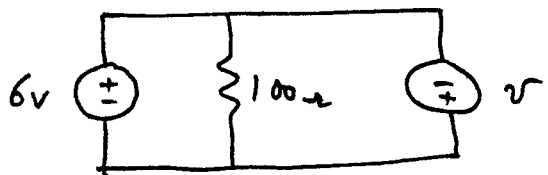
$$i_1 = 5, \quad i_2, i_3, i_4, i_5 = ?$$

Current division:

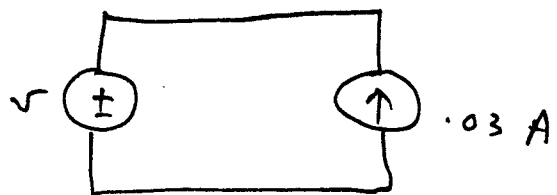
$$i_2 = \frac{2}{2+4} i_1 = \frac{5}{3} \text{ A}, \quad i_3 = \frac{4}{2+4} i_1 = \frac{10}{3} \text{ A}$$

$$i_4 = \frac{6}{6+3} i_3 = \frac{6}{9} \cdot \frac{10}{3} = \frac{20}{9} \text{ A}, \quad -i_5 = \frac{3}{6+3} i_3 \Rightarrow i_5 = -\frac{10}{9} \text{ A}.$$

#1.25 (Bobrow) :
43



(a)



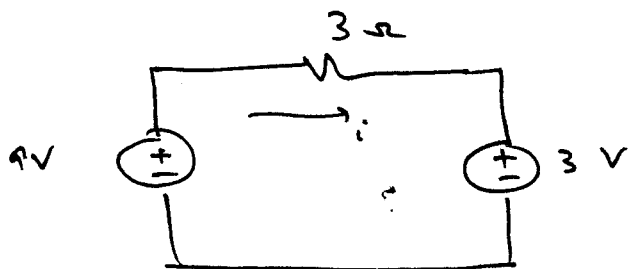
(b)

$v = -6\text{ V}$ for (a)

- Any value for v is permissible for (b).

(b) is both in series and in parallel.

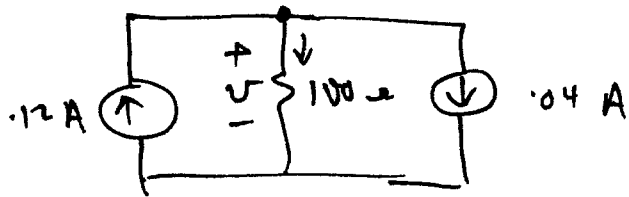
#1.27 (a) :
43



KVL:

$$-9 + 3i + 3 = 0 \implies \underline{i = 2\text{ A}}$$

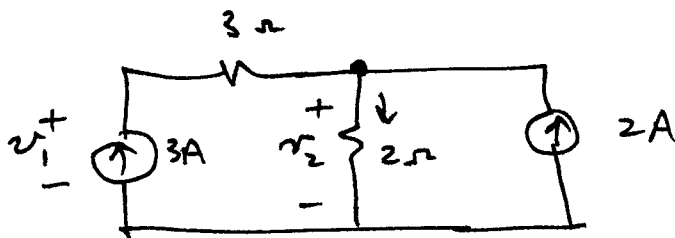
(b) :



KCL :

$$.12 = \frac{v}{100} + .04 \Rightarrow v = \underline{8V.}$$

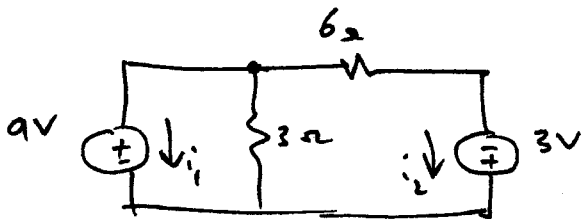
(c)



KCL: $3 + 2 = \frac{v_2}{2} \Rightarrow v_2 = \underline{10V}$

KVL: (Loop 1) $-v_1 + 3(3) + \frac{10}{2} = 0 \Rightarrow v_1 = \underline{19V}$

(d)

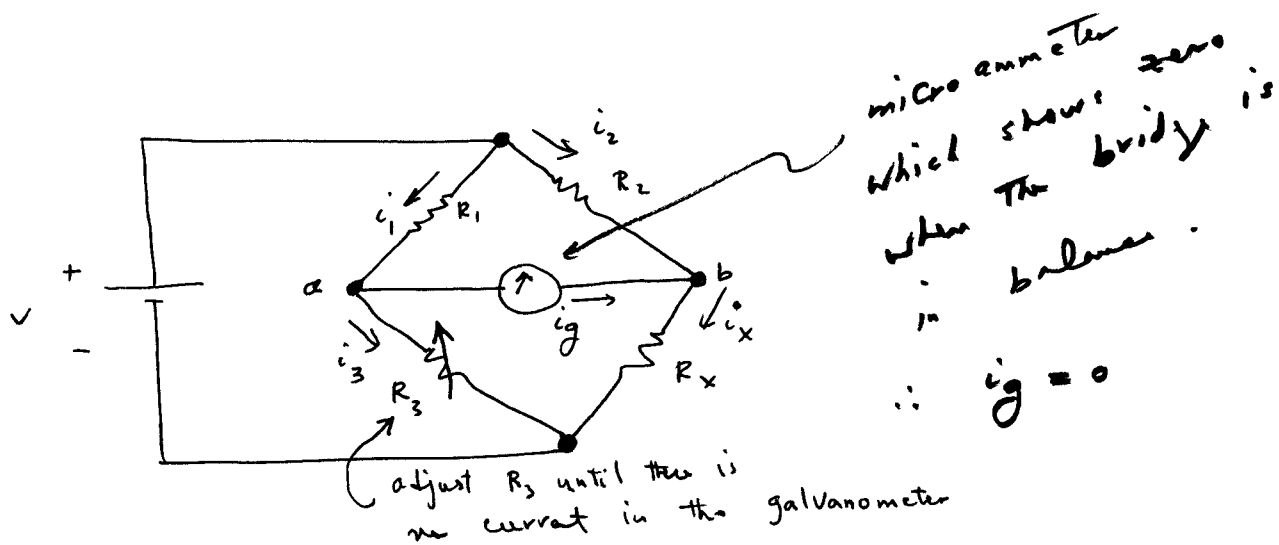


KVL (outer loop) : $-9 + 6i_2 - 3 = 0 \Rightarrow i_2 = \frac{12}{6} = \underline{2A}$

KCL: $i_1 + \frac{9}{3} + \frac{i_2}{2} = 0 \Rightarrow i_1 = \underline{-5A}$

The Wheatstone Bridge

The Wheatstone bridge circuit is a method used for the precision measurement of resistances of medium values, that is, resistances in the range of 1Ω to $1 M\Omega$.



when $i_g = 0 \Rightarrow i_1 = i_3$
 $i_2 = i_x$

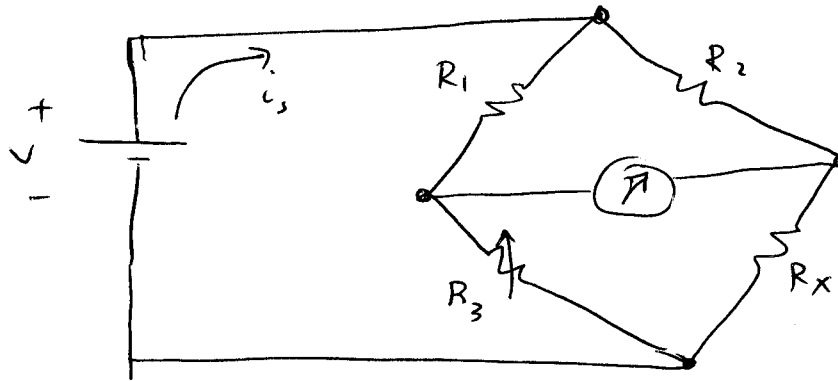
since $i_g = 0 \Rightarrow$ pts a and b have the same potential;

Thus,

$$\begin{aligned} i_1 R_1 &= i_2 R_2 \\ i_3 R_3 &= i_x R_x \end{aligned} \Rightarrow \frac{i_1 R_1}{i_3 R_3} = \frac{i_2 R_2}{i_x R_x}$$

$$\therefore R_1 R_x = R_2 R_3$$

$$DE \frac{3.5}{77} ;$$



$$R_1 = 100 \Omega$$

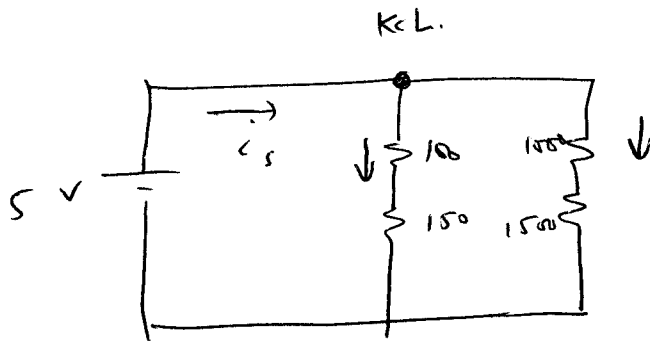
$$R_2 = 1000 \Omega$$

$$R_3 = 150 \Omega$$

$$V = 5 \text{ V}$$

$$(a) \quad \frac{100}{R_1} R_X = \frac{1000}{R_2} \frac{150}{R_3} \Rightarrow R_X = 1.5 \text{ K.}$$

(b) Suppose each bridge resistor is capable of dissipating 250 mW. Can the bridge be left in the balanced state without exceeding the power-dissipating capacity of the resistors, thereby damaging the bridge?



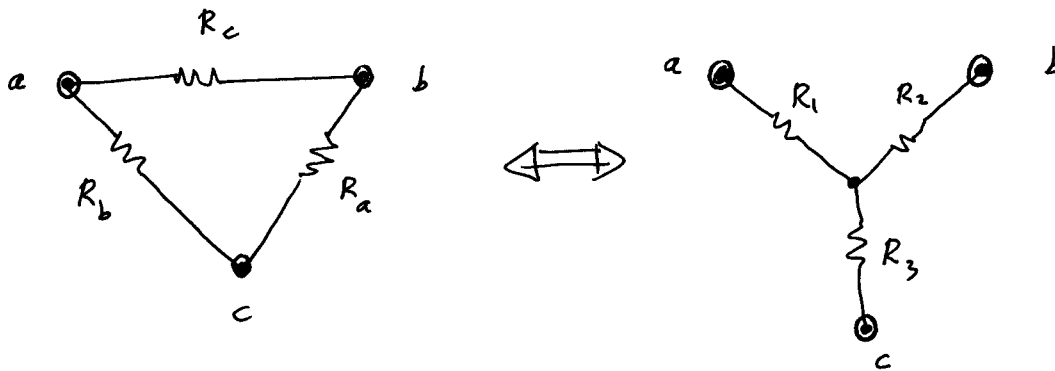
$$i_s = \frac{5}{250} + \frac{5}{2500} = 0.022 \text{ A} = 22 \text{ mA}$$

$$\therefore P_{\text{delivered}} = (5 \text{ V})(22 \text{ mA}) = 110 \text{ mW} < 250 \text{ mW}$$

\therefore Bridge will not be damaged.

Δ to Y Equivalent Circuits

The bridge configuration introduced an interconnection of R's that suggests further discussion. Consider



Note that, by definition, the two circuits are equivalent w.r.t their terminal behavior.

$$\therefore R_{ab} = R_1 + R_2 = (R_b + R_a) \parallel R_c$$

$$R_{bc} = R_2 + R_3 = (R_b + R_c) \parallel R_a$$

$$R_{ca} = R_1 + R_3 = (R_c + R_a) \parallel R_b$$

$$\therefore R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

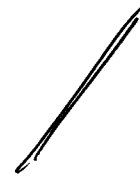
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

or

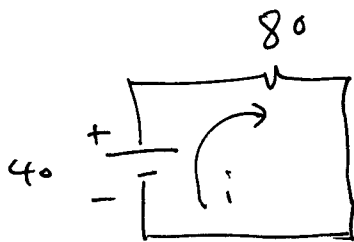
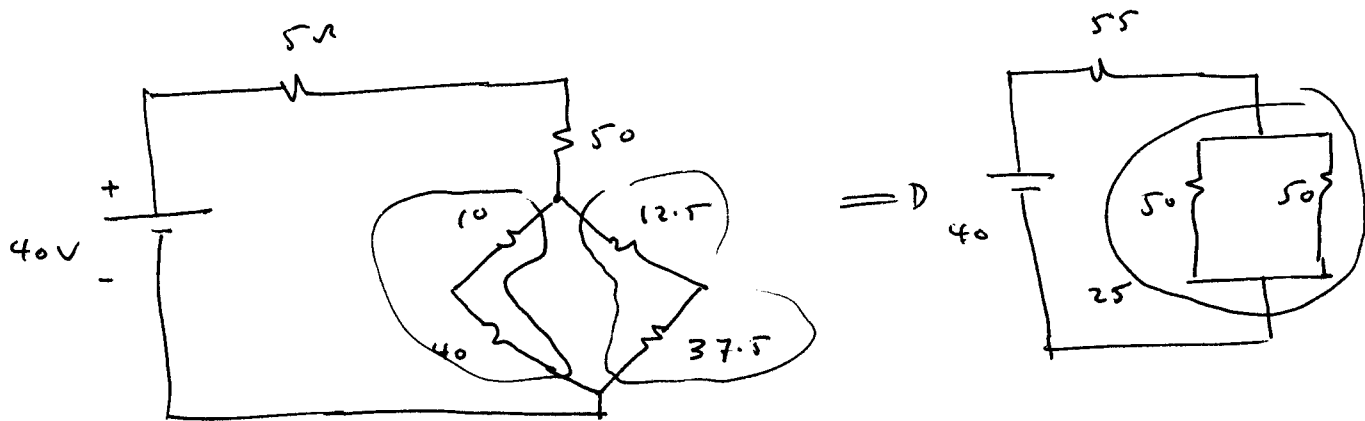
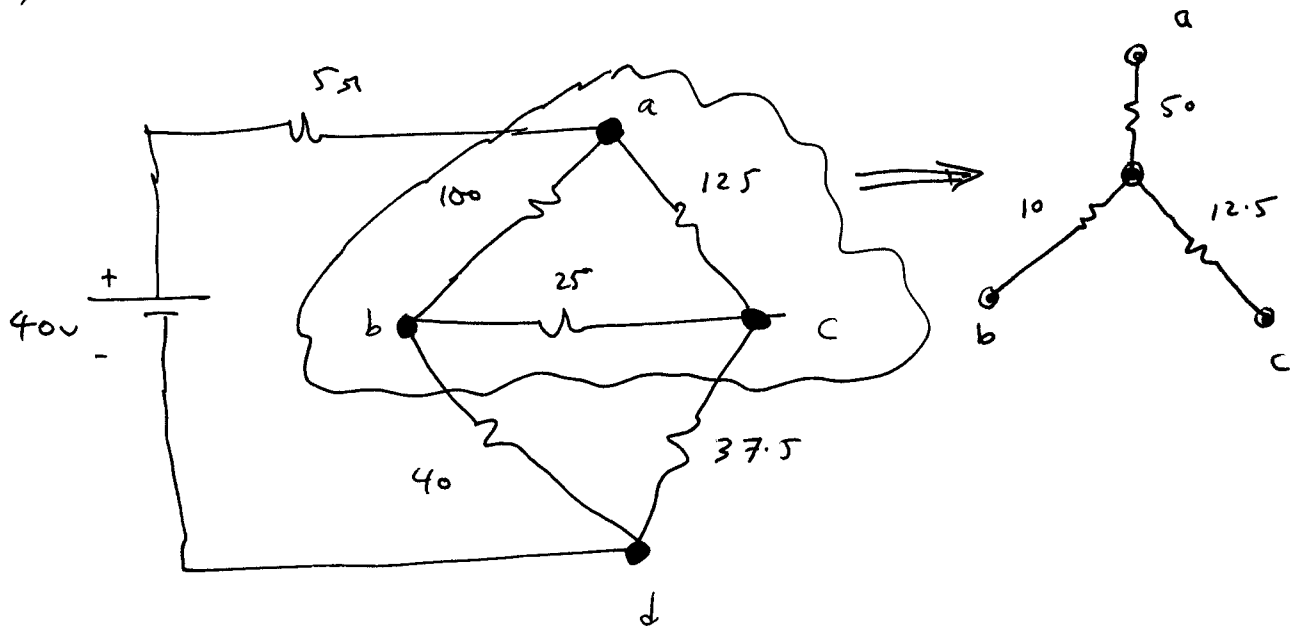
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} .$$



EXA: $\frac{3.6}{79}$



$\Rightarrow i = \underline{\underline{\frac{1}{2} \text{ A}}}$

$P_{del.} = (40) \left(\frac{1}{2} \right) = \underline{\underline{20 \text{ W}}}$