Chapter 4

Techniques of Circuit Analysis

So far we have been able to analyze relatively simple resistive circuits by the use of KVL and KCL in combination with Ohm's law. This approach can be used for all circuits, but as the circuits become more complicated, we will soon find this direct method quite cumbersome. In this chapter, we introduce two powerful methods of circuit analysis: the node-voltage method and the mesh-current method. We have already seen how series-parallel reductions and Δ-to-Y transformations can be used to simplify a given structure. We will now add source transformations and Thevenin-Norton equivalent circuits to our list of simplification techniques.

Recall

Branch: A branch is part of a circuit with two terminals to which connections can be made.

Node: A node is the point where two or more branches come together.

Loop: A loop is a closed path formed by connecting branches.

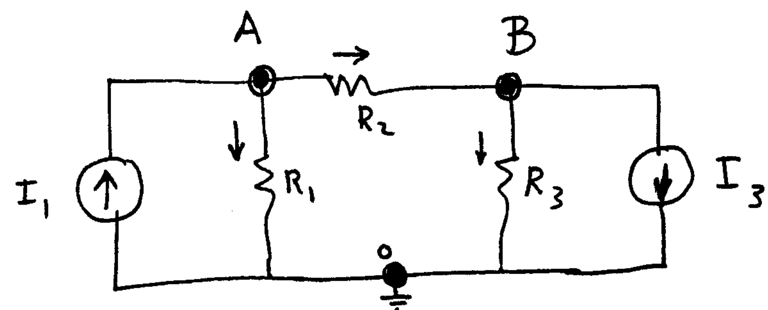
Mesh A mesh is a special type of loop; that is, it does not contain any other loops within it.

Example

Node - voltage method :

It is a method in which the KVL eqns are written implicitly on the circuit so that only the KCL eqns need be solved.

Exa :



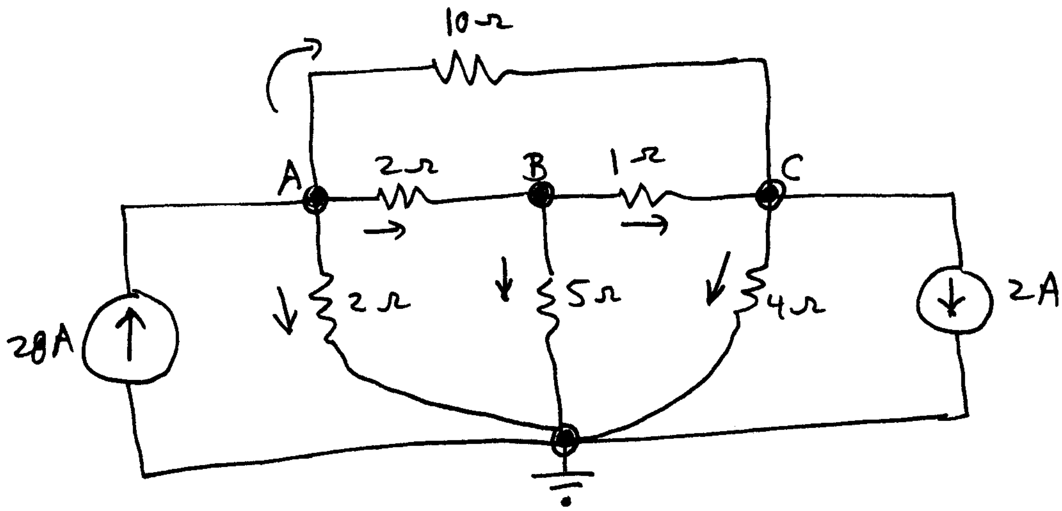
KCL A : $$I_1 = \frac{V_A}{R_1} + \frac{V_A - V_B}{R_2}$$

KCL B : $$\frac{V_A - V_B}{R_2} = \frac{V_B}{R_3} + I_3$$

Rearranging the terms gives:

$$\begin{cases} \dfrac{1}{R_1+R_2} V_A - \dfrac{1}{R_2} V_B = I_1 \\ -\dfrac{1}{R_2} V_A + \dfrac{1}{R_2+R_3} V_B = -I_3 \end{cases}$$

Exa: Use the node-voltage method to find the voltages v_A, v_B, v_C.



KCL A: $$28 = \frac{v_A}{2} + \frac{v_A - v_B}{2} + \frac{v_A - v_C}{10}$$

KCL B: $$\frac{v_A - v_B}{2} = \frac{v_B}{5} + \frac{v_B - v_C}{1}$$

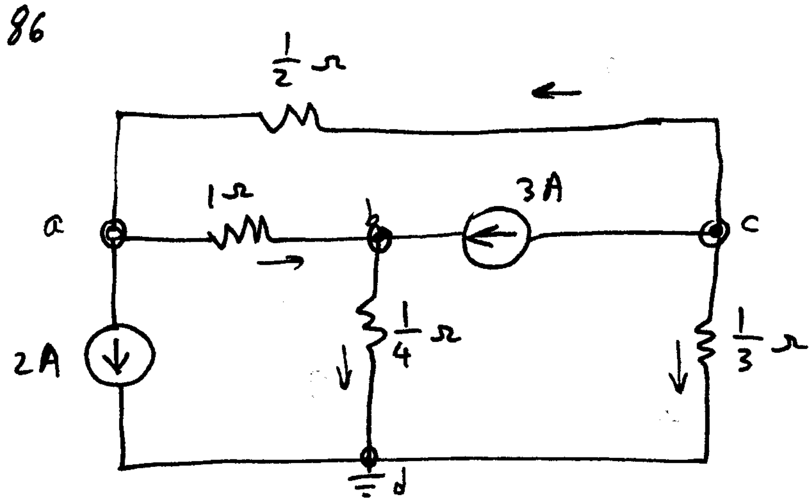
KCL C: $$\frac{v_B - v_C}{1} = \frac{v_C}{4} + 2 - \frac{v_A - v_C}{10}$$

or

$$\begin{cases} 1.1\, v_A - .5\, v_B - .1\, v_C = 28 \\ -.5\, v_A + 1.7\, v_B - v_C = 0 \\ -.1\, v_A - v_B + 1.35\, v_C = -2 \end{cases}$$

v_A, v_B, v_C = ...

$\frac{2.1}{86}$: (Bobrow):



Nodal Analysis :

KCL a.

$$\frac{V_a - V_b}{1} + 2 = \frac{V_c - V_a}{1/2}$$

$$\therefore \quad 3V_a - V_b - 2V_c = -2.$$

KCL b.

$$\frac{V_a - V_b}{1} + 3 = \frac{V_b}{1/4}$$

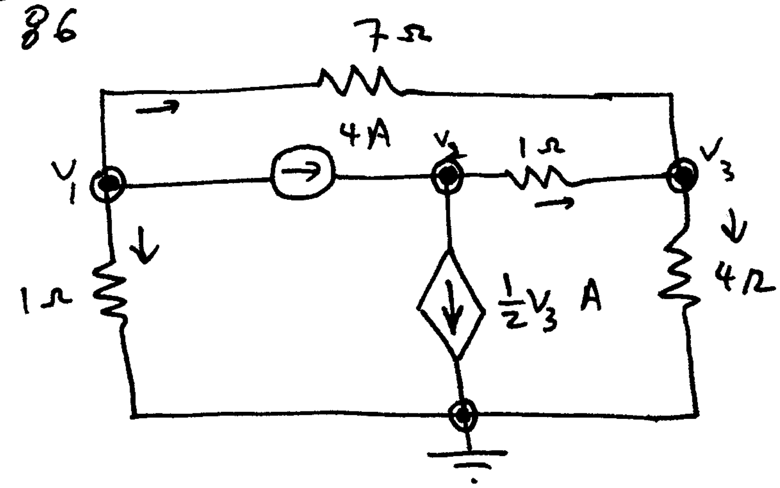
$$\therefore \quad V_a - 5V_b = -3.$$

KCL c.

$$\frac{V_c - V_a}{1/2} + \frac{V_c}{1/3} + 3 = 0 \implies 2V_a - 5V_c = 3.$$

$V_a = -1.3\ V$, $V_b = .34V$, $V_c = -1.12\ V$

$\# \frac{2.4}{86}$:



$$\frac{V_1}{1} + \frac{V_1 - V_3}{7} + 4 = 0 \implies 8V_1 - V_3 = -28$$

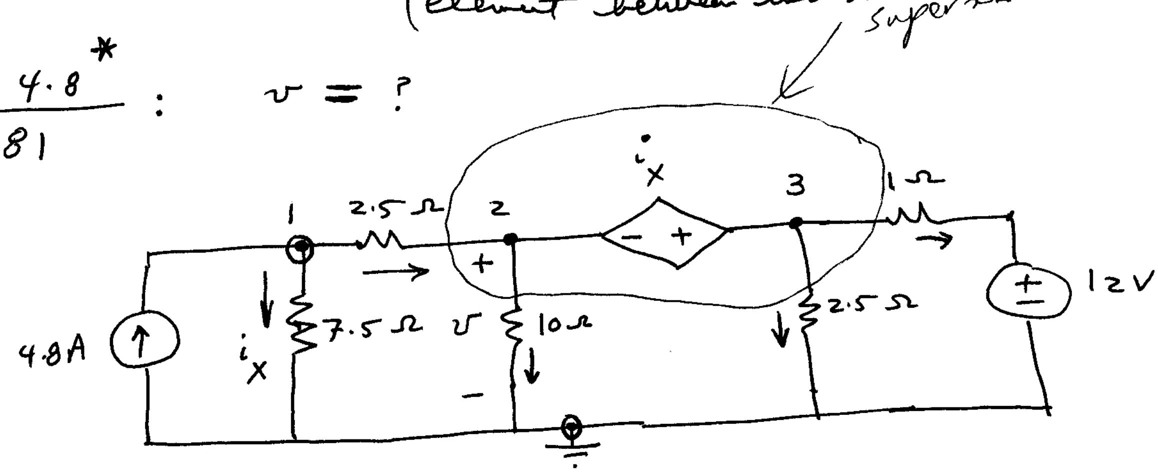
$$4 = \frac{1}{2}V_3 + \frac{V_2 - V_3}{1} \implies 2V_2 - V_3 = 8$$

$$\frac{V_1 - V_3}{7} + \frac{V_2 - V_3}{1} = \frac{V_3}{4} \implies -4V_1 - 28V_2 + 39V_3 = 0$$

$V_1 = \underline{-3\ V}$, $V_2 = \underline{6\ V}$, $V_3 = \underline{4\ V}$

Special case ; (A voltage source is the only element between two nodes.)

D 4.8* / 81 : $v = ?$



node 1 : $$4.8 = \frac{V_1}{7.5} + \frac{V_1 - V_2}{2.5} \quad (1)$$

node 2 : We see that node-voltage eqns at either node 2 or 3 can't be written because we cannot express the current in the dependent voltage source. To solve the pbm we combine nodes 2 and 3 and call it a "supernode". We now write the KCL eqns for the supernode.

$$\frac{V_1 - V_2}{2.5} = \frac{V_2}{10} + \frac{V_3}{2.5} + \frac{V_3 - 12}{1} \quad (2)$$

Also note that

$$V_3 - V_2 = i_x \quad (3)$$

and $$i_x = \frac{V_1}{7.5} \qquad (4)$$

(4) in (3) $\Rightarrow$

$$V_3 - V_2 = \frac{V_1}{7.5}$$

$$\therefore \quad V_3 = V_2 + \frac{V_1}{7.5} \qquad (5)$$

$\therefore$ (1), (2), (5) $\Rightarrow$

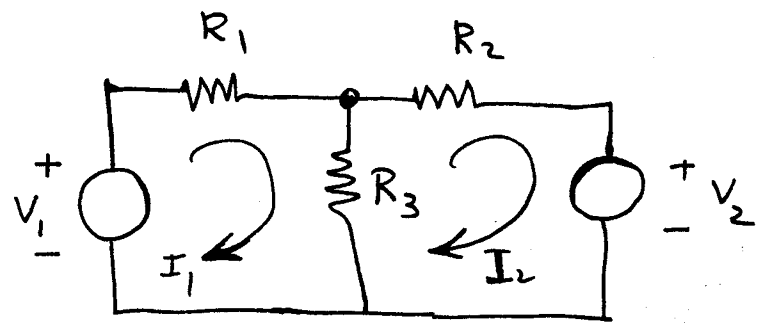
$$\begin{cases} \overset{0.53}{\cancel{\left(\frac{1}{2.5}+\frac{1}{7.5}\right)}} V_1 - \overset{0.4}{\cancel{\frac{1}{2.5}}} V_2 + 0\, V_3 = 4.8 \\ \overset{0.4}{\cancel{\frac{1}{2.5}}} V_1 - \overset{0.5}{\cancel{\left(\frac{1}{2.5}+\frac{1}{10}\right)}} V_2 - \overset{1.4}{\cancel{\left(\frac{1}{2.5}+1\right)}} V_3 = -12 \\ \overset{0.13}{\cancel{\frac{1}{7.5}}} V_1 + V_2 - V_3 = 0 \end{cases}$$

$$\therefore \quad \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 15 \\ 8 \\ 10 \end{pmatrix} \longleftarrow v$$

Loop- Current method : (mesh analysis).

It is a method of solving circuit pbms in which the KCL eqns are written implicitly and KVL eqns are written explicitly and solved.

Exa :



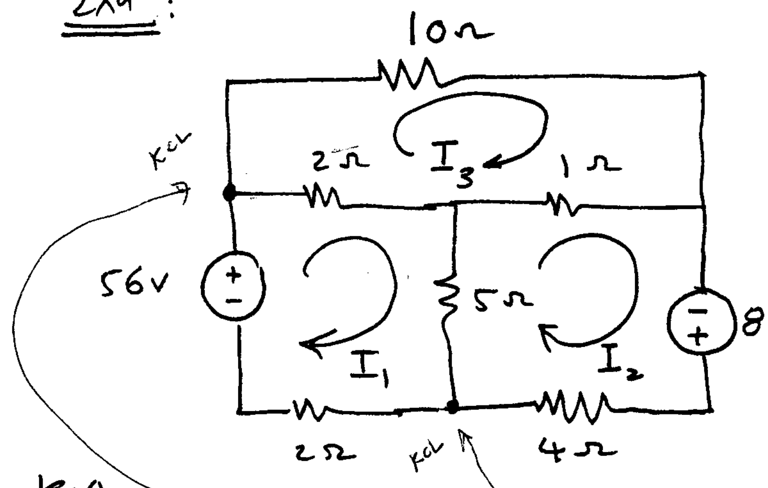
$$\begin{cases} -V_1 + R_1 I_1 + R_3 (I_1 - I_2) = 0 \\ +V_2 + R_3 (I_2 - I_1) + R_2 I_2 = 0 \end{cases}$$

$$\therefore \quad \begin{cases} (R_1 + R_3) I_1 - R_3 I_2 = V_1 \\ -R_3 I_1 + (R_2 + R_3) I_2 = -V_2 \end{cases}$$

$I_1 , I_2 = \cdots$

Using Loop-current method find I_1, I_2, I_3:

Exa:



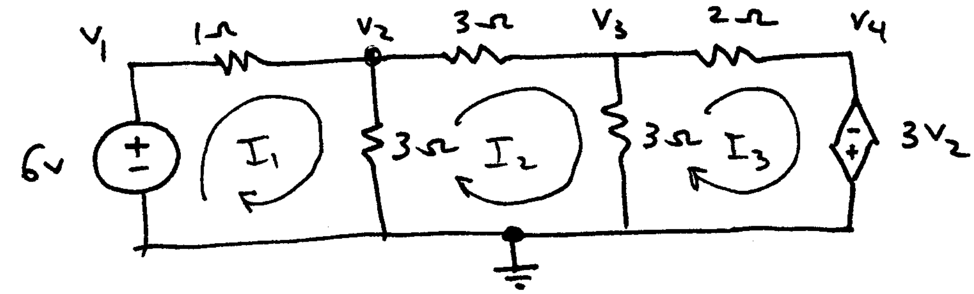
KVL:

$$\begin{cases} -56 + 2(I_1 - I_3) + 5(I_1 - I_2) + 2I_1 = 0. \\ -8 + 4I_2 + 5(I_2 - I_1) + (I_2 - I_3) = 0. \\ 2(I_3 - I_1) + 10I_3 + (I_3 - I_2) = 0. \end{cases}$$

I_1, I_2, I_3 = ---

2.15 (Bobrow)

90



$$-6 + I_1 + 3(I_1 - I_2) = 0 \qquad (1)$$

$$\cancel{3}(I_2 - I_1) + \cancel{3}I_2 + \cancel{3}(I_2 - I_3) = 0 \qquad (2)$$

$$3(I_3 - I_2) + 2I_3 - 3\overset{3(I_1-I_2)}{\cancel{V_2}} = 0 \qquad (3)$$

$$V_2 = 3(I_1 - I_2) \qquad (4)$$

$(1) \Rightarrow \quad 4I_1 - 3I_2 = 6$

$(2) \Rightarrow \quad -I_1 + 3I_2 - I_3 = 0$

$(4) \text{ in } (3) \Rightarrow \quad -9I_1 + 6I_2 + 5I_3 = 0$

$$I_1 = \frac{\begin{vmatrix} 6 & -3 & 0 \\ 0 & 3 & -1 \\ 0 & 6 & 5 \end{vmatrix}}{\begin{vmatrix} 4 & -3 & 0 \\ -1 & 3 & -1 \\ -9 & 6 & 5 \end{vmatrix}} = \frac{126}{42} = 3A, \quad I_2 = 2A, \quad I_3 = 3$$

$\therefore \quad V_1 = \underline{6}\ V.$

$V_2 = 3(I_1 - I_2) = \underline{3}\ V.$

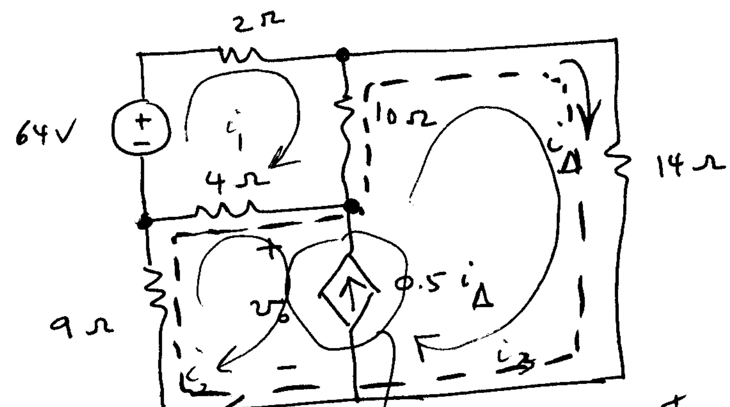
$V_3 = 3(I_2 - I_3) = \underline{-3}\ V$

$V_4 = \underline{-9V}$

Special case

When a branch includes a current source we need extra things to do.

Prob 4.19 / 115 : $v_o = ?$



supermesh

We can't write KVL for this current source. So, remove it "mentally" and form a "supermesh" and write mesh eqs.

mesh eqs:

$$\begin{cases} -64 + 2i_1 + 10(i_1 - i_3) + 4(i_1 - i_2) = 0 & (1) \\ 9i_2 + 4(i_2 - i_1) + 10(i_3 - i_1) + 14 i_3 = 0 & (2) \end{cases}$$

Also, $\begin{cases} i_3 - i_2 = 0.5\, i_\Delta \\ i_\Delta = i_3 \end{cases} \Longrightarrow \quad 0.5\, i_3 - i_2 = 0. \qquad (3)$

(1), (2), (3) $\Rightarrow$

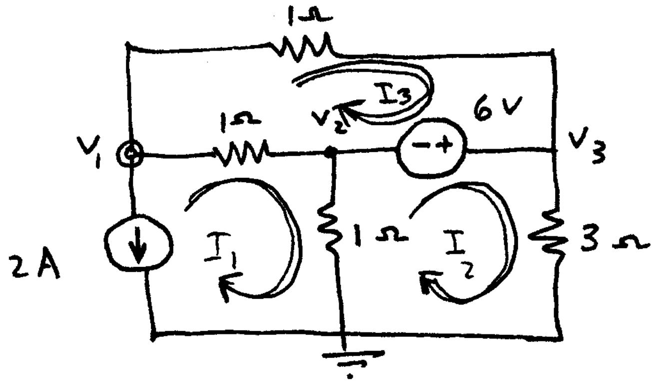
$$\begin{cases} 16 i_1 - 4 i_2 - 10 i_3 = 64 \\ -14 i_1 + 13 i_2 + 24 i_3 = 0 \\ 0 i_1 - i_2 + 0.5 i_3 = 0 \end{cases}$$

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 6.1 \\ 1.4 \\ 2.8 \end{pmatrix} \text{ A.}$$

$$9 i_2 + 4 (i_2 - i_1) + v_o = 0 \Rightarrow \underline{\underline{v_o = 6.20 \text{ V}}}$$

#2.17 (Bobrow)

90



$-V_1 + (I_1 - I_3) + 1(I_1 - I_2) = 0.$ (1)

$(I_2 - I_1) - 6 + 3I_2 = 0.$ (2)

$I_3 + 6 + (I_3 - I_1) = 0.$ (3)

$I_1 = -2A$ (4)

(4) in (2) $\Rightarrow$ $4I_2 - \cancel{I_1}^{-2} = 6$ $\Rightarrow$ $I_2 = 1$ A (5)

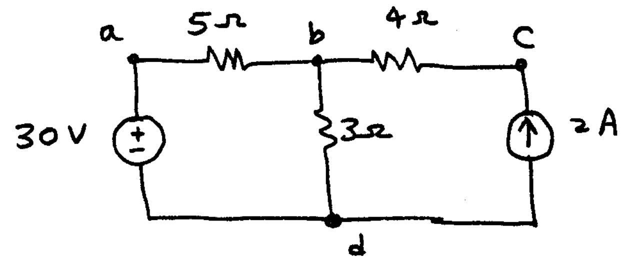
(4) in (3) $\Rightarrow$ $2I_3 = I_1 - 6 = -2 - 6 = -8 \Rightarrow I_3 = -4$

(1) $\Rightarrow$ $V_1 = 2I_1 - I_2 - I_3 = -4 - 1 + 4 = -1$ V

$V_2 = 1(I_1 - I_2) = -2 - 1 = -3$ V.

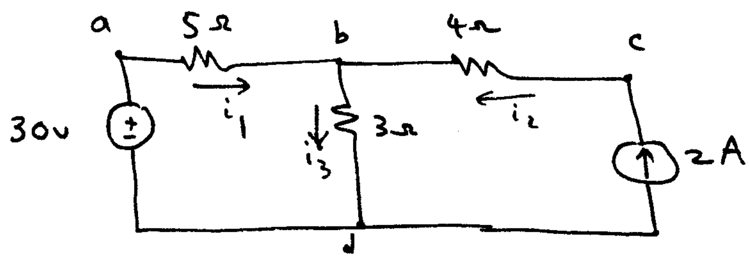
$V_3 = 3I_2 = 3$ V.

E10 / 54 :



$v_{3\Omega} = ?$ $\quad$ $v_{2A} = ?$

(a) Element Currents :



KCL b : $i_1 + \overset{2}{\cancel{i_2}} = i_3 \Rightarrow i_1 = i_3 - 2$

KVL (abd) : $-30 + 5i_1 + 3i_3 = 0$

$\Rightarrow -30 + 5(i_3 - 2) + 3i_3 = 0$

$8i_3 = 40 \Rightarrow \underline{i_3 = 5 \text{ A}}$

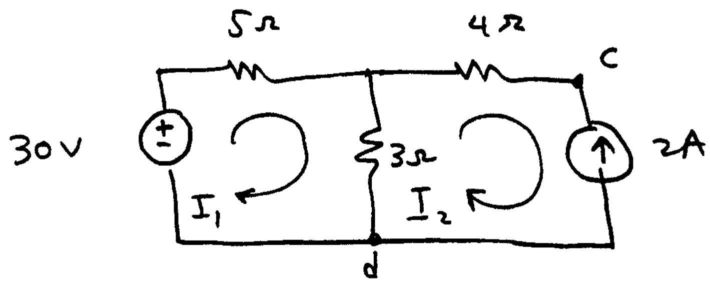
$v_{3\Omega} = (3)(5) = 15$ V.

KVL (bcd) : $V_{bc} + \overset{v_{2A}}{\cancel{V_{cd}}} + V_{db} = 0$

$\therefore \quad V_{2A} = \underbrace{-V_{db}}_{V_{bd}} \underbrace{-V_{bc}}_{V_{cb}}$

$= V_{bd} + V_{cb} = 15 + 4\overset{2}{\cancel{i_2}} = \underline{23 \text{ V}}$

(b) Loop Currents :



KVL : $\begin{cases} -30 + 5I_1 + 3(I_1 - I_2) = 0. & (1) \\ 4I_2 + v_{cd} + 3(I_2 - I_1) = 0. & (2) \end{cases}$

But $I_2 = -2\,A$

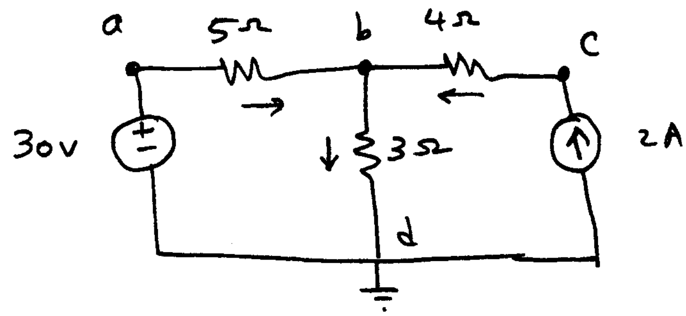
$\therefore \quad 8I_1 - 3\cancel{I_2}^{-2} = 30 \Rightarrow I_1 = \underline{3\,A}$

$v_{3\Omega} = 3(\cancel{I_1}^{3} - \cancel{I_2}^{-2}) = \underline{15\ \text{V}}$

(2) $\Rightarrow \quad v_{cd} = -4\cdot(-2) - 3(-2-3) = 8 + 15 = 23$

$\therefore \quad v_{2A} = \underline{23\ \text{V}}$

(c) <u>Node voltages</u> :



KCL b : $\frac{30 - V_b}{5} + 2 - \frac{V_b}{3} = 0.$

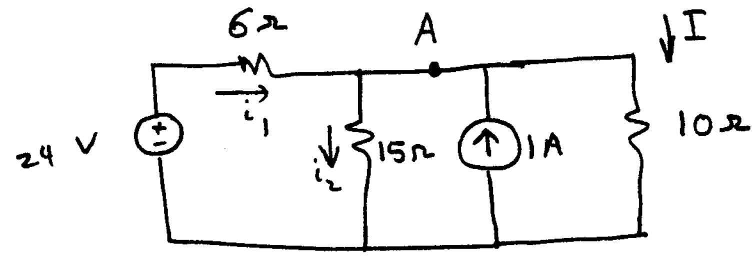
$v_b = v_{3\Omega} = \underline{15\ \Omega}$

KVL L_2 : $\cancel{V_{cb}}^{\,8} + \cancel{V_{bd}}^{\,15} + V_{dc} = 0$

$\therefore \quad v_{cd} = \underline{23\ V}$

$\frac{E11}{54}$: $I = ?$

(a) <u>Element currents</u>:



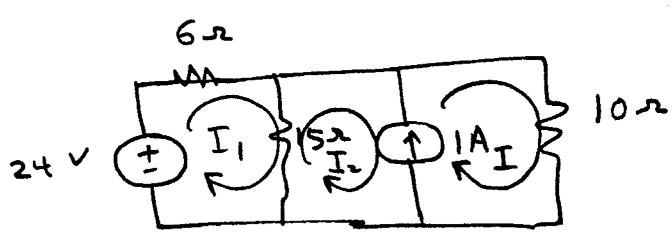
KCL A : $i_1 + 1 - i_2 - I = 0$

KVL outside Loop : $-24 + 6 i_1 + 10 I = 0$

Also : $15 i_2 = 10 I$

$$\underline{I = 1.5 \text{ A}}$$

(b) <u>Loop currents</u>:



KVL :

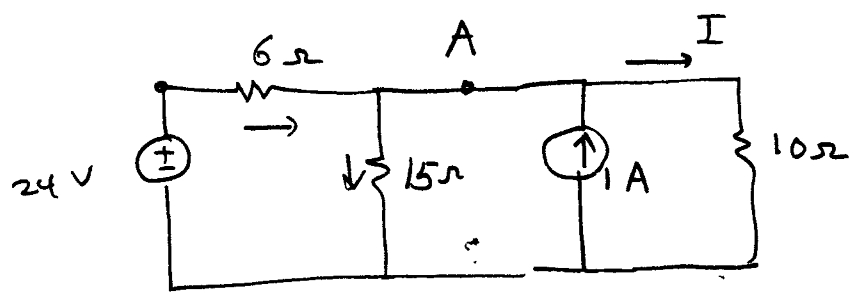
$$\begin{cases} -24 + 6 I_1 + 15(I_1 - I_2) = 0 \\ -24 + 6 I_1 + 10 I = 0 \\ I_2 + 1 = I \end{cases}$$

KVL outside loop :

Also :
(For the branch containing the current source)

$I = \underline{1.5 \text{ A}}$.

(c) <u>Node voltages</u> :



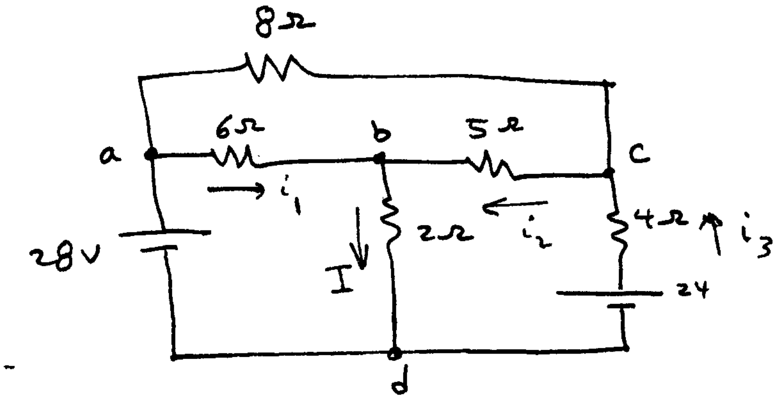
KCL A :

$$\frac{24 - V_A}{6} + 1 - \frac{V_A}{15} - \frac{V_A}{10} = 0$$

$V_A = 1\underline{5 \text{ V}}$: $\Rightarrow$ $I = \frac{V_A}{10} = \underline{1.5 \text{ A}}$

$\frac{E12}{54}$: $I_{2\Omega} = ?$

(a) element currents :



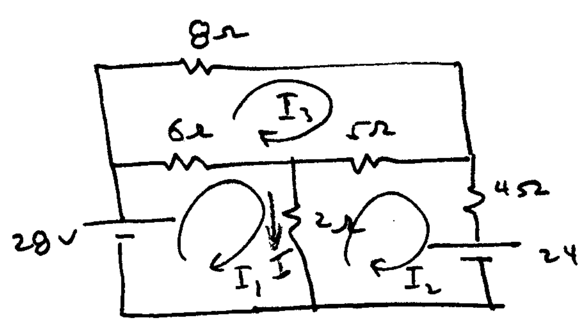
$$
\begin{cases}
-28 + 6i_1 + 2I = 0 \\
-24 + 4i_3 + 5i_2 + 2I = 0 \\
i_1 + i_2 - I = 0
\end{cases}
$$

KVL :

KCL :

$\underline{I = 5\ A}$

(b) Loop currents :



KVL: $-28 + 6(I_1 - I_3) + 2(I_1 - I_2) = 0$

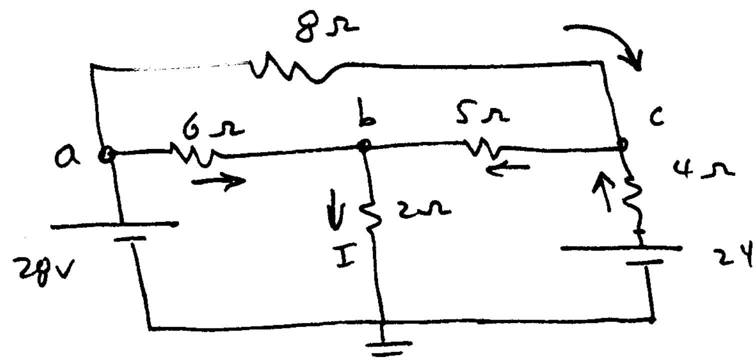
$$2(I_2 - I_1) + 5(I_2 - I_3) + 4I_2 + 24 = 0$$

$$8I_3 + 5(I_3 - I_2) + 6(I_3 - I_1) = 0$$

$I_1 = 4$ $\quad I_2 = -1$ $\quad I_3 =$

$\underline{I} = I_1 - I_2 = \underline{5A}$

(c) <u>Node voltages</u>:



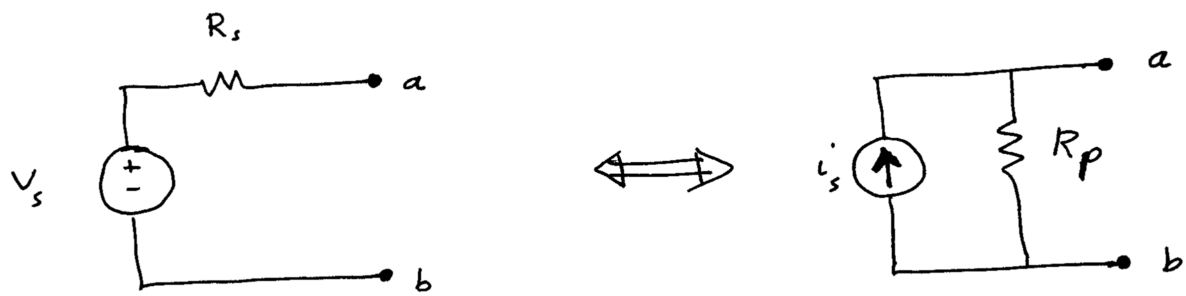
KCL b: $\dfrac{28 - V_b}{6} + \dfrac{V_c - V_b}{5} - \dfrac{V_b}{2} = 0$

KCL c: $\dfrac{24 - V_c}{4} + \dfrac{28 - V_c}{8} - \dfrac{V_c - V_b}{5} = 0$

$V_b = 10\,V \Rightarrow I = \frac{10}{2} = \underline{5\ A}$

Source Transformation

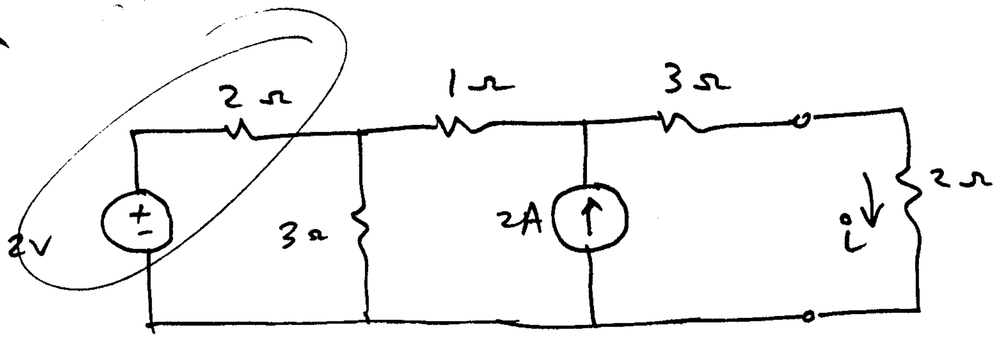
A source transformation shown below allows us to replace a voltage source in series with a resistor by a current source in parallel with the same resistor, or vice versa.

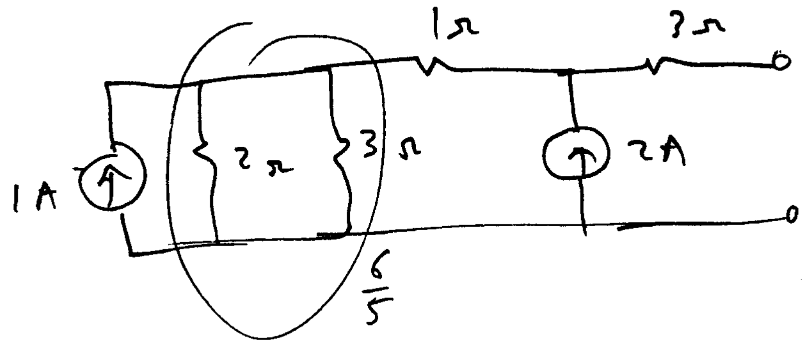


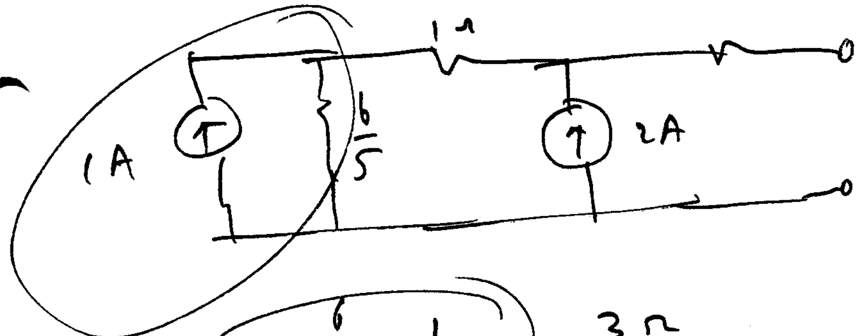
$$i_s = \frac{V_s}{R_s}$$

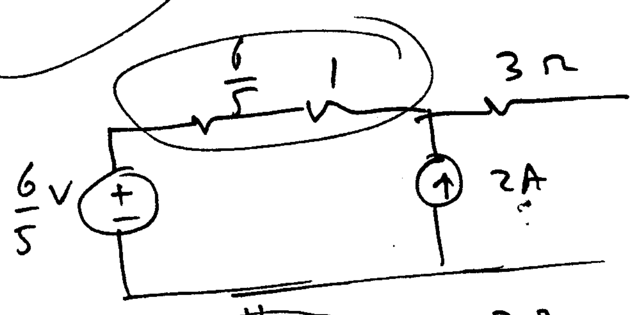
$$R_s = R_p .$$

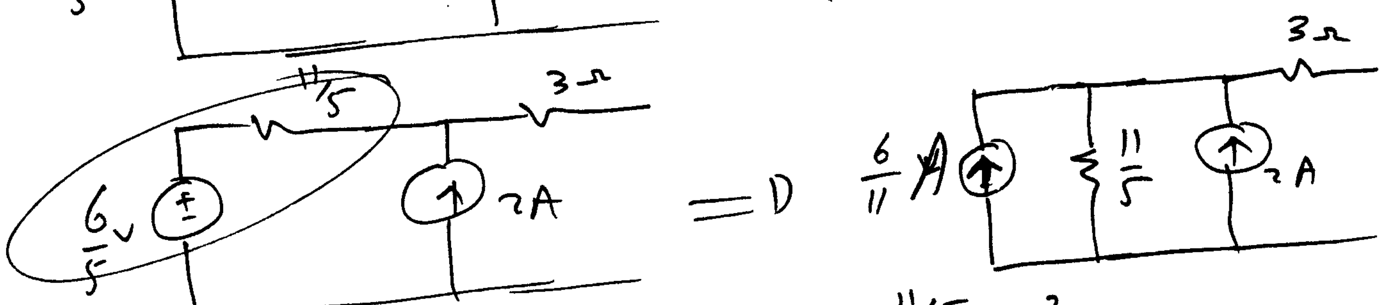
Example use source trans to simplify the given circuit.

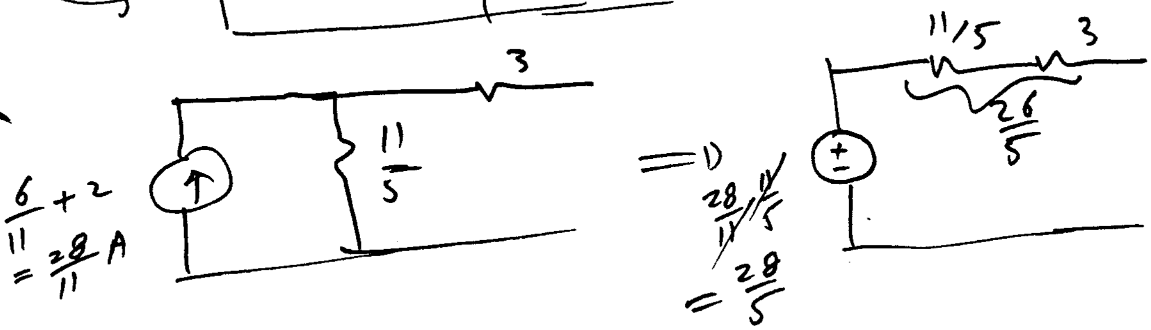






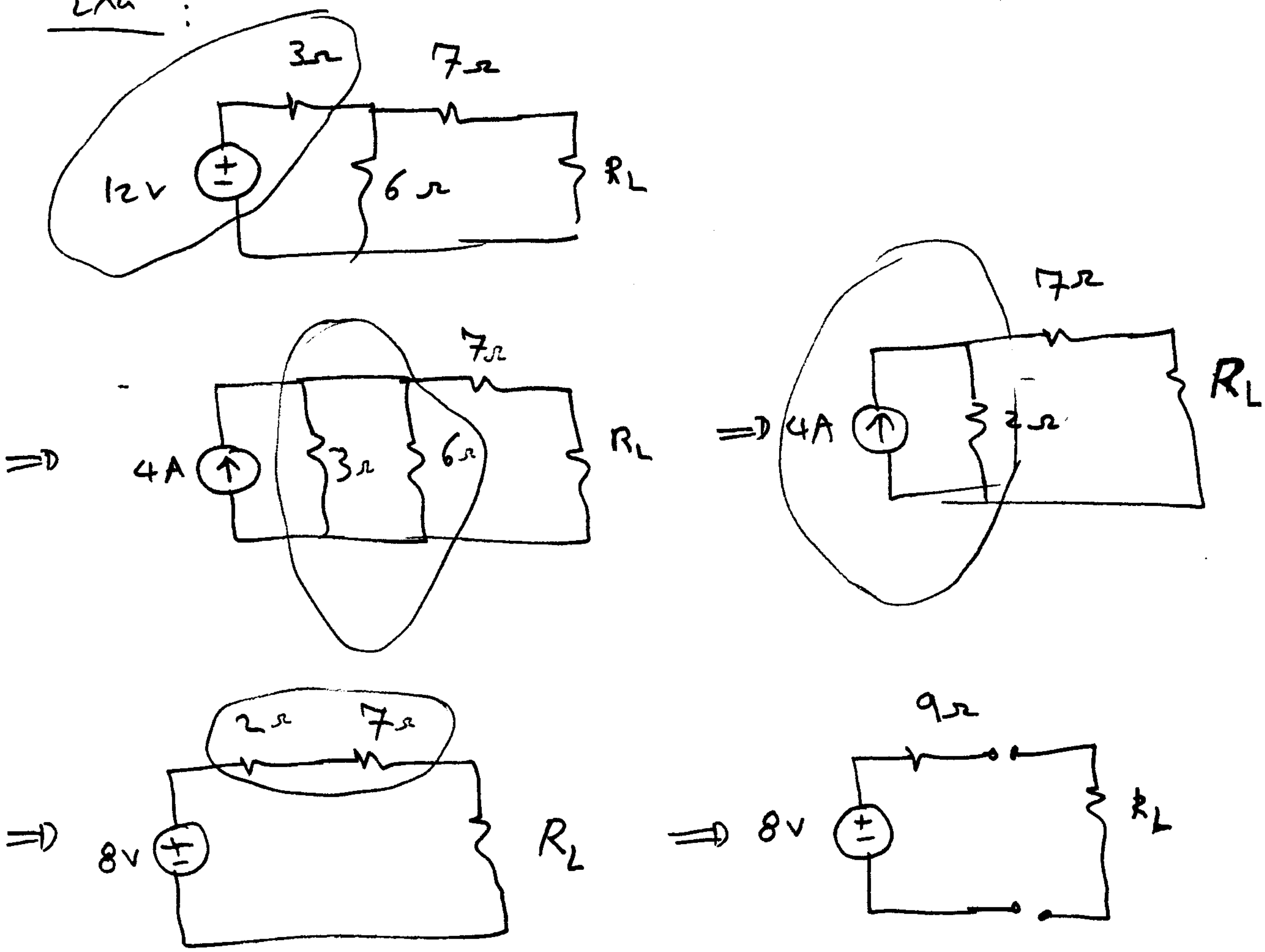




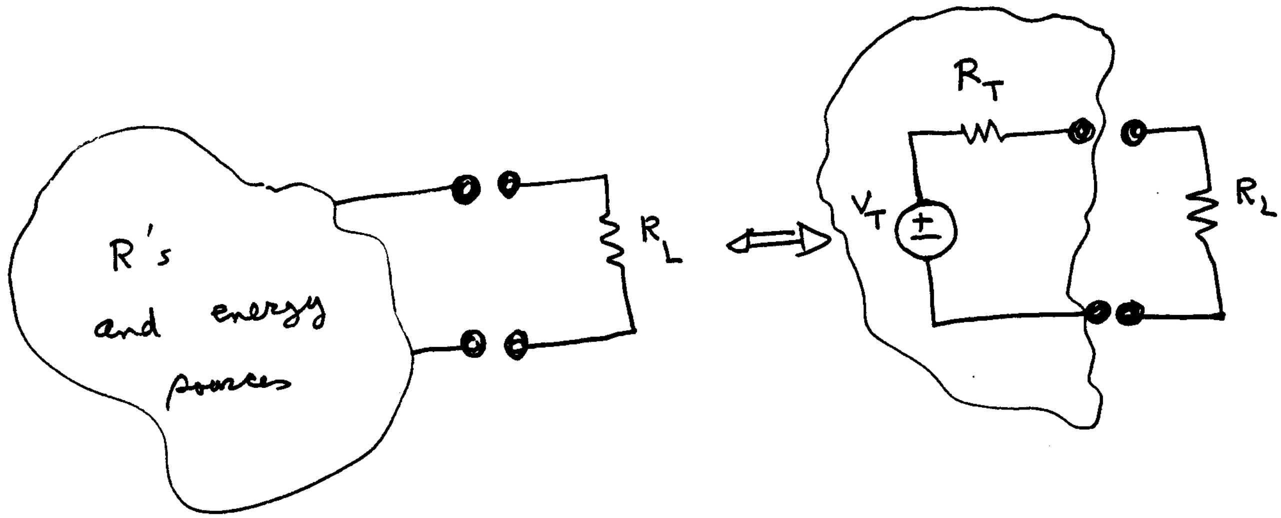


The source transformation principle can be very useful in network analysis.

Exa:



Thevenin's Theorem

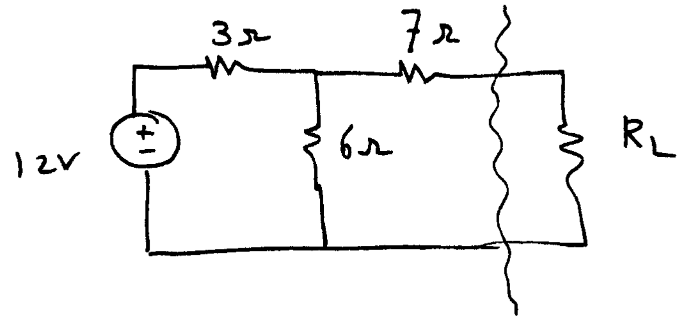


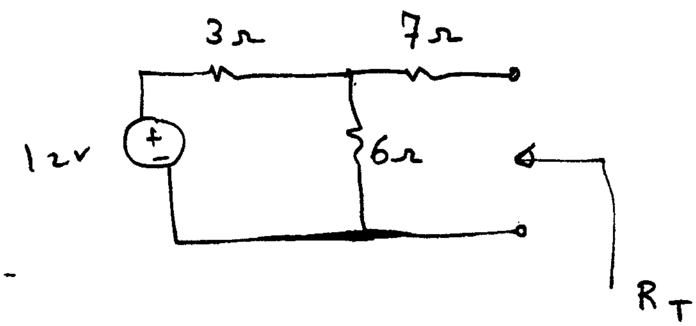
where $V_T = V_{oc}$

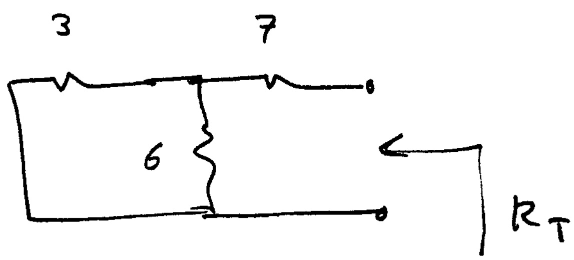
$$R_T = \frac{V_T}{I_{sc}} = \frac{V_{oc}}{I_{sc}}$$

OR R_T is the "resistance seen by looking in" at the terminals with all voltage sources short-circuited and current sources open-circuited

Exa :



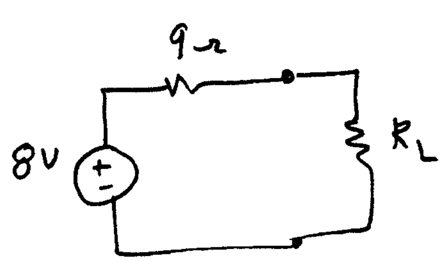




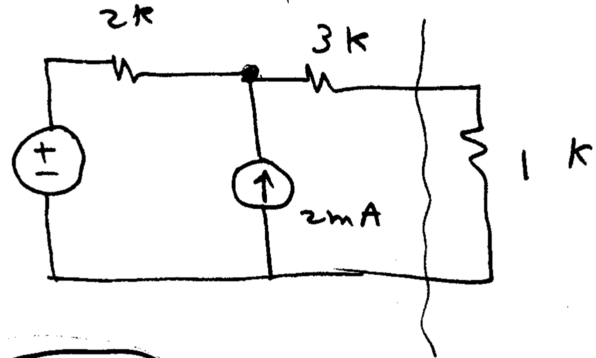
$$R_T = 3 // 6 + 7 = \frac{18}{9} + 7 = 9\,\Omega$$

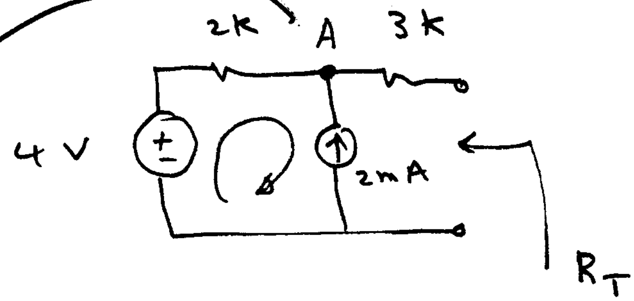
$$V_T = V_{6\Omega} = \frac{6}{3+6}\,12 = \frac{\cancel{6}^{\,2}}{\cancel{9}_{\,3}}\cancel{(12)}^{\,4} = 8\ \text{V}$$

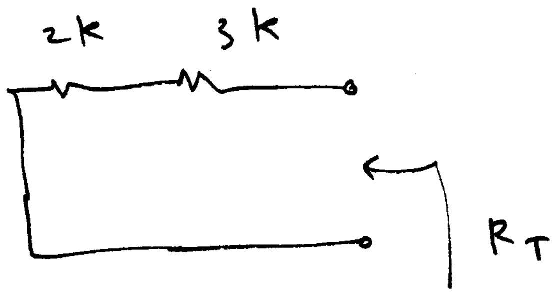
$\therefore$



Exa :







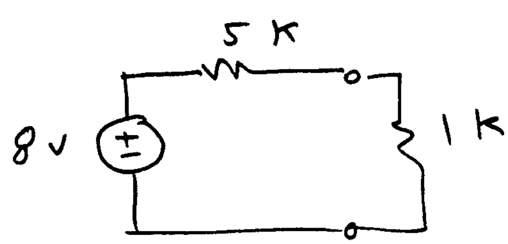
$R_T = 5$ k

KCL A : $\dfrac{4 - V_A}{2} + 2 = 0$

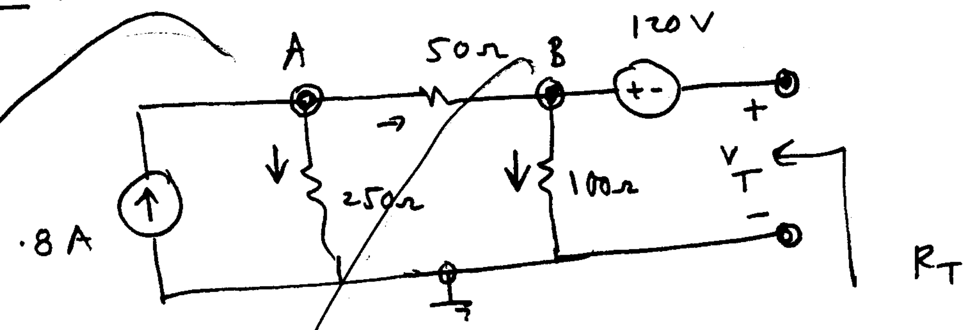
$4 - V_A + 4 = 0 \Rightarrow V_A = 8$

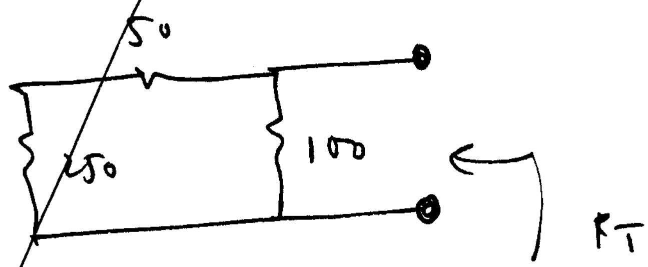
$\therefore \; V_T = 8$ V

$\therefore$



Exa :





$$R_T = (250+50) \,//\, 100 = \frac{300 \times 100}{\underbrace{300+100}_{400}} = \underline{\underline{75\,\Omega}}.$$

KCL A :

$$\left\{\begin{array}{l} .8 = \dfrac{V_A}{250} + \dfrac{V_A - V_B}{50} \Rightarrow 200 = \cancel{V_A} + \cancel{5V_A} - 5V_B \\ \text{KCL B: } \dfrac{V_A - V_B}{50} = \dfrac{V_B}{100} \Rightarrow 2V_A - 2V_B = V_B \Rightarrow 3V_B = 2V_A \quad (2) \end{array}\right.$$

$$(1) \Rightarrow \left\{\begin{array}{l} 6V_A - 5V_B = 200 \\ 3V_B = 2V_A \end{array}\right. \Rightarrow \left(\frac{3}{2}V_B\right)6 - 5V_B = 200$$

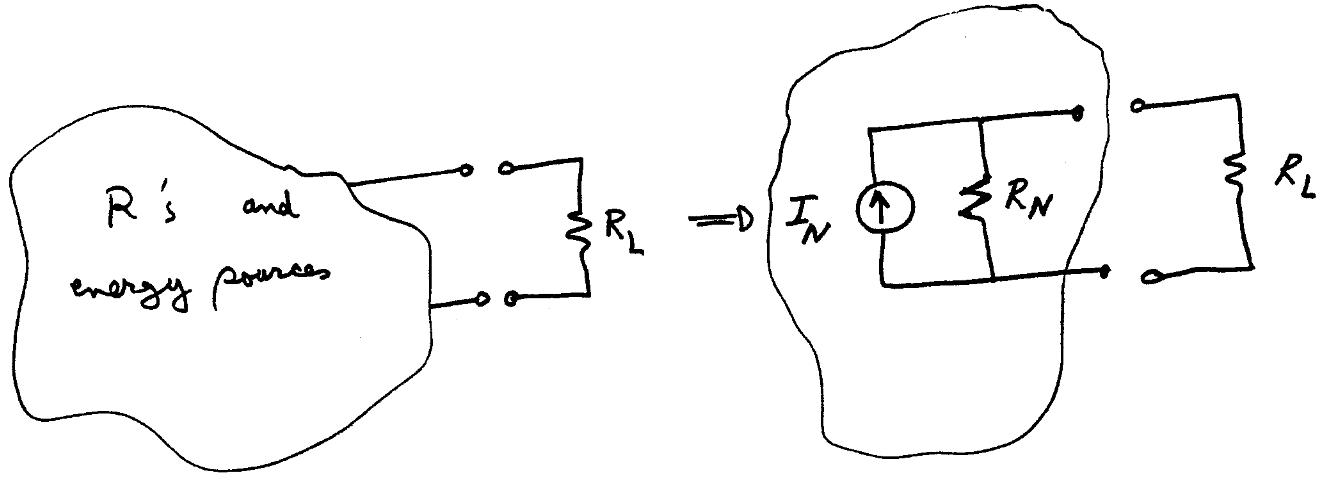
$$4V_B = 200$$

$$\underline{V_B = 50 \text{ V}}$$

$$-\overset{50}{\cancel{V_B}} + 120 + V_T = 0$$

$$\underline{\underline{V_T = -70 \text{ V}}}$$

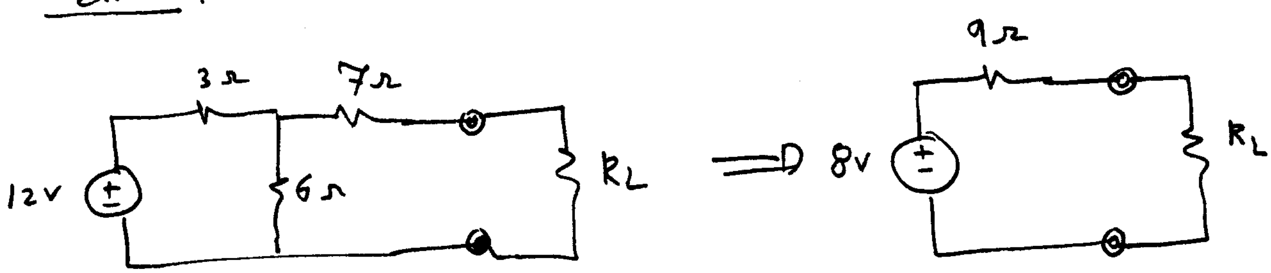
NorThon's Theorem



$$I_N = \frac{V_T}{R_T} \quad , \quad R_N = R_T$$

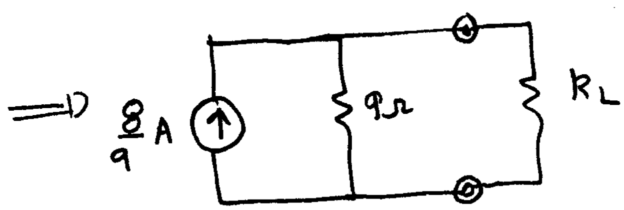
It is a "source transformation" principle.

Exa: From previous example we got:



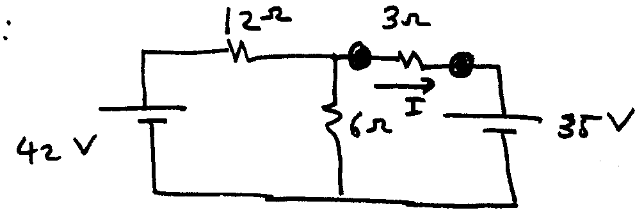
Thovenin

So:

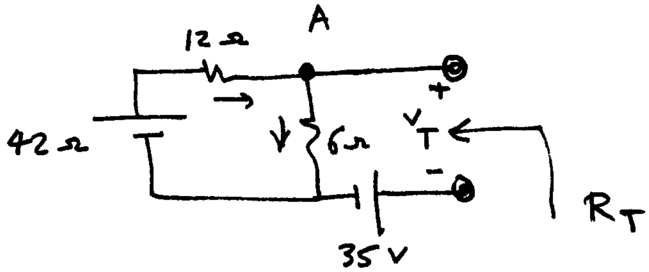


NorThon

E25 / 55 :



$I = ?$

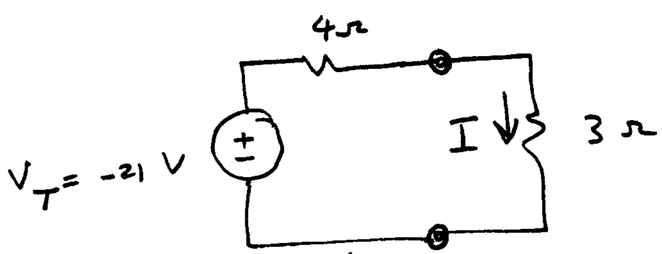


$$R_T = 12 // 6 = \frac{(12)(6)}{12+6} = 4\,\Omega$$

KCL A:
$$\frac{42 - V_A}{\cancel{12}_{\,2}} = \frac{V_A}{\cancel{6}}$$

$$42 - V_A = 2V_A \Rightarrow V_A = 14\ \text{V}$$

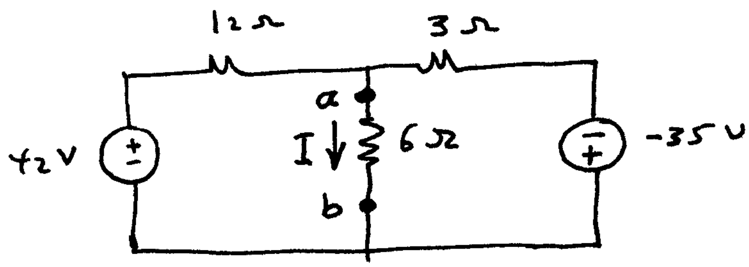
KVL:
$$35 - \overset{14}{\cancel{V_A}} + V_T = 0 \Rightarrow V_T = -21\ \text{V}$$



$$-V_T + (4+3)\,I = 0$$
$$21 + 7I = 0 \Rightarrow \underline{I = -3\ \text{A}}$$

Q2 ENGR 212 Name — Key —

Consider the following circuit:

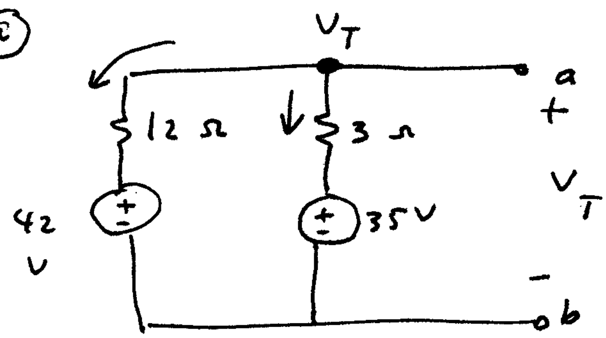


a. Find the Thevenin's equivalent circuit across a and b.

b. Using part a, find I.

Sol$^{\underline{n}}$:

ⓐ

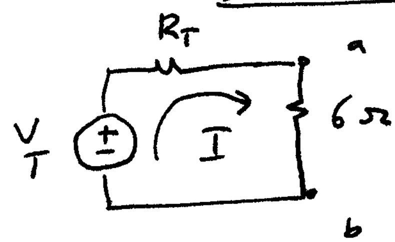


$$\boxed{R_T = 3 // 12 = 2.4\ \Omega}$$

$$\frac{V_T - 42}{12} + \frac{V_T - 35}{3} = 0 \implies \boxed{V_T = 36.4\ V}$$

$$V_T - 42 + 4V_T - 140 = 0$$

$$5V_T = 182$$

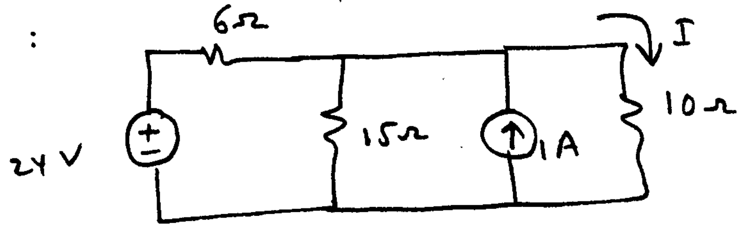


ⓑ

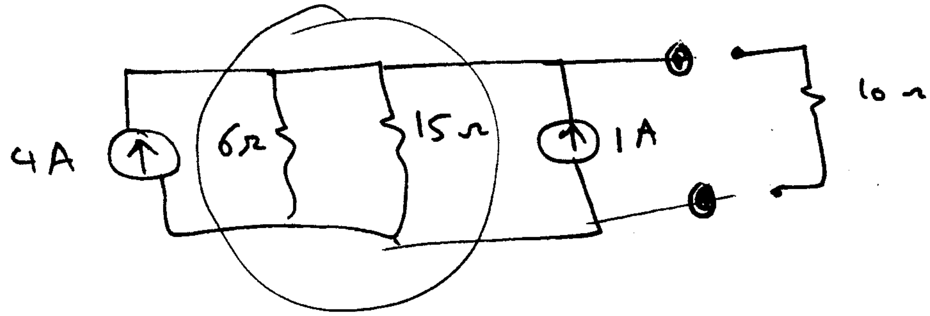
$$-\overset{-36.4}{\cancel{V_T}} + \overset{2.4}{\cancel{R_T}} I + 6I = 0$$

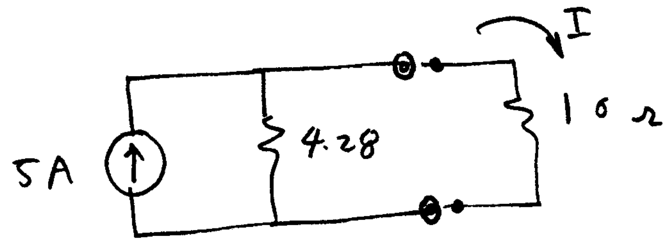
$$8.4 I = 36.4 \implies \boxed{I = 4.33\ A}$$

$\frac{E\,27}{56}$:



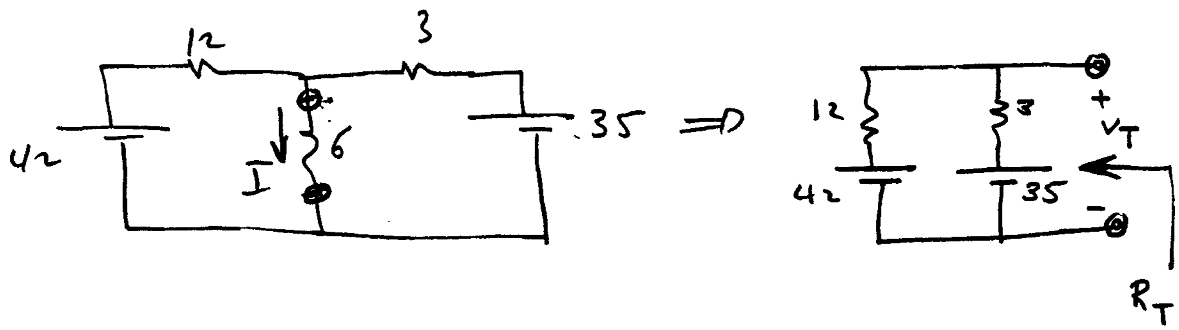
$I = ?$



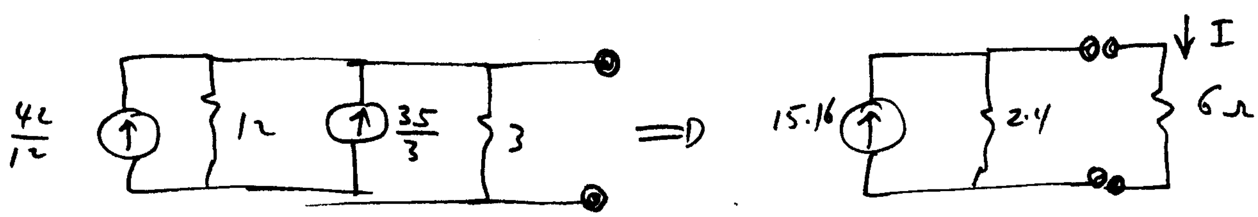


$$I = \frac{4.28}{4.28+10}(5) = \underline{1.5\ A}$$

$\frac{E\,28}{56}$:



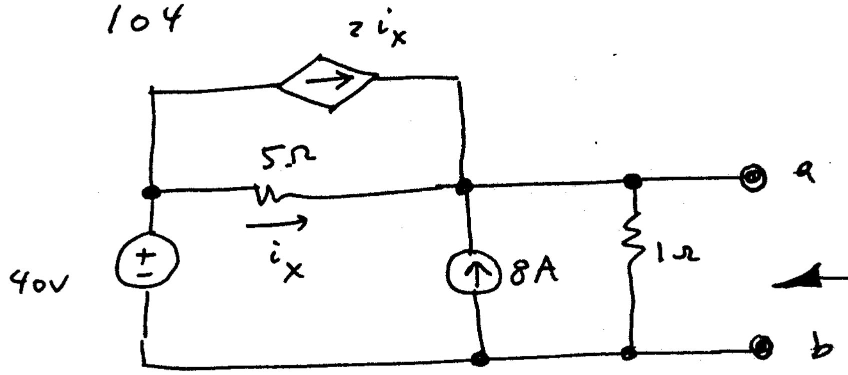
$$R_T = 3 /\!/ 12 = \frac{\cdot\cdot}{\cdot\cdot} = 2.4\ \Omega$$



$$I = \frac{2.4}{2.4+6}(15.16) = \underline{4.33\ A}$$

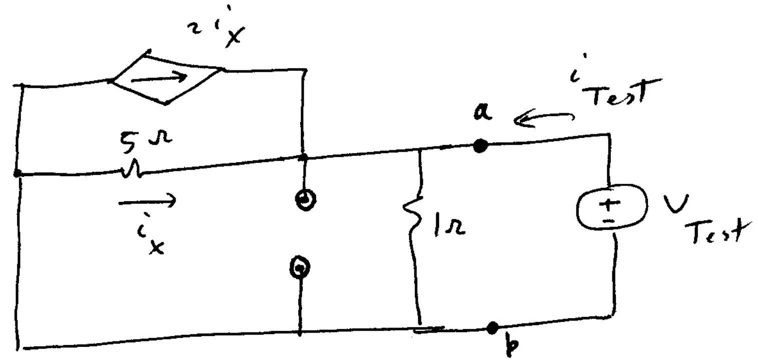
Thevenin theorem — Dependent sources.

$\frac{D\,4.22}{104}$:

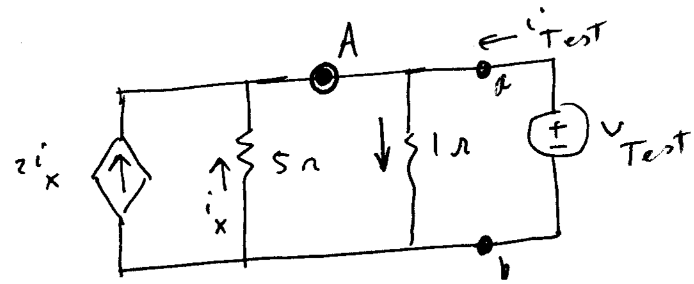


$V_T, R_T = ?$

To find R_T, we first deactivate the indep sources and then excite the circuit from a, b with either a test voltage source or a test current source.



$\Rightarrow R_T = \frac{V_{Test}}{i_{Test}}$



KCL A :

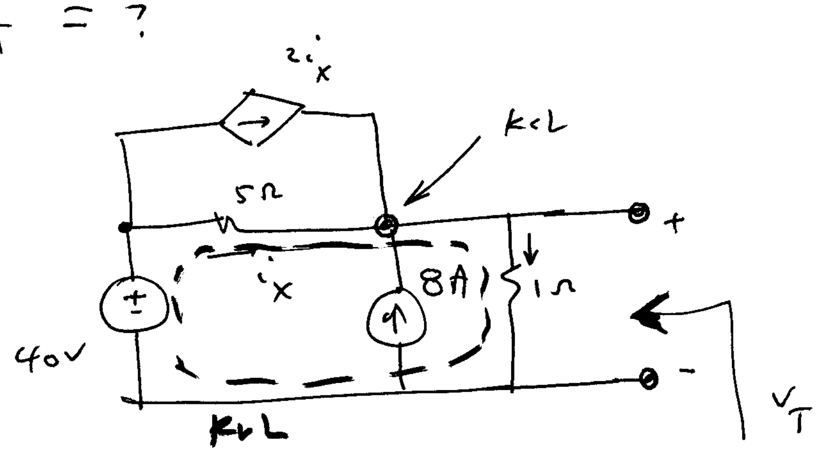
$$2i_x + i_x - \frac{V_{Test}}{1} + i_{Test} = 0$$

Also, $$i_x = -\frac{V_{Test}}{5}$$

$$\therefore \quad -\frac{2}{5}V_{Test} - \frac{1}{5}V_{Test} - V_{Test} + i_{Test} = 0$$

$$+\frac{8}{5}V_{Test} = i_{Test} \Rightarrow \frac{V_{Test}}{i_{Test}} = \frac{5}{8} = \underbrace{0.625\,\Omega}_{R_T}$$

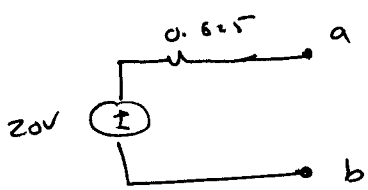
$V_T = ?$



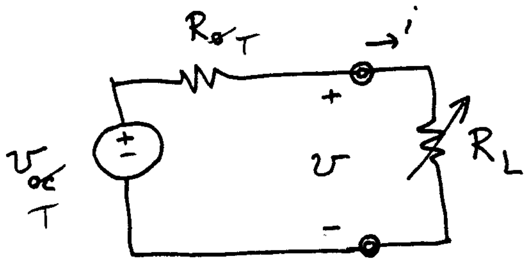
$$\begin{cases} 2i_x + i_x + 8 = \frac{V_T}{1} \\ -40 + 5i_x + V_T = 0 \end{cases} \Rightarrow \begin{aligned} 3i_x - V_T &= -8 \\ 5i_x + V_T &= 40 \end{aligned}$$

$$8i_x = 32 \Rightarrow \underline{i_x = 4}$$

$$\therefore \quad \underline{V_T = 20\text{ V}}$$



Maximum power Transfer.



wish To determine R_L that will absorb the max power.

$$P = i^2 R_L = \left(\frac{v_{oc_T}}{R_{oc_T}+R_L}\right)^2 R_L = \frac{v_{oc_T}^2 R_L}{(R_{oc_T}+R_L)^2}$$

Find $\frac{dP}{dR_L} = 0. \Longrightarrow R_L = R_{oc_T}.$ ← optimum Load

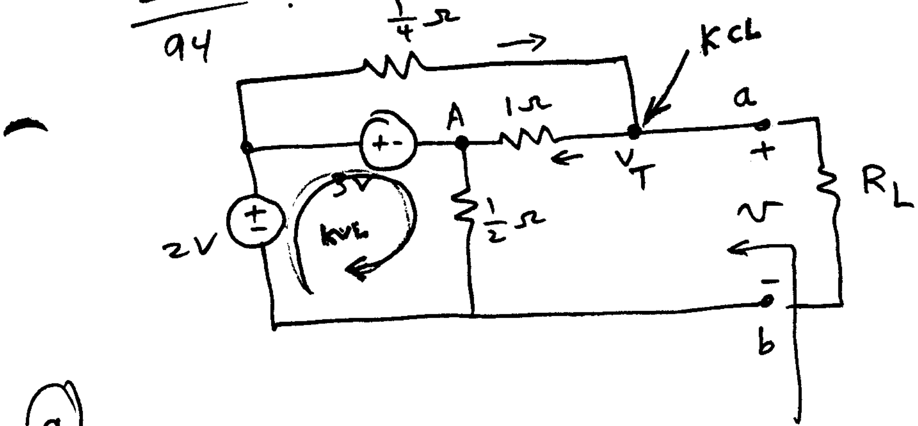
and $$P_{max} = \frac{v_{oc_T}^2 R_L}{(2R_L)^2} = \frac{v_{oc_T}^2}{4R_L} = \frac{v_{oc_T}^2}{4R_{oc_T}}.$$

$$\frac{dP}{dR_L} = \frac{v_T^2(R_T+R_L)^2 - 2(R_T+R_L)v_T^2 R_L}{(\quad)^4} = 0.$$

$$\therefore\ v_T^2(R_T+R_L)\left[(R_T+R_L) - 2R_L\right] = 0.$$

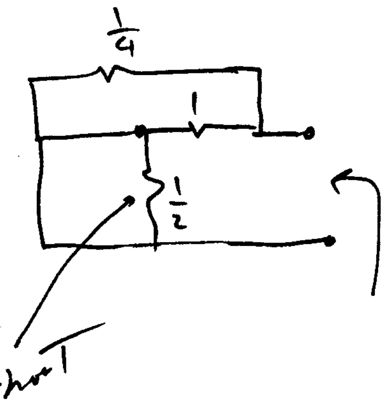
$$R_T - R_L = 0 \Longrightarrow \boxed{R_L = R_T}.$$

$\frac{2.31}{94}$:



(a)

$R_T = ?$



$R_T = 1 // \frac{1}{4} = \frac{1}{5}$

$V_T = ?$

KVL: $-2 + 3 + V_A = 0. \Rightarrow V_A = -1$ V.

KCL: $\frac{2 - V_T}{1/4} = \frac{V_T - \cancel{V_A}^{-1}}{1} \Rightarrow \underline{V_T = \frac{7}{5} \text{ V}}$

(b)

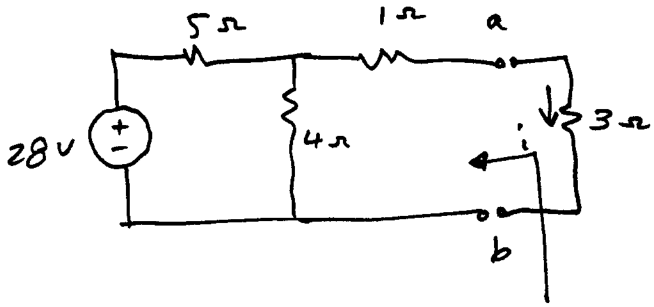
$\Rightarrow V = \frac{1/3}{1/3 + \frac{1}{5}} \left(\frac{7}{5}\right) = \frac{7}{8}$ V.

(c) $R_L = \frac{1}{5} \leftarrow$ optimum load

(d) $P = \frac{V_T^2}{4 R_T} = \frac{(7/5)^2}{4(\frac{1}{5})} = \frac{49}{20}$ W. $= \underline{2.45 \text{ W}}$.

2.29 (Bobrow)

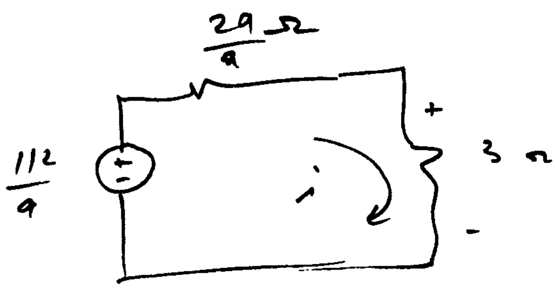
93



a) $R_T = 5 /\!/ 4 + 1 = \frac{20}{9} + 1 = \frac{20+9}{9} = \frac{29}{9}\ \Omega$

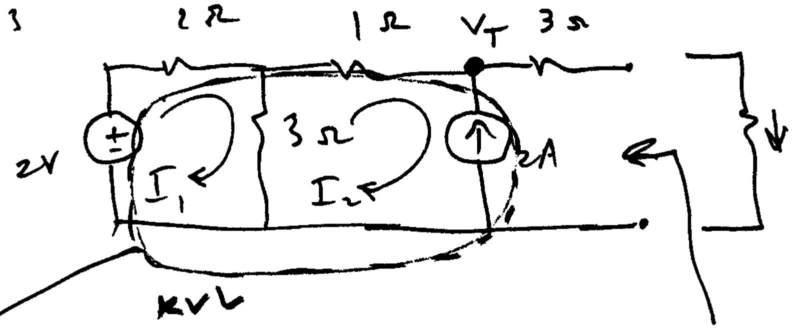
$v_T = \frac{4}{4+5}(28) = \frac{112}{9}$ V.

b)



$-\frac{112}{9} + \left(\frac{29}{9} + 3\right) i = 0. \Rightarrow i = 2A.$

$\frac{2.30}{93}$:



$$R_T = 2 /\!/ 3 + 1 + 3 = \frac{2\cdot 3}{2+3} + 4 = \frac{6}{5} + 4 = \frac{26}{5}.$$

$V_T = ?$

$$-2 + 2I_1 + 3(I_1 - \cancel{I_2}^{\,-2}) = 0 \Rightarrow I_1 = -\frac{4}{5}\text{ A}.$$

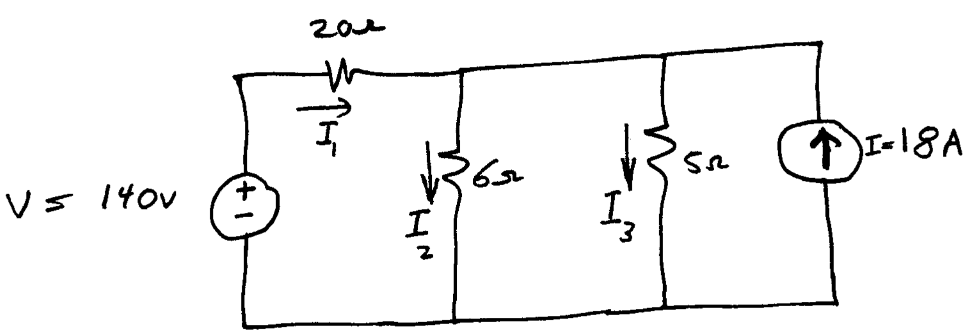
$$-2 + 2\cancel{I_1}^{\,-\frac{4}{5}} + \cancel{I_2}^{\,-2} + V_T = 0 \Rightarrow V_T = \frac{28}{5}\text{ V}.$$

Principle of Superposition

In any electric network, the element voltages and currents are the effects produced by the applied sources, which are regarded as the causes. If superposition exits between cause and effect it means:

$$\overset{\text{effect}}{y} = f(\overset{\text{cause}}{x}) \implies \underbrace{f(x_1)}_{y_1} + \underbrace{f(x_2)}_{y_2} = f(x_1 + x_2)$$

Exa: Use the principle of superposition to find I_1, I_2, I_3

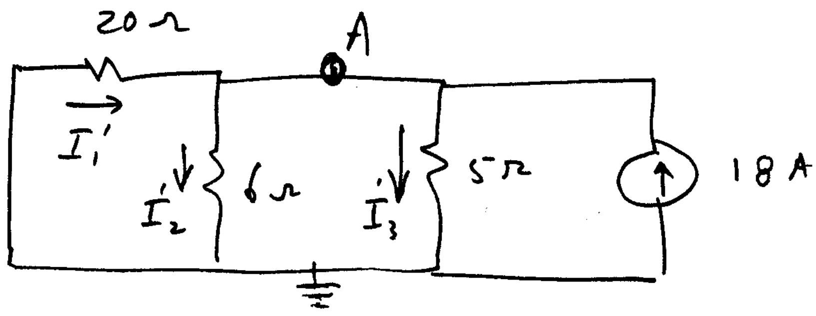


First, let $V = 0 \Rightarrow I_1'$, I_2', I_3'

Second, let $I = 0 \Rightarrow I_1''$, I_2'', I_3''

$\therefore \quad I_1 = I_1' + I_1''$, $I_2 = I_2' + I_2''$, $I_3 = I_3' + I_3''$

(1)



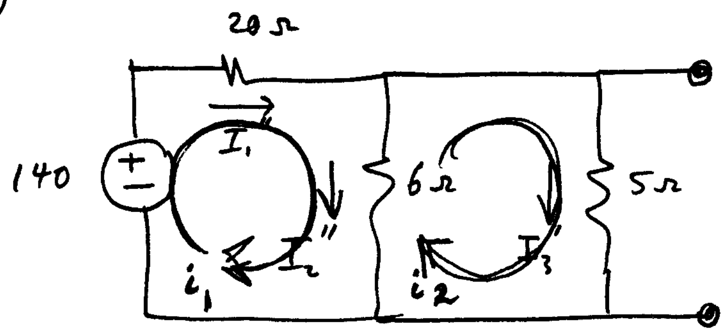
KCL A : (node voltages)

$$\overbrace{\frac{V_A}{6}}^{I_2'} + \overbrace{\frac{V_A}{5}}^{I_3'} = \overbrace{-\frac{V_A}{20}}^{I_1'} + 18$$

$V_A = 43.2$ V

$I_1' = \underline{-2.16 \text{ A}}$, $I_2' = \underline{7.20 \text{ A}}$, $I_3' = \underline{8.64 \text{ A}}$.

(2)



$$
\begin{cases}
-140 + 20 i_1 + 6(i_1 - i_2) = 0 \\
6(i_2 - i_1) + 5 i_2 = 0
\end{cases}
$$

$i_1 = 6.16\ A$, $i_2 = 3.36\ A$

$\therefore$ $i_1 \nearrow$ $I_1'' = \underline{6.16}$

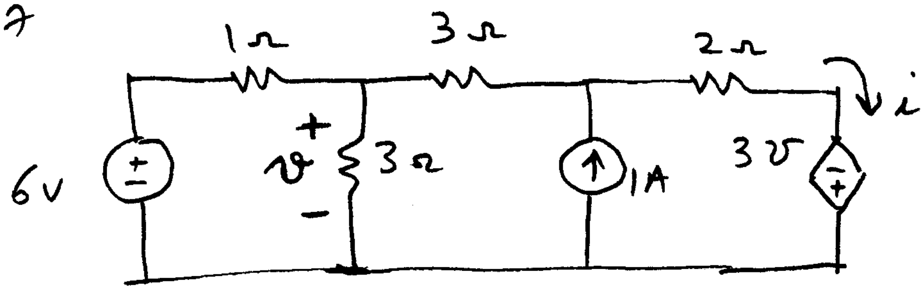
$i_1 - i_2 \swarrow$ $I_2'' = i_1 - i_2 = \underline{2.80\ A}$

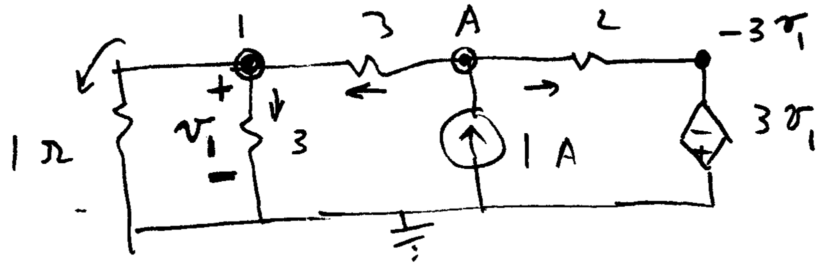
$i_2 \swarrow$ $I_3'' = \underline{3.36 A}$

$$
\therefore
\begin{cases}
I_1 = 4.\ A \\
I_2 = 10\ A \\
I_3 = 12 A
\end{cases}
$$

2.49 :

97



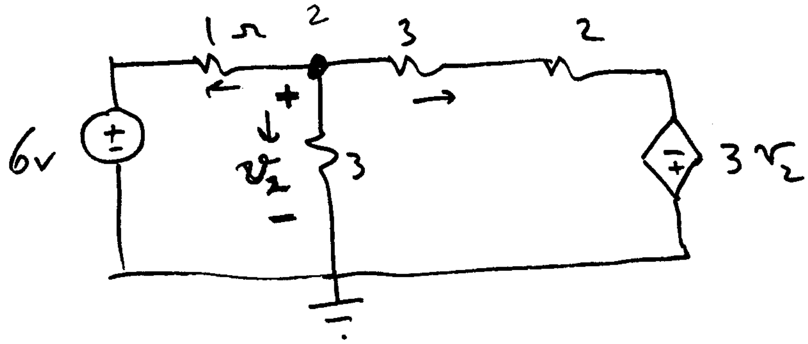


KCL 1: $\dfrac{v_1}{1} + \dfrac{v_1}{3} = \dfrac{v_A - v_1}{3}$ $\Rightarrow 5v_1 = v_A.$

KCL A: $\dfrac{v_A - v_1}{3} + \dfrac{v_A - (-3v_1)}{2} = 1$

$2v_A - 2v_1 + 3v_A + 9v_1 = 6 \Rightarrow 5v_A + 7v_1 = 6.$

$\therefore \quad v_1 = \dfrac{3}{16}$ v.



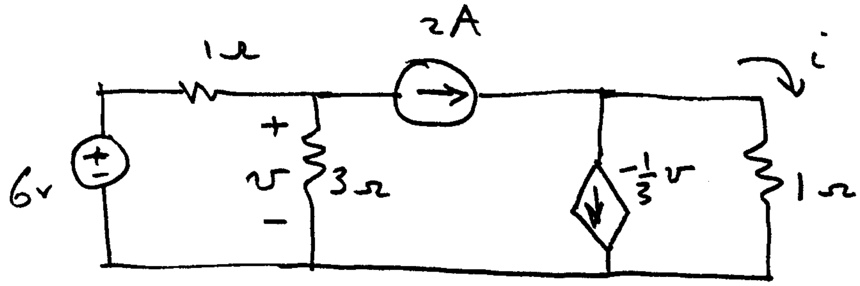
$$\frac{v_2 - 6}{1} + \frac{v_2}{3} + \frac{V_2 - (-3v_2)}{5} = 0$$

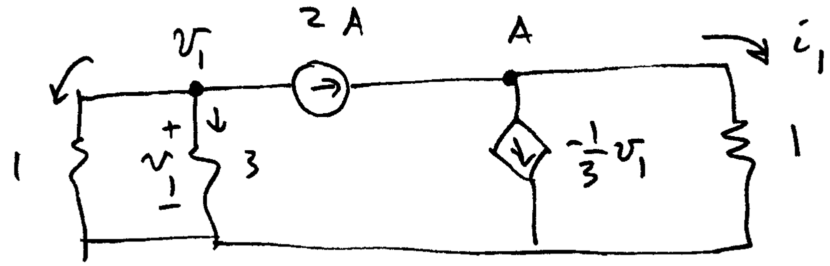
$$\Rightarrow \quad v_2 = \frac{45}{16} \text{ . V.}$$

$$\therefore \quad v = v_1 + v_2 = \frac{3}{16} + \frac{45}{16} = \underline{3\text{ V}}.$$

#2.50 :

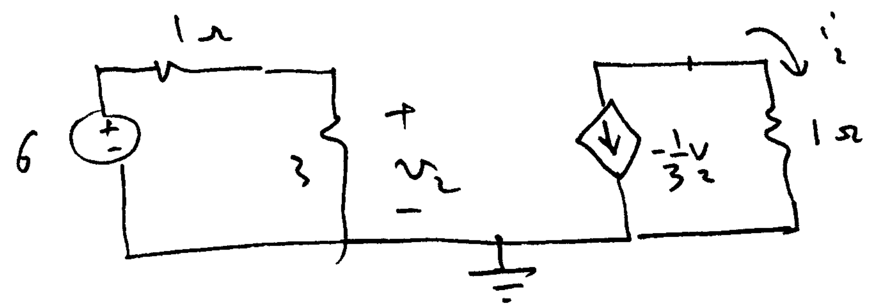
98





$$\frac{v_1}{1} + \frac{v_1}{3} + 2 = 0. \quad \Rightarrow \quad v_1 = -\tfrac{3}{2}\ \text{V}.$$

KCL A: $$2 = -\frac{1}{3}\cancel{v_1}^{-\frac{3}{2}} + i_1 \Rightarrow i_1 = 2 + \frac{1}{2} = \frac{3}{2}\ \text{A}.$$



voltage division :

$$v_2 = \frac{3}{1+3} 6 = \frac{9}{2} \text{ V.}$$

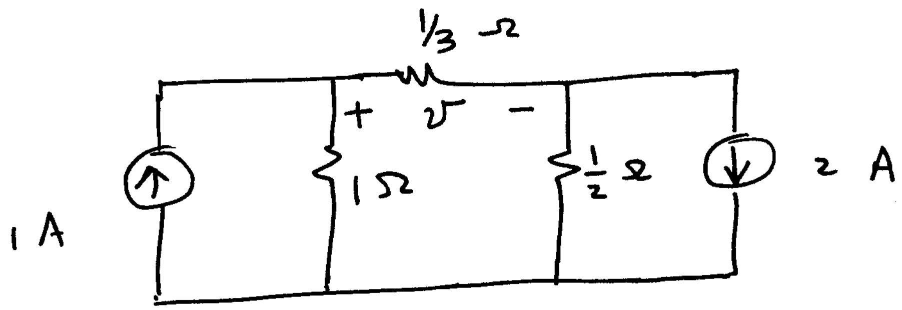
also :

$$i_2 = -\left(-\frac{1}{3} v_2\right) = \frac{v_2}{3} = \frac{9}{2} \cdot \frac{1}{3} = \frac{3}{2} \text{ A.}$$

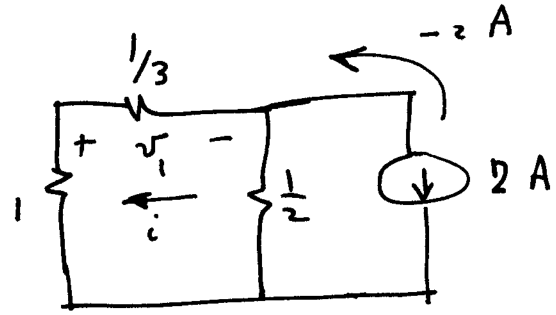
$$\therefore \quad v = v_1 + v_2 = -\frac{3}{2} + \frac{9}{2} = 3 \text{ V.}$$

$$i = i_1 + i_2 = \frac{3}{2} + \frac{3}{2} = 3 \text{ A.}$$

Exa: Find v using superposition.



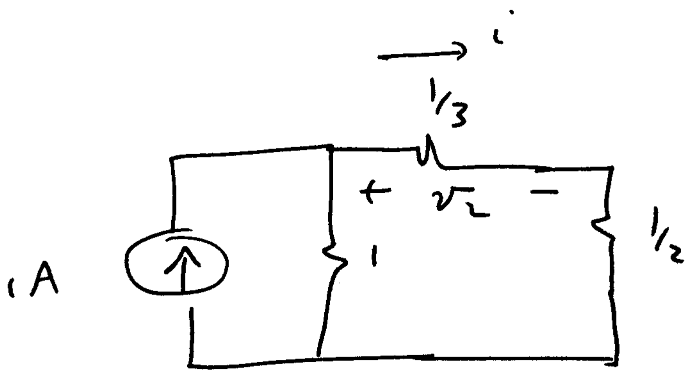
1.



$$i = \frac{\frac{1}{2}}{1 + \frac{1}{3} + \frac{1}{2}} (-2) = -\frac{6}{11} \text{ A}$$

$$v_1 = \left(-\frac{1}{3}\right)\left(-\frac{6}{11}\right) = \frac{2}{11} \text{ V}$$

2.



$$i = \frac{1}{1 + \frac{1}{2} + \frac{1}{3}}(1) = \frac{6}{11} \text{ A}$$

$$\therefore \quad v_2 = \left(\frac{1}{3}\right)\left(\frac{6}{11}\right) = \frac{2}{11} \text{ V}$$

$$\therefore \quad V = v_1 + v_2 = \frac{2}{11} + \frac{2}{11} = \frac{4}{11} \text{ V}$$