

Chapter 4

Techniques of Circuit Analysis

So far we have been able to analyze relatively simple resistive circuits by the use of KVL and KCL in combination with Ohm's law. This approach can be used for all circuits, but as the circuits become more complicated, we will soon find this direct method quite cumbersome. In this chapter, we introduce two powerful methods of circuit analysis: the node-voltage method and the mesh-current method. We have already seen how series-parallel reductions and Δ -to-Y transformations can be used to simplify a given structure. We will now add source transformations and Thevenin-Norton equivalent circuits to our list of simplification techniques.

Recall

Branch: A branch is part of a circuit with two terminals to which connections can be made.

Node: A node is the point where two or more branches come together.

Loop: A loop is a closed path formed by connecting branches.

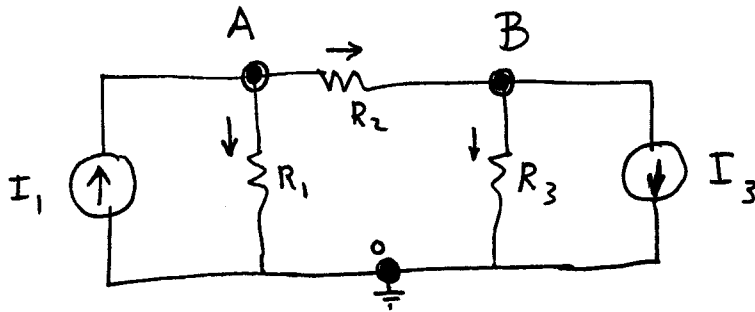
Mesh: A mesh is a special type of loop; that is, it does not contain any other loops within it.

Example

Node-voltage method :

It is a method in which the KVL eqⁿs are written implicitly on the circuit so that only the KCL eq^s need be solved.

Exa :



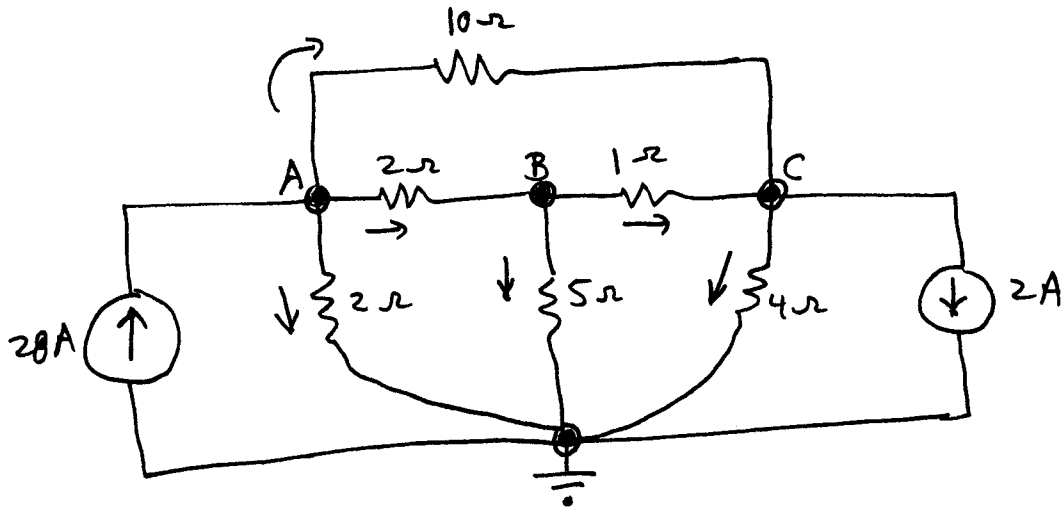
$$\text{KCL A : } I_1 = \frac{V_A}{R_1} + \frac{V_A - V_B}{R_2}$$

$$\text{KCL B : } \frac{V_A - V_B}{R_2} = \frac{V_B}{R_3} + I_3$$

Rearranging the terms gives:

$$\begin{cases} \frac{1}{R_1 + R_2} V_A - \frac{1}{R_2} V_B = I_1 \\ -\frac{1}{R_2} V_A + \frac{1}{R_2 + R_3} V_B = -I_3 \end{cases}$$

Exa: Use the node-voltage method to find the voltages v_A , v_B , v_C .



$$\text{KCL A: } 28 = \frac{v_A}{2} + \frac{v_A - v_B}{2} + \frac{v_A - v_C}{10}$$

$$\text{KCL B: } \frac{v_A - v_B}{2} = \frac{v_B}{5} + \frac{v_B - v_C}{1}$$

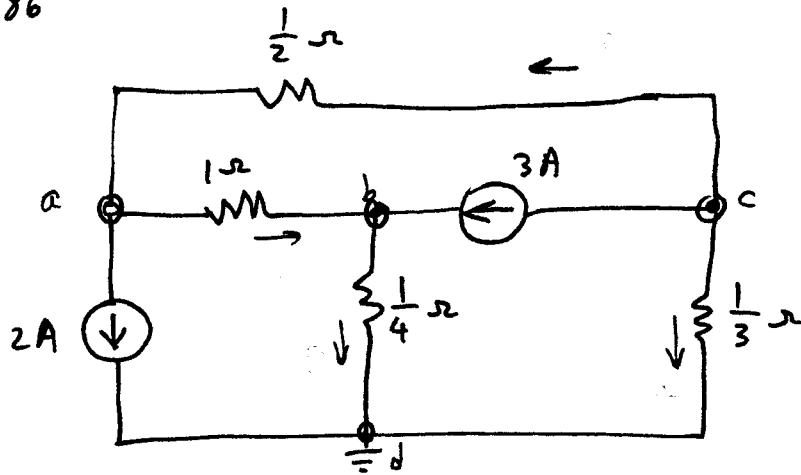
$$\text{KCL C: } \frac{v_B - v_C}{1} = \frac{v_C}{4} + 2 - \frac{v_A - v_C}{10}$$

or

$$\begin{cases} 1.1 v_A - 0.5 v_B - 0.1 v_C = 28 \\ -0.5 v_A + 1.7 v_B - v_C = 0 \\ -0.1 v_A - v_B + 1.35 v_C = -2 \end{cases}$$

$$v_A, v_B, v_C = \dots$$

2.1 : (Bobrow) :
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Nodal Analysis :

KCL a.

$$\frac{V_a - V_b}{1} + 2 = \frac{V_c - V_a}{1/2}$$

$$\therefore 3V_a - V_b - 2V_c = -2.$$

KCL b.

$$\frac{V_a - V_b}{1} + 3 = \frac{V_b}{1/4}$$

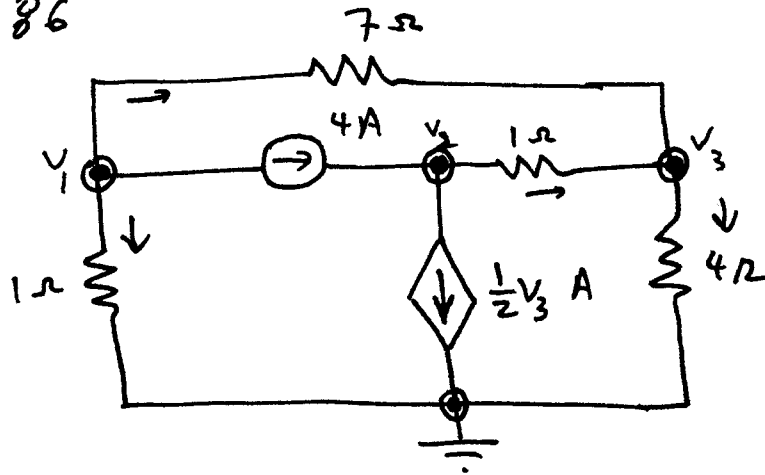
$$\therefore V_a - 5V_b = -3.$$

KCL c.

$$\frac{V_c - V_a}{1/2} + \frac{V_c}{1/3} + 3 = 0 \Rightarrow 2V_a - 5V_c = 3.$$

$$V_a = -1.3V, \quad V_b = .34V, \quad V_c = -1.12V$$

2.4 :
86



$$\frac{V_1}{1} + \frac{V_1 - V_3}{7} + 4 = 0 \quad \Rightarrow \quad 8V_1 - V_3 = -28$$

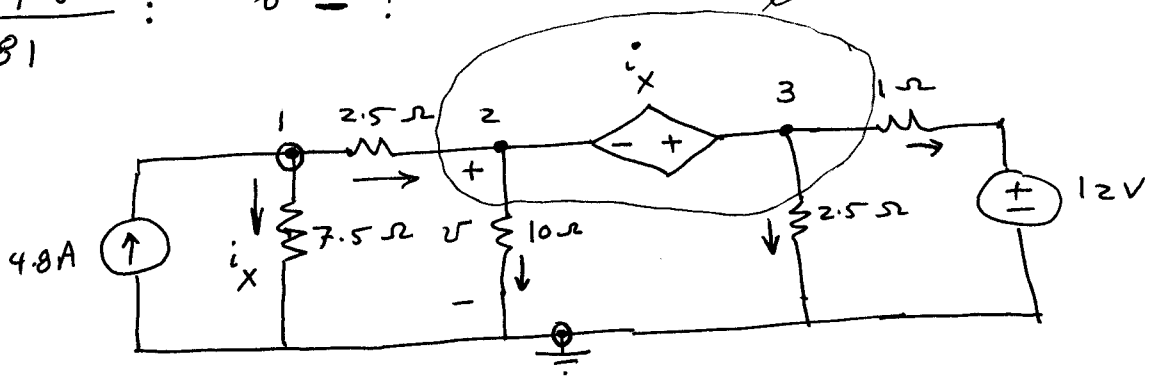
$$4 = \frac{1}{2} V_3 + \frac{V_2 - V_3}{1} \quad \Rightarrow \quad 2V_2 - V_3 = 8$$

$$\frac{V_1 - V_3}{7} + \frac{V_2 - V_3}{1} = \frac{V_3}{4} \quad \Rightarrow \quad -4V_1 - 28V_2 + 39V_3 = 0$$

$$V_1 = \underline{-3\text{ V}}, \quad V_2 = \underline{6\text{ V}}, \quad V_3 = \underline{4\text{ V}}$$

Special case ; (A voltage source is the only element between two nodes. Supernode)

D 4.8* : $v = ?$
81



node 1 : $4.8 = \frac{V_1}{7.5} + \frac{V_1 - V_2}{2.5}$ (1)

node 2 : We see that node-voltage eqⁿ at either node 2 or 3 can't be written because of we cannot express the current in the dependent voltage source. To solve the pb we combine nodes 2 and 3 and call it a "supernode". We now write the KCL eqⁿ for the supernode.

$$\frac{V_1 - V_2}{2.5} = \frac{V_2}{10} + \frac{V_3}{2.5} + \frac{V_3 - 12}{1}$$
 (2)

Also note that

$$V_3 - V_2 = i_x$$
 (3)

and $i_x = \frac{V_1}{7.5}$ (4)

(4) in (3) \Rightarrow

$$V_3 - V_2 = \frac{V_1}{7.5}$$

$$\therefore V_3 = V_2 + \frac{V_1}{7.5} \quad (5)$$

\therefore (1), (2), (5) \Rightarrow

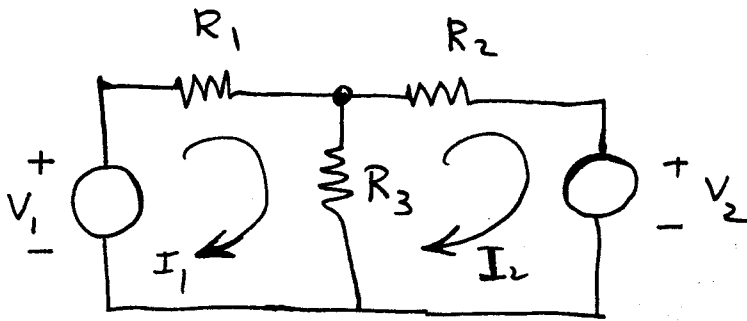
$$\begin{cases} \left(\frac{1}{2.5} + \frac{1}{7.5} \right) V_1 - \frac{1}{2.5} V_2 + 0 V_3 = 4.8 \\ \frac{1}{2.5} V_1 - \left(\frac{1}{2.5} + \frac{1}{10} \right) V_2 - \left(\frac{1}{2.5} + 1 \right) V_3 = -12 \\ \frac{1}{7.5} V_1 + V_2 - V_3 = 0 \end{cases}$$

$$\therefore \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 15 \\ 8 \\ 10 \end{pmatrix} \leftarrow v$$

Loop-Current method: (mesh analysis)

It is a method of solving circuit problems in which the KCL eqⁿs are written implicitly and KVL eqⁿs are written explicitly and solved.

Exa :



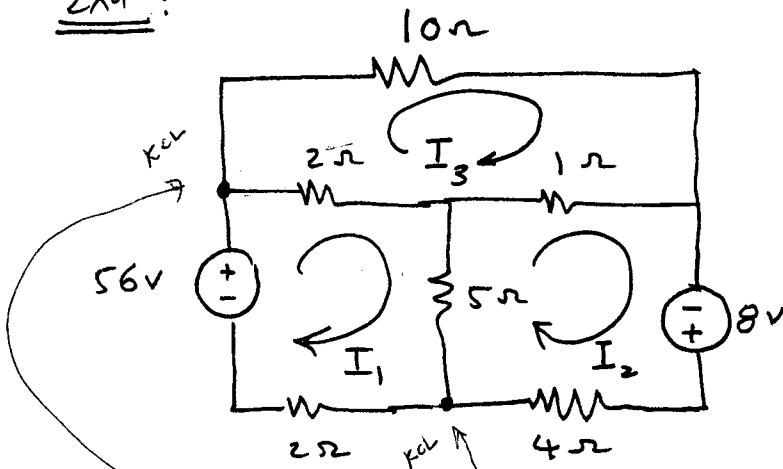
$$\begin{cases} -V_1 + R_1 I_1 + R_3 (I_1 - I_2) = 0 \\ +V_2 + R_3 (I_2 - I_1) + R_2 I_2 = 0 \end{cases}$$

$$\therefore \begin{cases} (R_1 + R_3) I_1 - R_3 I_2 = V_1 \\ -R_3 I_1 + (R_2 + R_3) I_2 = -V_2 \end{cases}$$

$$I_1, I_2 = \dots$$

Using Loop-current method find I_1, I_2, I_3 :

Exa:

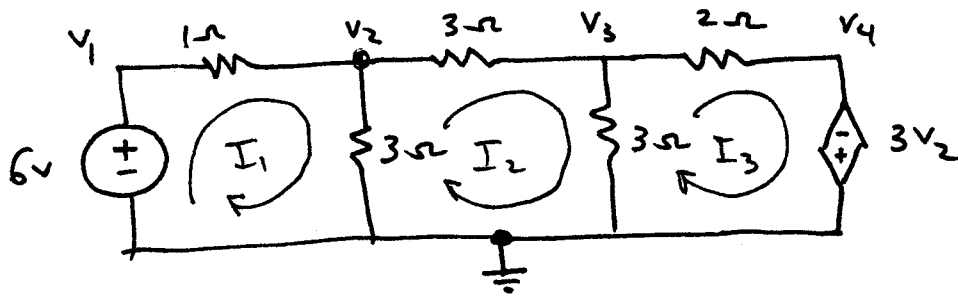


KVL:

$$\begin{cases} -56 + 2(I_1 - I_3) + 5(I_1 - I_2) + 2I_1 = 0 \\ -8 + 4I_2 + 5(I_2 - I_1) + (I_2 - I_3) = 0 \\ 2(I_3 - I_1) + 10I_3 + (I_3 - I_2) = 0 \end{cases}$$

$I_1, I_2, I_3 = \dots$

2.15 (Bobrow)
90



$$-6 + I_1 + 3(I_1 - I_2) = 0 \quad (1)$$

$$3(I_2 - I_1) + 3I_2 + 3(I_2 - I_3) = 0 \quad (2)$$

$$3(I_3 - I_2) + 2I_3 - \frac{3(I_1 - I_2)}{2} = 0 \quad (3)$$

$$V_2 = 3(I_1 - I_2) \quad (4)$$

$$(1) \Rightarrow 4I_1 - 3I_2 = 6$$

$$(2) \Rightarrow -I_1 + 3I_2 - I_3 = 0$$

$$(4) \text{ in } (3) \Rightarrow -9I_1 + 6I_2 + 5I_3 = 0$$

$$I_1 = \frac{\begin{vmatrix} 6 & -3 & 0 \\ 0 & 3 & -1 \\ 0 & 6 & 5 \end{vmatrix}}{\begin{vmatrix} 4 & -3 & 0 \\ -1 & 3 & -1 \\ -9 & 6 & 5 \end{vmatrix}} = \frac{126}{42} = 3A, \quad I_2 = 2A, \quad I_3 = 3A$$

$$\begin{vmatrix} 4 & -3 & 0 \\ -1 & 3 & -1 \\ -9 & 6 & 5 \end{vmatrix}$$

$$\therefore V_1 = 6V$$

$$V_2 = 3(I_1 - I_2) = 3V$$

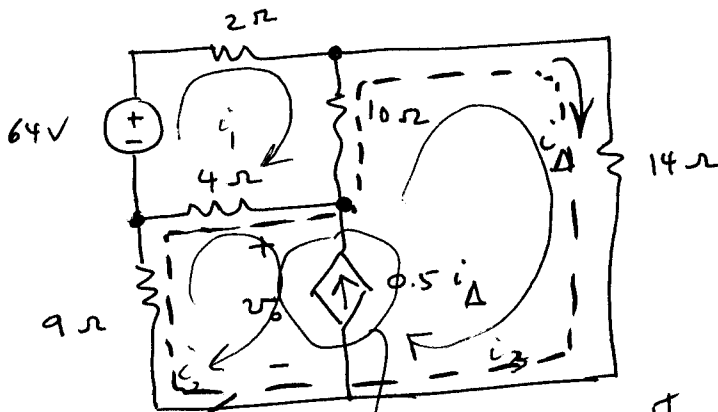
$$V_3 = 3(I_2 - I_3) = -3V$$

$$V_4 = -9V$$

Special case

When a branch includes a current source we need extra things to do.

Prob 4.19 : $v_o = ?$
115



super mesh

We can't write KVL for this current source. So, remove it "mentally" and form a super mesh and write mesh eqⁿ.

mesh eqⁿ:

$$\begin{cases} -64 + 2i_1 + 10(i_1 - i_3) + 4(i_1 - i_2) = 0 & (1) \\ 9i_2 + 4(i_2 - i_1) + 10(i_3 - i_1) + 14i_3 = 0 & (2) \end{cases}$$

Also,

$$\begin{cases} i_3 - i_2 = 0.5 i_{\Delta} \\ i_{\Delta} = i_3 \end{cases} \Rightarrow 0.5 i_3 - i_2 = 0 \quad (3)$$

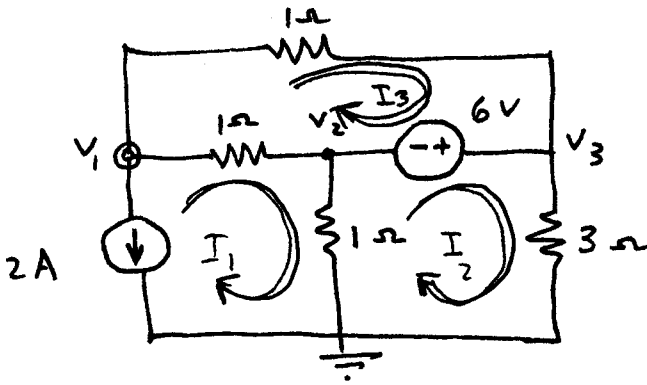
$$(1), (2), (3) \Rightarrow$$

$$\begin{cases} 16 i_1 - 4 i_2 - 10 i_3 = 64 \\ -14 i_1 + 13 i_2 + 24 i_3 = 0 \\ 0 i_1 - i_2 + 0.5 i_3 = 0 \end{cases}$$

$$\begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 6.1 \\ 1.4 \\ 2.8 \end{pmatrix} \text{ A.}$$

$$9 i_2 + 4 (i_2 - i_1) + v_o = 0 \Rightarrow v_o = \underline{\underline{6.20 \text{ V}}}$$

2.17 (Babrow)
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$$-V_1 + (I_1 - I_3) + 1(I_1 - I_2) = 0 \quad (1)$$

$$(I_2 - I_1) - 6 + 3I_2 = 0 \quad (2)$$

$$I_3 + 6 + (I_3 - I_1) = 0 \quad (3)$$

$$I_1 = -2A \quad (4)$$

$$(4) \text{ in } (2) \Rightarrow 4I_2 - \cancel{I_1}^{-2} = 6 \Rightarrow \underline{I_2 = 1A} \quad (5)$$

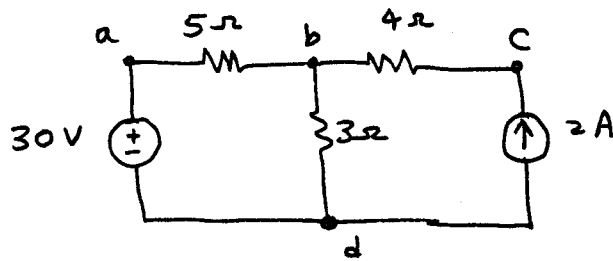
$$(4) \text{ in } (3) \Rightarrow 2I_3 = I_1 - 6 = -2 - 6 = -8 \Rightarrow \underline{I_3 = -4}$$

$$(1) \Rightarrow V_1 = 2I_1 - I_2 - I_3 = -4 - 1 + 4 = \underline{\underline{-1V}}$$

$$V_2 = 1(I_1 - I_2) = -2 - 1 = \underline{\underline{-3V}}$$

$$V_3 = 3I_2 = \underline{\underline{3V}}$$

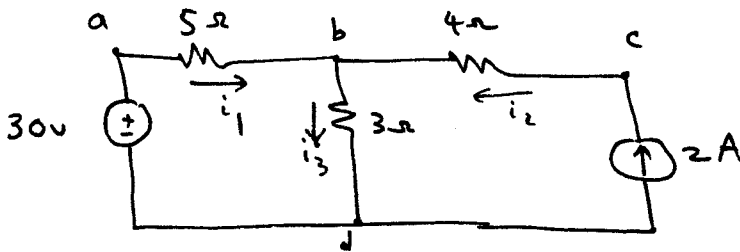
$$\frac{E_{10}}{54} :$$



$$V_{3\Omega} = ?$$

$$V_{2A} = ?$$

(a) Element Currents :



$$\text{KCL } b : i_1 + i_2 = i_3 \Rightarrow i_1 = i_3 - 2$$

$$\text{KVL (abd)} : -30 + 5i_1 + 3i_3 = 0$$

$$\Rightarrow -30 + 5(i_3 - 2) + 3i_3 = 0$$

$$8i_3 = 40 \Rightarrow i_3 = \underline{5 \text{ A}}$$

$$V_{3\Omega} = (3)(5) = 15 \text{ V.}$$

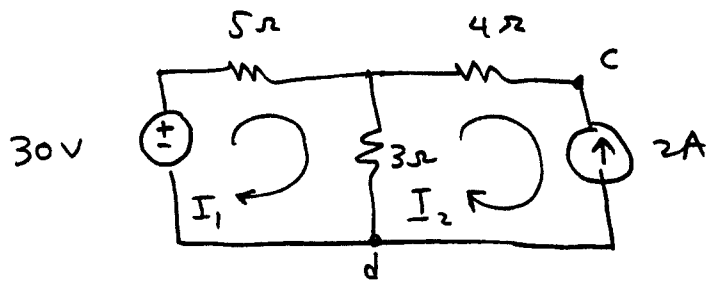
KVL (bcd)

$$V_{bc} + V_{cd} + V_{db} = 0$$

$$\therefore V_{2A} = \underbrace{-V_{db}}_{V_{bd}} - \underbrace{V_{bc}}_{V_{cb}}$$

$$= V_{bd} + V_{cb} = 15 + 4 \cdot \frac{2}{2} = \underline{23 \text{ V}}$$

(b) Loop Currents :



$$\text{KVL : } \begin{cases} -30 + 5I_1 + 3(I_1 - I_2) = 0 & (1) \\ 4I_2 + v_{cd} + 3(I_2 - I_1) = 0 & (2) \end{cases}$$

$$\text{But } I_2 = -2 \text{ A}$$

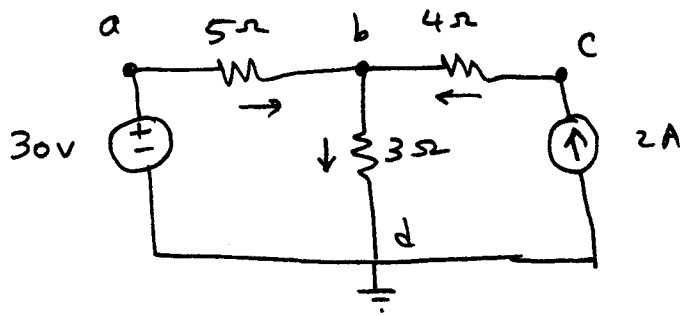
$$\therefore 8I_1 - 3\cancel{I_2}^{-2} = 30 \Rightarrow I_1 = \underline{3 \text{ A}}$$

$$v_{3\Omega} = 3(\cancel{I_1}^3 - \cancel{I_2}^{-2}) = \underline{15 \text{ V}}$$

$$(2) \Rightarrow v_{cd} = -4(-2) - 3(-2-3) = 8 + 15 = 23$$

$$\therefore v_{2A} = \underline{23 \text{ V}}$$

(c) Node voltages :



$$\text{KCL } b: \quad \frac{30 - V_b}{5} + 2 - \frac{V_b}{3} = 0.$$

$$V_b = V_{3\Omega} = \underline{15 \text{ V}}$$

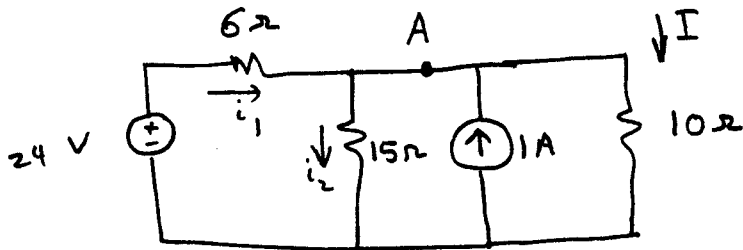
$$\text{KVL } L_2: \quad \frac{8}{V_{cb}} + \frac{15}{V_{bd}} + V_{dc} = 0.$$

$$\therefore V_{cd} = \underline{23 \text{ V}}$$

$$\# \frac{E11}{54} :$$

$$I = ?$$

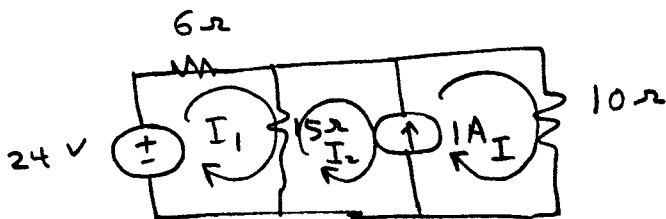
(a) Element currents:



$$\begin{cases} \text{kcl A :} & i_1 + 1 - i_2 - I = 0 \\ \text{kvl outside Loop :} & -24 + 6i_1 + 10I = 0 \\ \text{Also :} & 15i_2 = 10I \end{cases}$$

$$\underline{I = 1.5 A}$$

(b) Loop currents:



$$\text{KVL : } \left\{ \begin{array}{l} -24 + 6 I_1 + 15 (I_1 - I_2) = 0 \\ -24 + 6 I_1 + 10 I = 0 \end{array} \right.$$

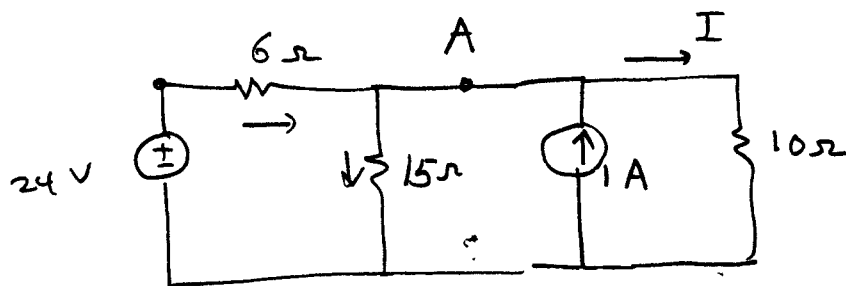
$$\text{KVL outside loop : } -24 + 6 I_1 + 10 I = 0$$

$$I_2 + 1 = I$$

Also :
For the branch
containing the
current source

$$\underline{I = 1.5 \text{ A}}$$

(c) node voltages :



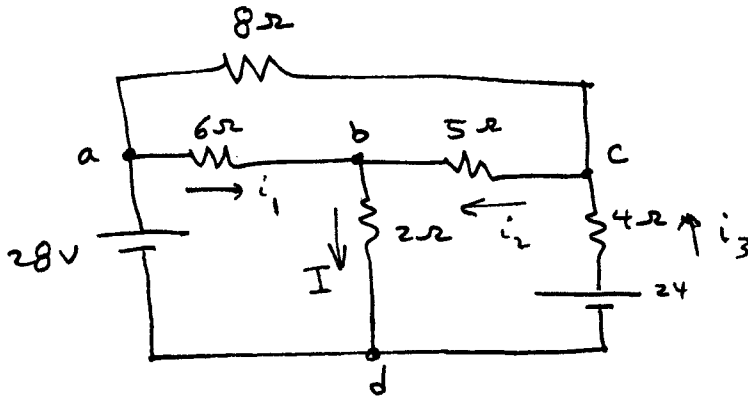
$$\text{KCL A : } \frac{24 - V_A}{6} + 1 - \frac{V_A}{15} - \frac{V_A}{10} = 0$$

$$V_A = \underline{15 \text{ V}} : \Rightarrow I = \frac{V_A}{10} = \underline{1.5 \text{ A}}$$

$$\frac{E_{12}}{54}$$

$$I_{2\Omega} = ?$$

(a) element currents :

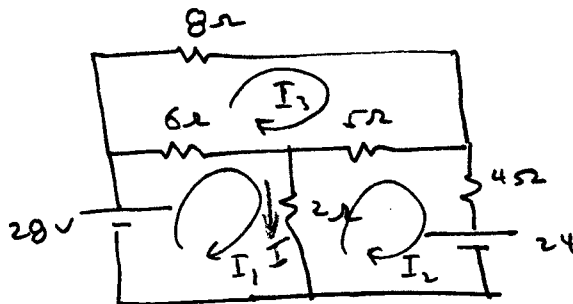


$$\text{KVL: } \begin{cases} -28 + 6i_1 + 2I = 0 \\ -24 + 4i_3 + 5i_2 + 2I = 0 \end{cases}$$

$$\text{KCL: } i_1 + i_2 - I = 0$$

$$\underline{I = 5 \text{ A}}$$

(b) Loop currents :



$$\text{KVL: } -28 + 6(I_1 - I_3) + 2(I_1 - I_2) = 0.$$

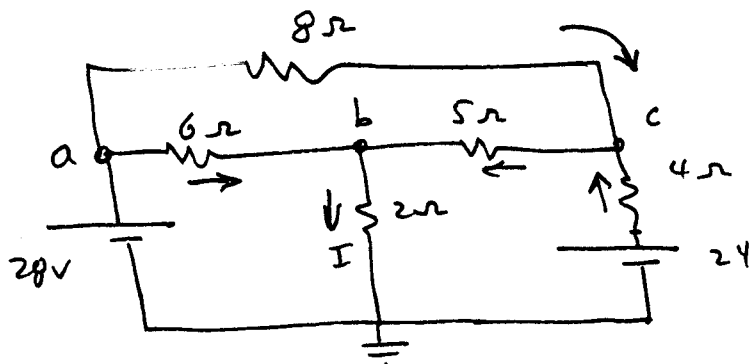
$$2(I_2 - I_1) + 5(I_2 - I_3) + 4I_2 + 24 = 0.$$

$$8I_3 + 5(I_3 - I_2) + 6(I_3 - I_1) = 0.$$

$$I_1 = 4 \quad I_2 = -1 \quad I_3 =$$

$$I = I_1 - I_2 = \underline{5 \text{ A}}$$

(c) Node voltages:



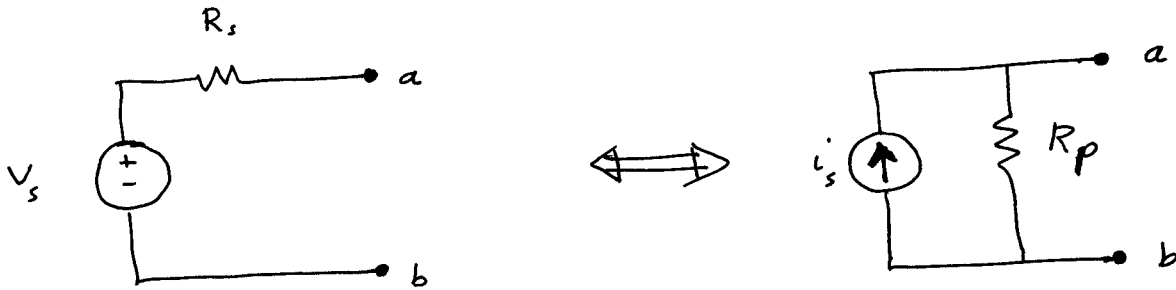
$$\text{KCL } b: \left\{ \begin{array}{l} \frac{28 - V_b}{6} + \frac{V_c - V_b}{5} - \frac{V_b}{2} = 0. \end{array} \right.$$

$$\text{KCL } c: \left\{ \begin{array}{l} \frac{24 - V_c}{4} + \frac{28 - V_c}{8} - \frac{V_c - V_b}{5} = 0. \end{array} \right.$$

$$V_b = 10 \text{ V} \quad \Rightarrow \quad I = \frac{10}{2} = \underline{5 \text{ A}}$$

Source Transformation

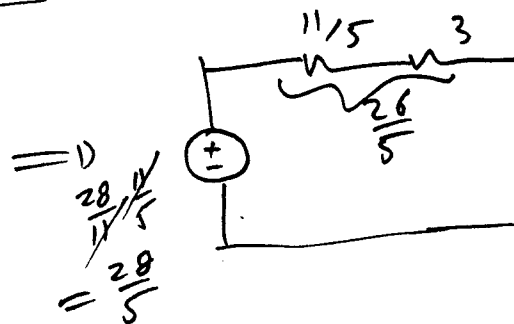
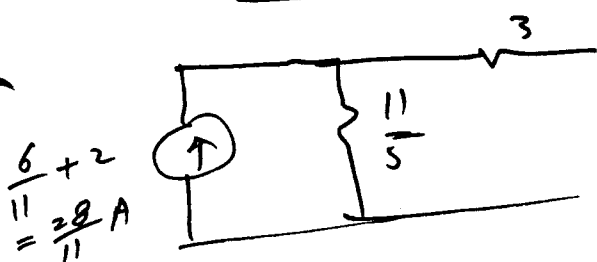
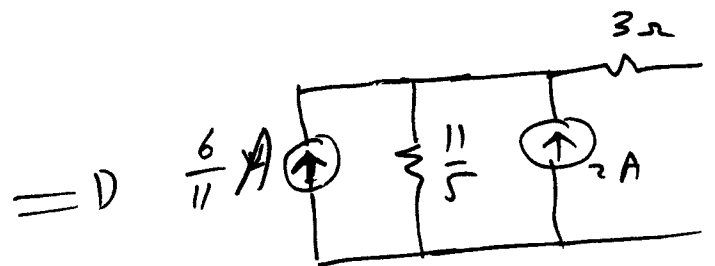
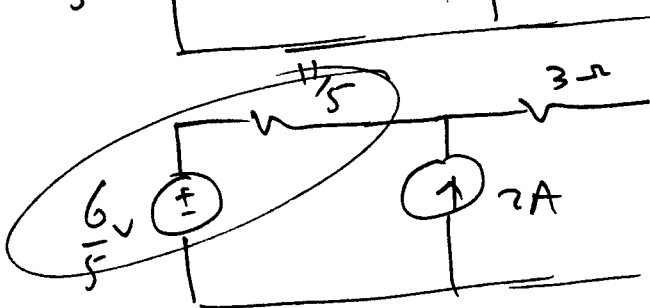
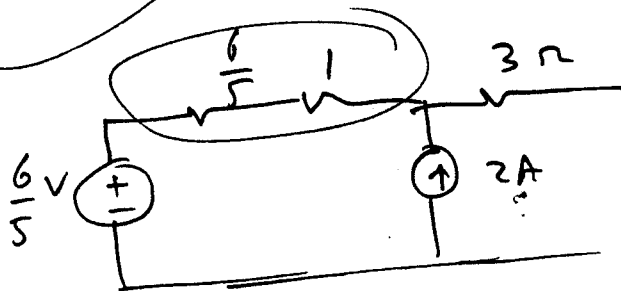
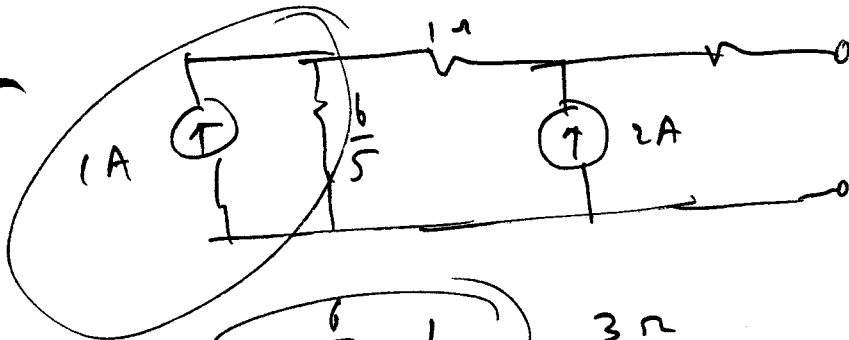
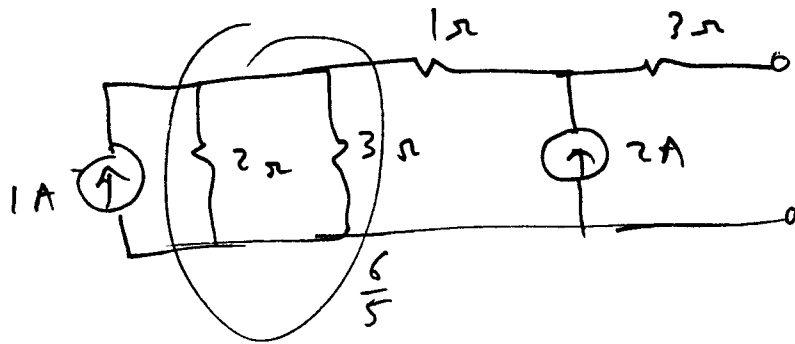
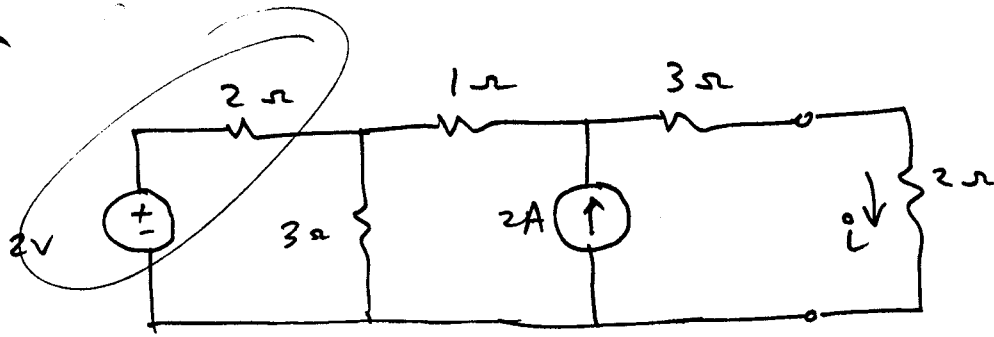
A source transformation shown below allows us to replace a voltage source in series with a resistor by a current source in parallel with the same resistor, or vice versa.



$$i_s = \frac{V_s}{R_s}$$

$$R_s = R_p.$$

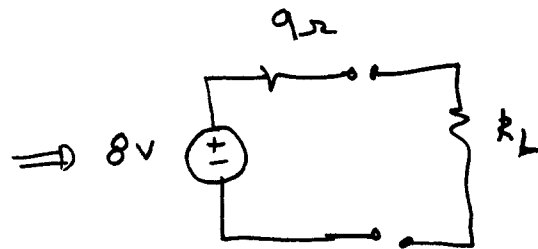
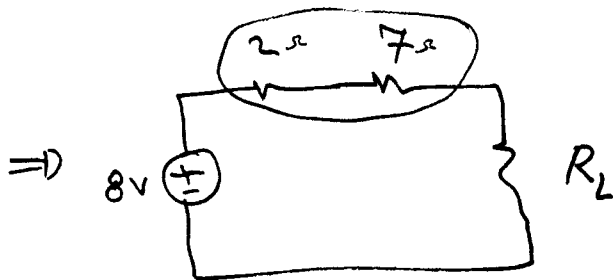
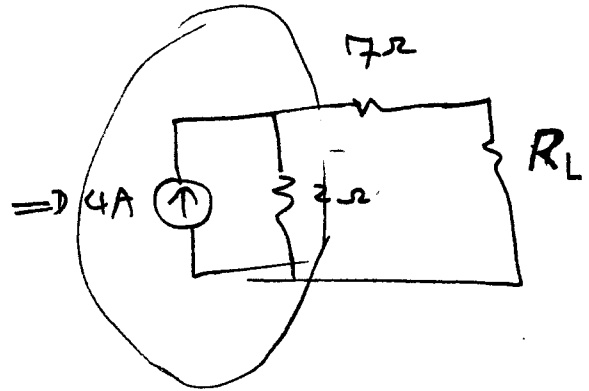
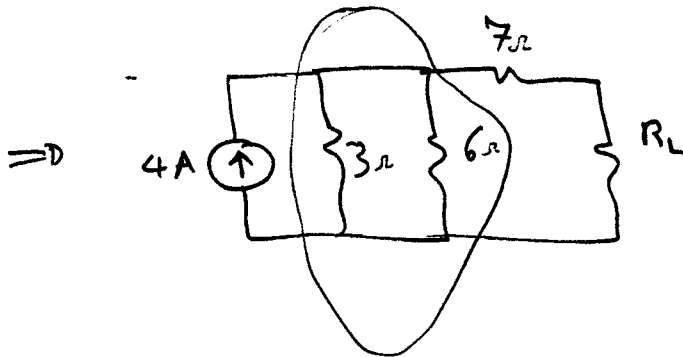
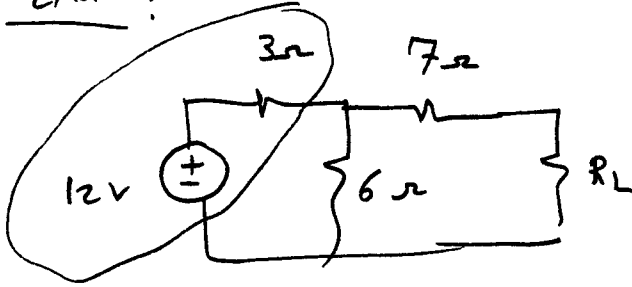
Example use source trans to simplify the given circuit.



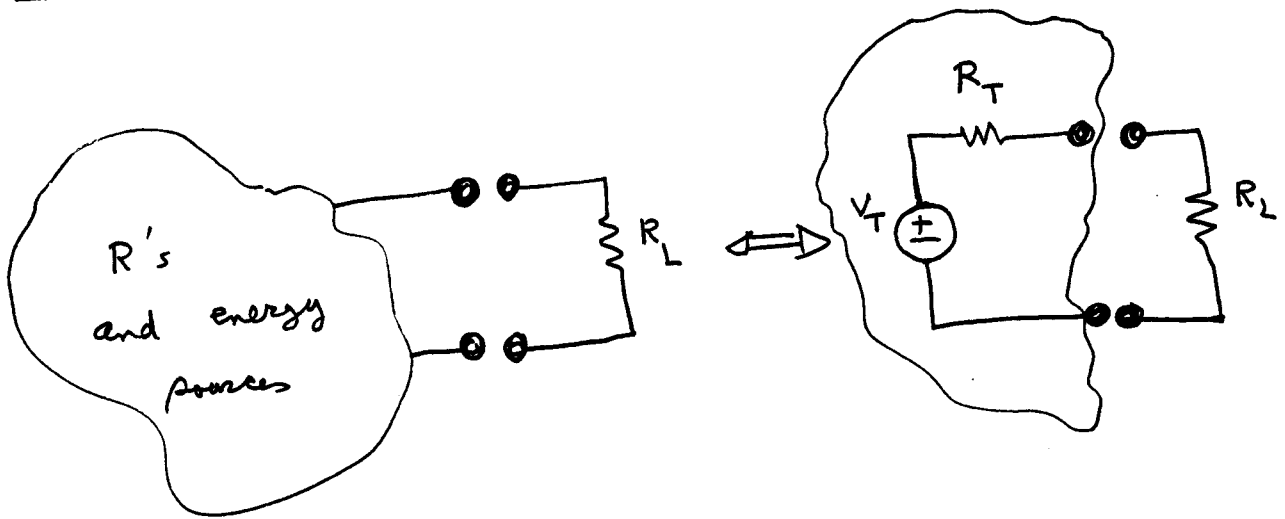
The source transformation principle can be very useful

in network analysis.

EXA:



Thevenin's Theorem



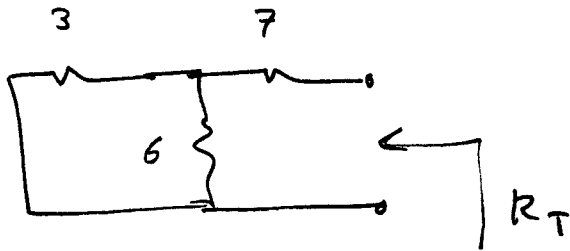
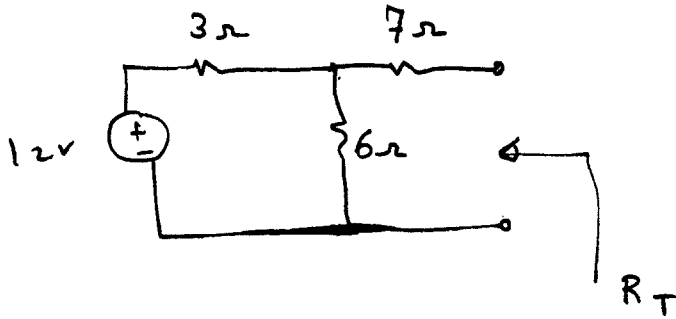
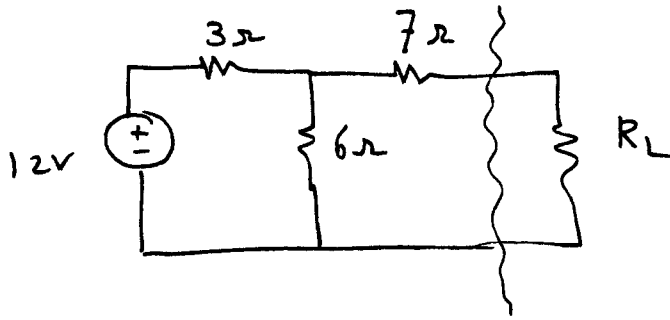
where

$$V_T = V_{oc}$$

$$R_T = \frac{V_T}{I_{sc}} = \frac{V_{oc}}{I_{sc}}$$

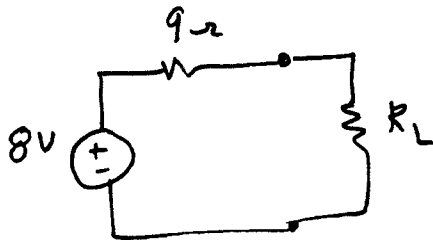
OR R_T is the "resistance seen by looking in" at the terminals with all voltage sources short-circuited and current sources open-circuited

Exa :

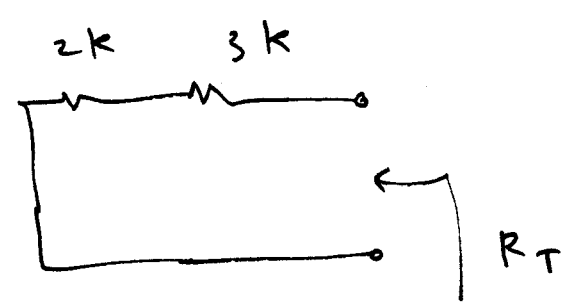
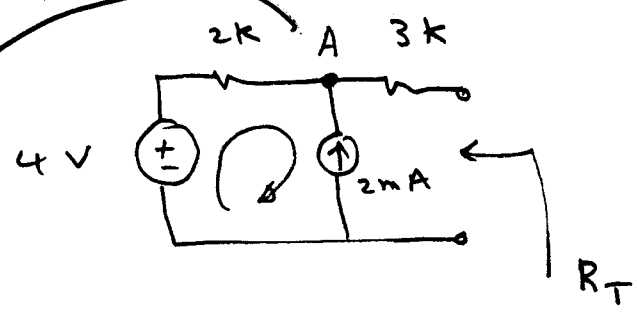
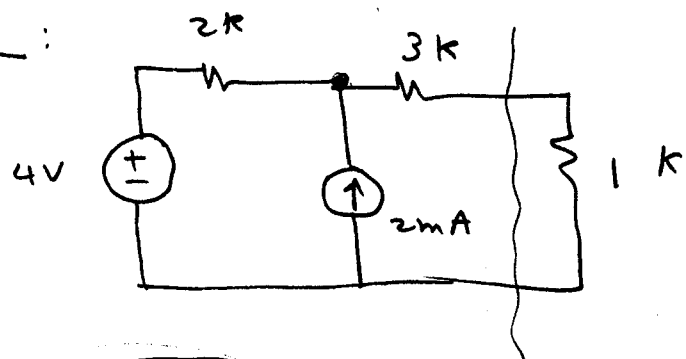


$$R_T = 3 // 6 + 7 = \frac{18}{9} + 7 = 9 \Omega$$

$$V_T = V_{6\Omega} = \frac{6}{3+6} 12 = \frac{\cancel{6}^2}{\cancel{9}_3} (\cancel{12})^4 = 8 \text{ V}$$



EX9 :

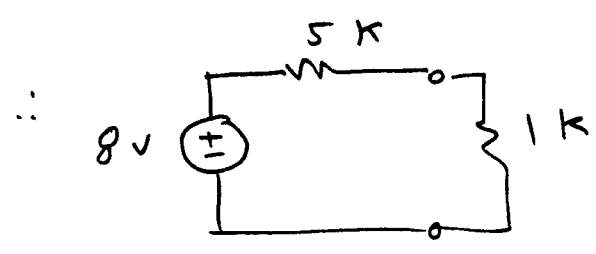


$R_T = 5 \text{ k}$

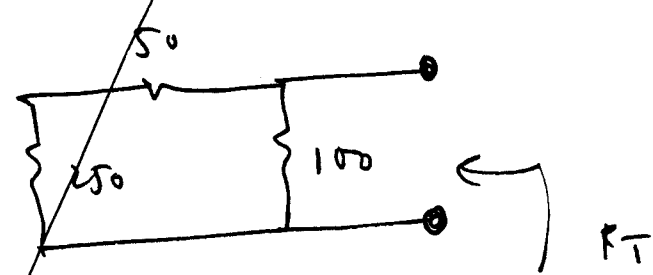
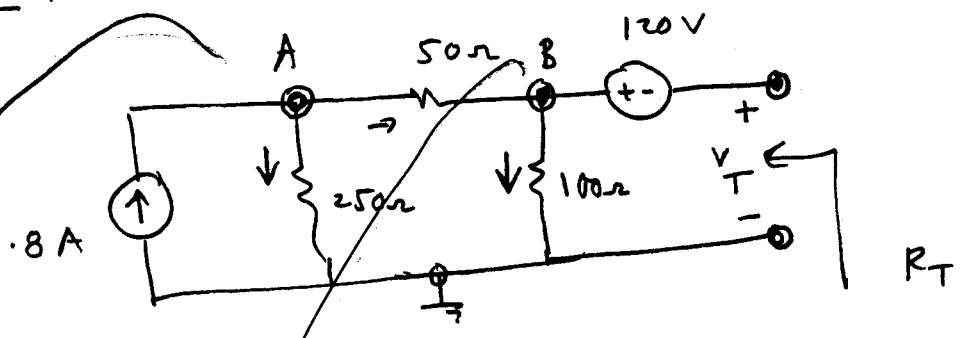
KCL A: $\frac{4 - V_A}{2} + 2 = 0$

$4 - V_A + 4 = 0 \Rightarrow V_A = 8$

$\therefore V_T = 8 \text{ V}$



Exa :



$$R_T = (250 + 50) \parallel 100 = \frac{300 \times 100}{300 + 100} = \underline{\underline{75 \Omega}}$$

KCL A :

$$.8 = \frac{V_A}{250} + \frac{V_A - V_B}{50} \Rightarrow 200 = \cancel{V_A} + \cancel{5V_A} - 5V_B$$

KCL B :

$$\frac{V_A - V_B}{50} = \frac{V_B}{100} \Rightarrow 2V_A - 2V_B = V_B \Rightarrow 3V_B = 2V_A \quad (2)$$

(1) \Rightarrow

$$\begin{cases} 6V_A - 5V_B = 200 \\ 3V_B = 2V_A \end{cases}$$

$$\Rightarrow 2 \left(\frac{3}{2} V_B \right) 6 - 5V_B = 200$$

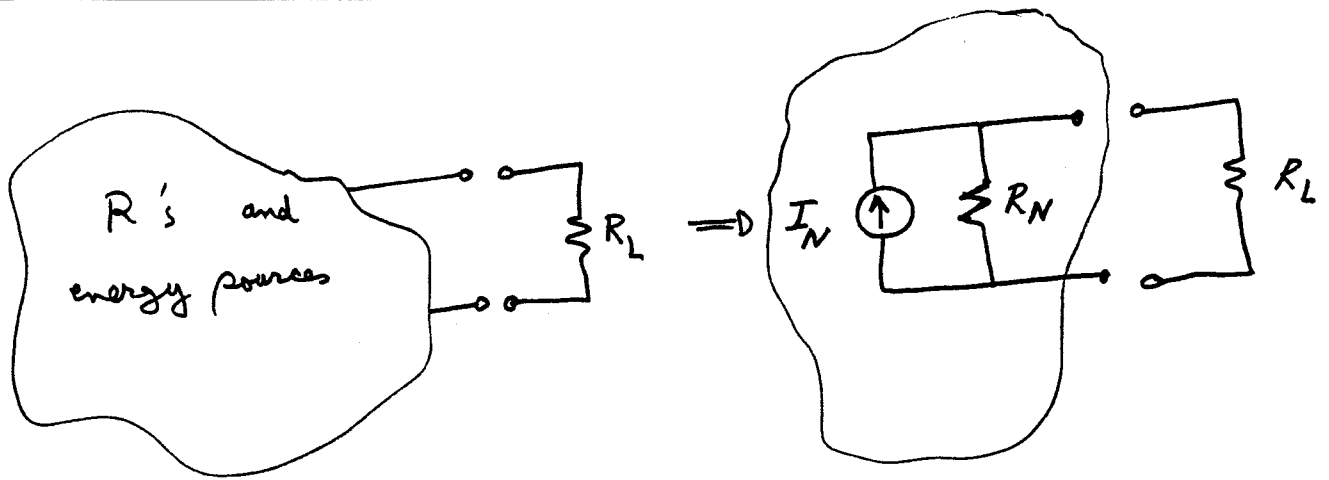
$$4V_B = 200$$

$$\underline{\underline{V_B = 50 \text{ V}}}$$

$$-\cancel{V_B} + 120 + V_T = 0$$

$$\underline{\underline{V_T = -70 \text{ V}}}$$

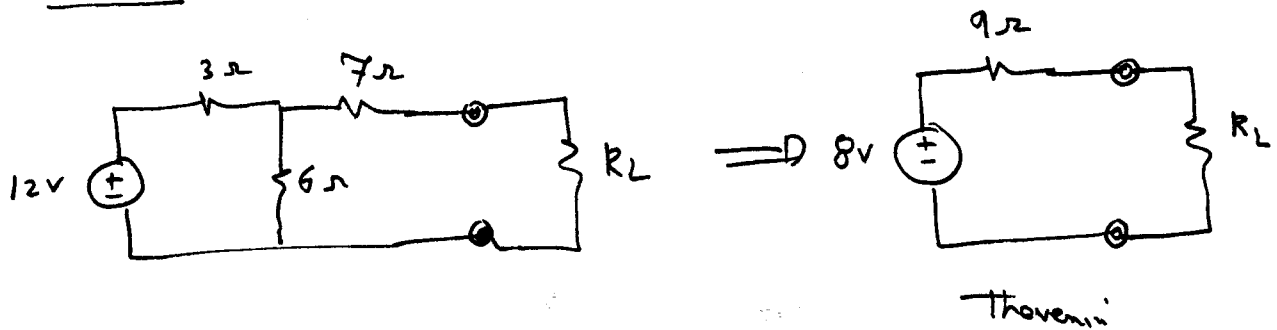
Norton's Theorem



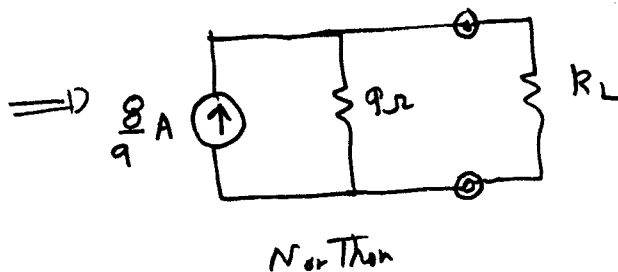
$$I_N = \frac{V_T}{R_T}, \quad R_N = R_T$$

It is a "source transformation" principle.

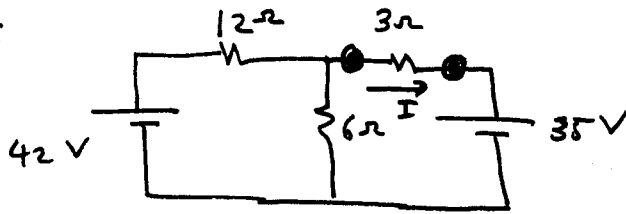
Exa: From previous example we get:



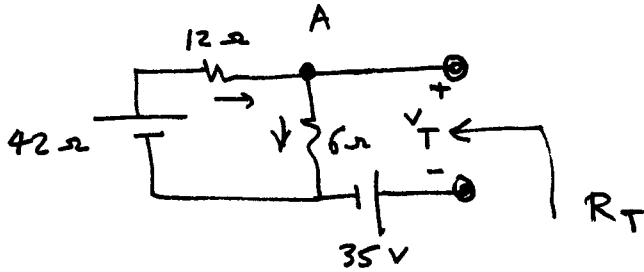
So:



$$\frac{E_{25}}{55}$$



$$I = ?$$

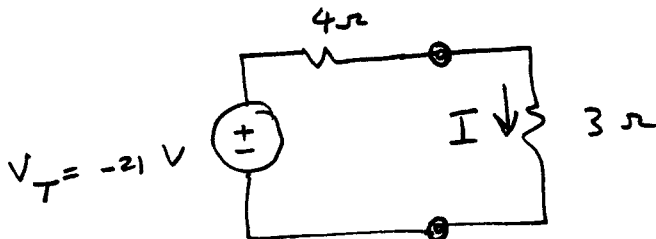


$$R_T = 12 \parallel 6 = \frac{(12)(6)}{12+6} = 4 \Omega$$

$$\text{KCL A: } \frac{42 - V_A}{12} = \frac{V_A}{6}$$

$$42 - V_A = 2V_A \Rightarrow V_A = 14 \text{ V}$$

$$\text{KVL: } 35 - \frac{14}{A} + V_T = 0 \Rightarrow V_T = -21 \text{ V}$$



$$-V_T + (4+3)I = 0$$

$$-21 + 7I = 0$$

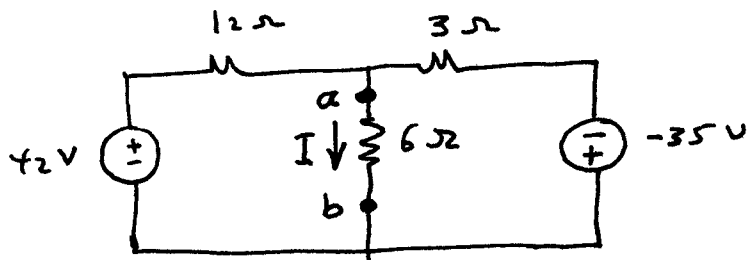
$$\Rightarrow \underline{I = -3 \text{ A}}$$

Q2

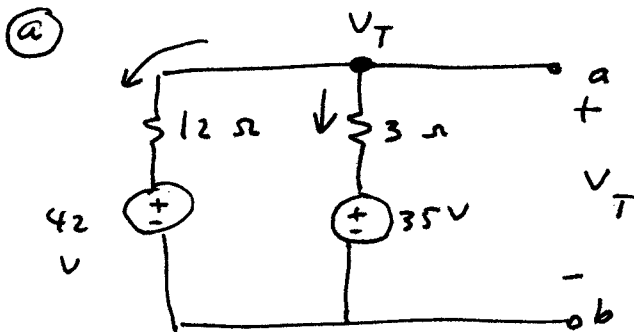
ENGR 212

Name _____ Key _____

Consider the following circuit:



- Find the Thevenin's equivalent circuit across a and b.
- Using part a, find I .

solⁿ:

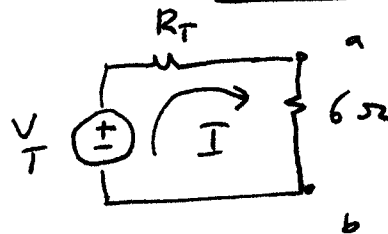
$$R_T = 3 \parallel 12 = 2.4 \Omega$$

$$\frac{V_T - 42}{12} + \frac{V_T - 35}{3} = 0$$

$$\Rightarrow V_T = 36.4 \text{ V}$$

$$V_T - 42 + 4V_T - 140 = 0$$

$$5V_T = 182$$

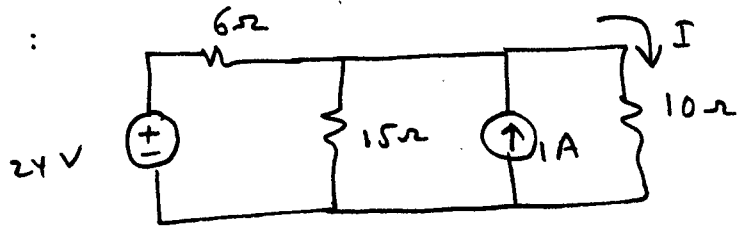


(b)

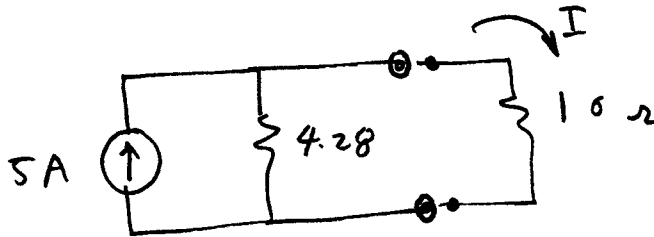
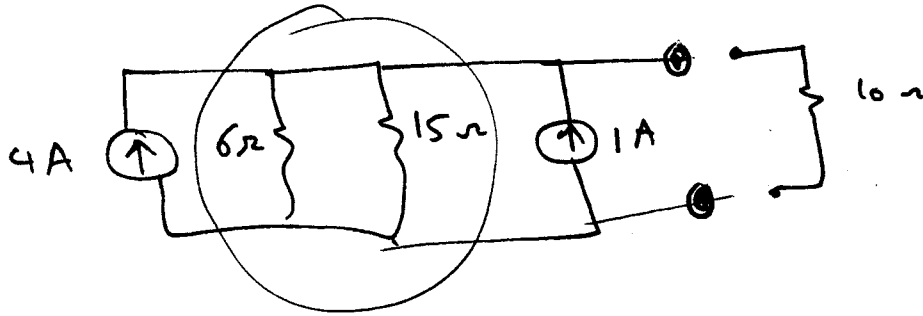
$$-\frac{36.4}{1} + \frac{2.4}{1} I + 6 I = 0$$

$$8.4 I = 36.4 \Rightarrow I = 4.33 \text{ A}$$

$\frac{E_{27}}{56}$:

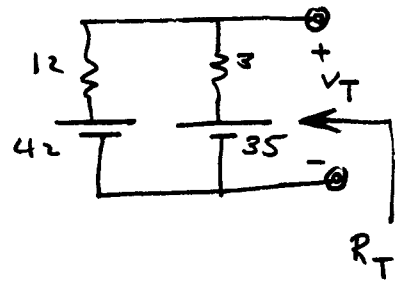
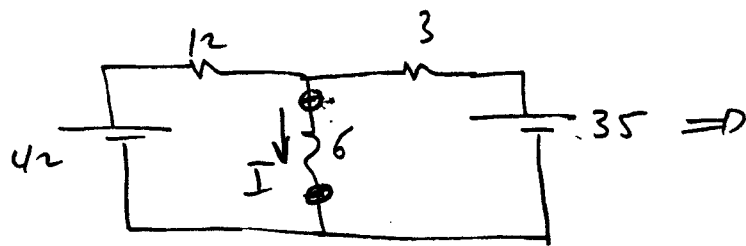


$I = ?$

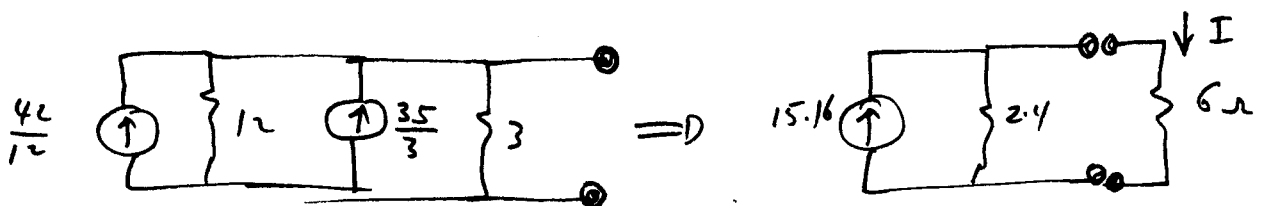


$$I = \frac{4.28}{4.28 + 10} (5) = 1.5 \text{ A}$$

$\frac{E_{28}}{56}$:



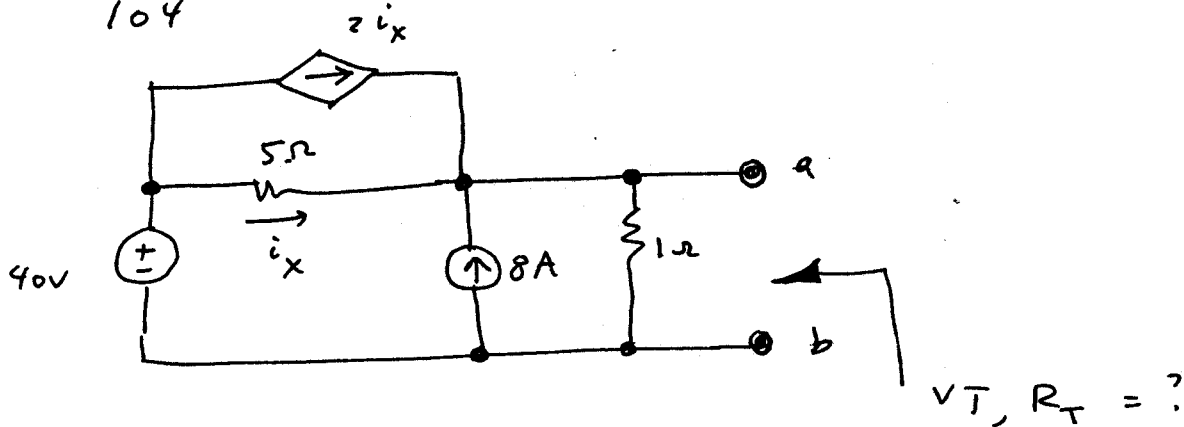
$$R_T = 3 \parallel 12 = \dots = 2.4 \Omega$$



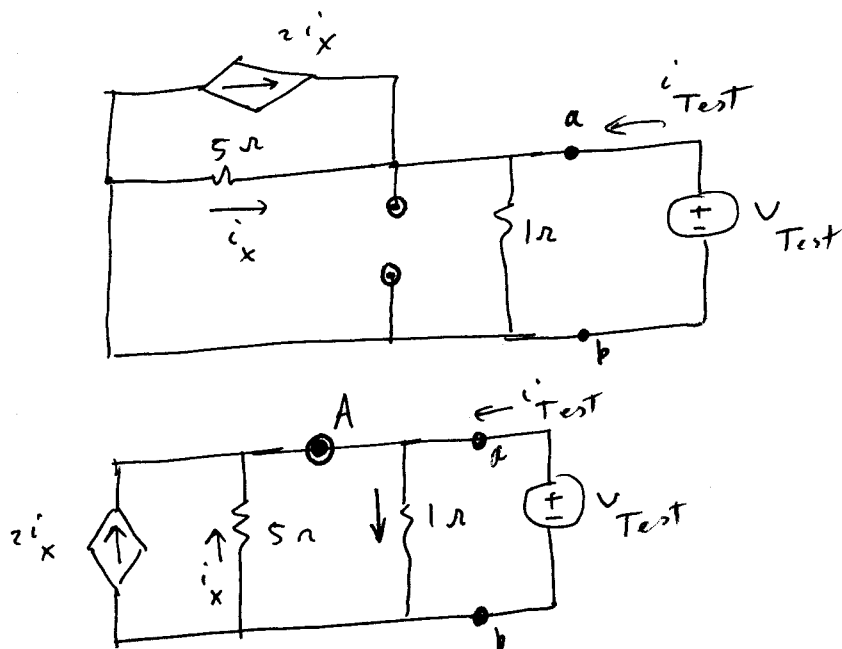
$$I = \frac{2.4}{2.4 + 6} (15.16) = 4.33 \text{ A}$$

Thevenin theorem - Dependent sources.

D4.22
104 :



To find R_T , we first deactivate the indep sources and then excite the circuit from a, b with either a test voltage source or a test current source.



$$\Rightarrow R_T = \frac{V_{Test}}{i_{Test}}$$

KCL A :

$$2i_x + i_x - \frac{V_{Test}}{1} + i_{Test} = 0$$

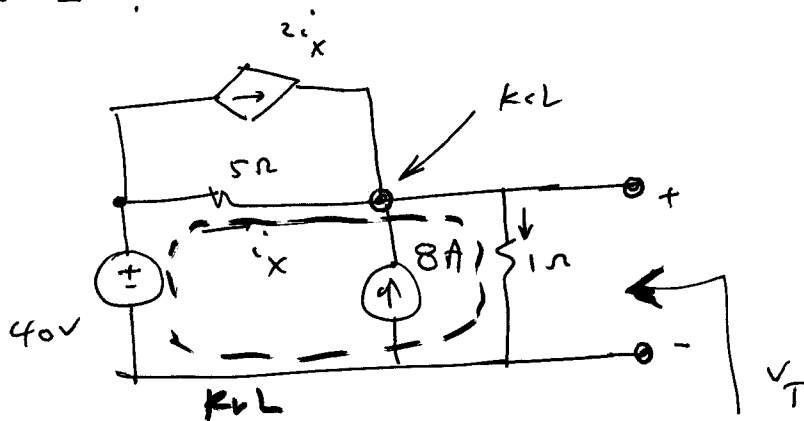
Also,
$$i_x = -\frac{V_{Test}}{5}$$

$$\therefore -\frac{2}{5} V_{Test} - \frac{1}{5} V_{Test} - V_{Test} + i_{Test} = 0$$

$$+\frac{8}{5} V_{Test} = i_{Test} \Rightarrow \frac{V_{Test}}{i_{Test}} = \frac{5}{8} = 0.625 \Omega$$

R_T

$V_T = ?$



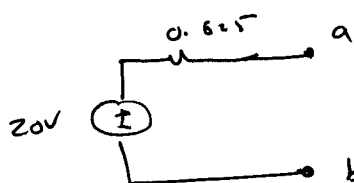
$$\begin{cases} 2i_x + i_x + 8 = \frac{V_T}{1} \\ -40 + 5i_x + V_T = 0 \end{cases} \Rightarrow$$

$$3i_x - V_T = -8$$

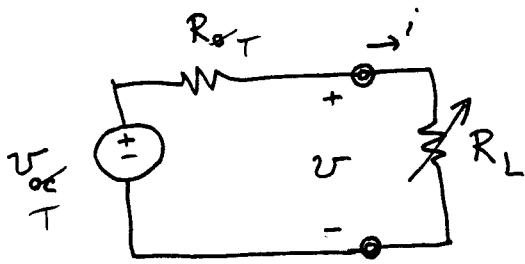
$$5i_x + V_T = 40$$

$$8i_x = 32 \Rightarrow \underline{i_x = 4}$$

$$\therefore \underline{V_T = 20V}$$



Maximum Power Transfer.



wish to determine R_L that will absorb the max power.

$$P = i^2 R_L = \left(\frac{V_{ocT}}{R_{\theta T} + R_L} \right)^2 R_L = \frac{V_{ocT}^2 R_L}{(R_{\theta T} + R_L)^2}$$

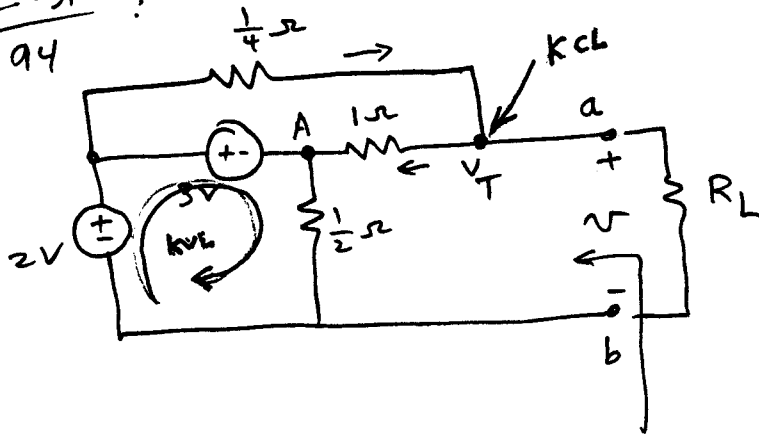
Find $\frac{dP}{dR_L} = 0 \implies R_L = R_{\theta T}$ ← optimum Load

and $P_{max} = \frac{V_{ocT}^2 R_L}{(2R_L)^2} = \frac{V_{ocT}^2}{4R_L} = \frac{V_{ocT}^2}{4R_{\theta T}}$

$$\frac{dP}{dR_L} = \frac{V_T^2 (R_T + R_L)^2 - 2R_L (R_T + R_L) V_T^2}{()^4} = 0$$

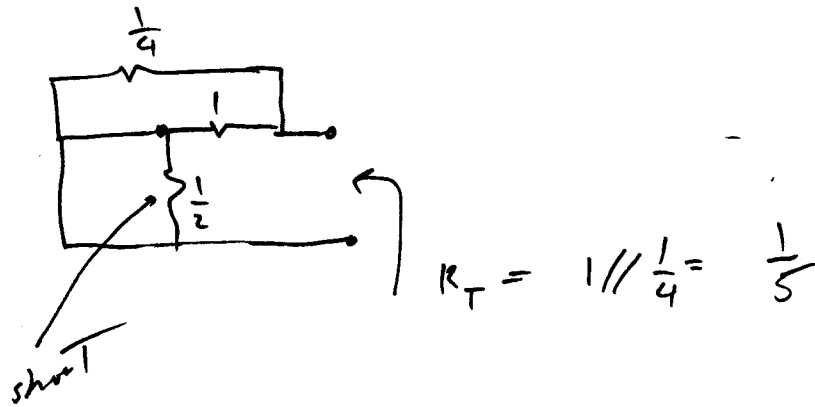
$$\therefore V_T^2 (R_T + R_L) [(R_T + R_L) - 2R_L] = 0$$
$$R_T - R_L = 0 \implies \boxed{R_L = R_T}$$

$$\frac{2 \cdot 31}{94} :$$



(a)

$$R_T = ?$$

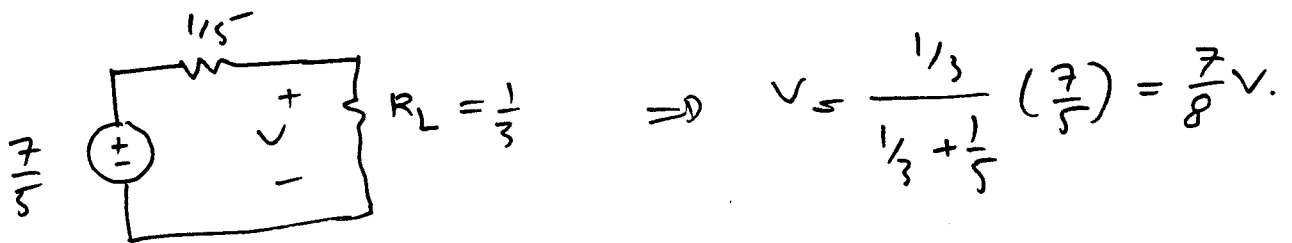


$$V_T = ?$$

$$\text{KVL: } -2 + 3 + V_A = 0 \Rightarrow V_A = -1 \text{ V.}$$

$$\text{KCL: } \frac{2 - V_T}{1/4} = \frac{V_T - \cancel{V_A}}{1} \Rightarrow \underline{V_T = \frac{7}{5} \text{ V}}$$

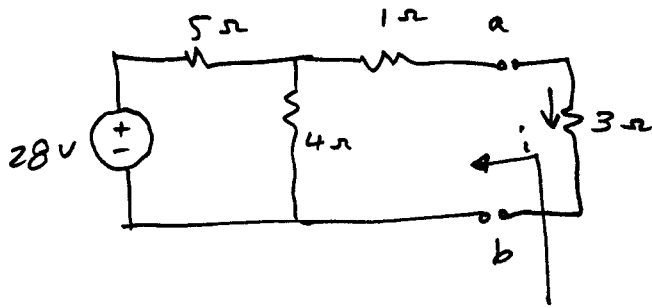
(b)



(c) $R_L = \frac{1}{5}$ ← optimum load

$$(d) P = \frac{V_T^2}{4 R_T} = \frac{(7/5)^2}{4(1/5)} = \frac{49}{20} \text{ W.} = \underline{2.45 \text{ W.}}$$

2.29 (Bobrow)
93



a) $R_T = 5 \parallel 4 + 1 = \frac{20}{9} + 1 = \frac{20+9}{9} = \frac{29}{9} \Omega$

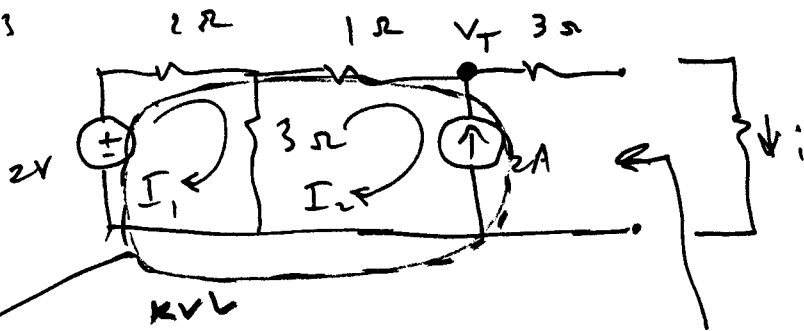
$V_T = \frac{4}{4+5} (28) = \frac{112}{9} \text{ V.}$

b)



$-\frac{112}{9} + \left(\frac{29}{9} + 3\right) i = 0 \Rightarrow \underline{i = 2 \text{ A}}$

$$\frac{2 \cdot 3}{2+3}$$



$$R_T = 2 \parallel 3 + 1 + 3 = \frac{2 \cdot 3}{2+3} + 4 = \frac{6}{5} + 4 = \frac{26}{5}$$

$$V_T = ?$$

$$-2 + 2I_1 + 3(I_1 - I_2) = 0 \quad \Rightarrow \quad I_1 = -\frac{4}{5} \text{ A}$$

$$-2 + 2I_1 + I_2 + V_T = 0 \quad \Rightarrow \quad V_T = \frac{28}{5} \text{ V}$$

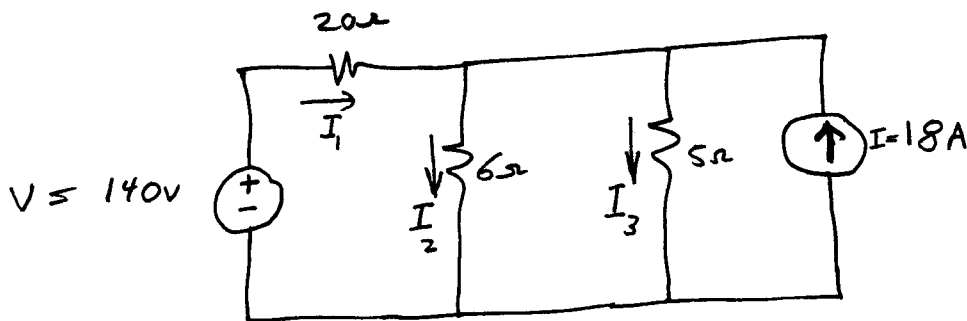
Principle of Superposition

In any electric network, the element voltages and currents are the effects produced by the applied sources, which are regarded as the causes. If superposition exists between cause and

effect it means:

$$\underset{\substack{\downarrow \\ \text{effect}}}{y} = f(\underset{\substack{\downarrow \\ \text{cause}}}{x}) \Rightarrow \underbrace{f(x_1)}_{y_1} + \underbrace{f(x_2)}_{y_2} = f(x_1 + x_2)$$

Exa: Use the principle of superposition to find I_1, I_2, I_3

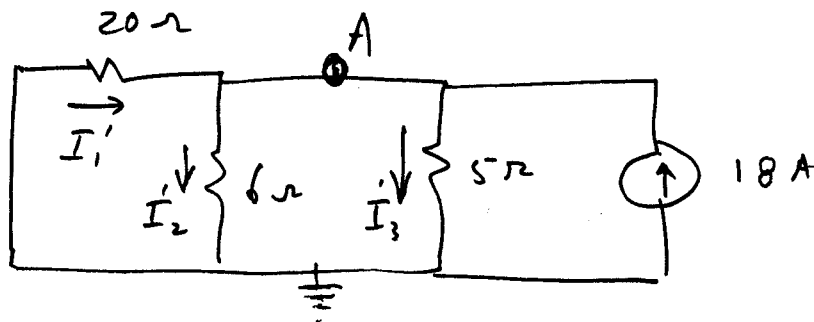


First, let $v=0 \Rightarrow I_1', I_2', I_3'$

Second, let $I=0 \Rightarrow I_1'', I_2'', I_3''$

$$\therefore I_1 = I_1' + I_1'', \quad I_2 = I_2' + I_2'', \quad I_3 = I_3' + I_3''$$

①



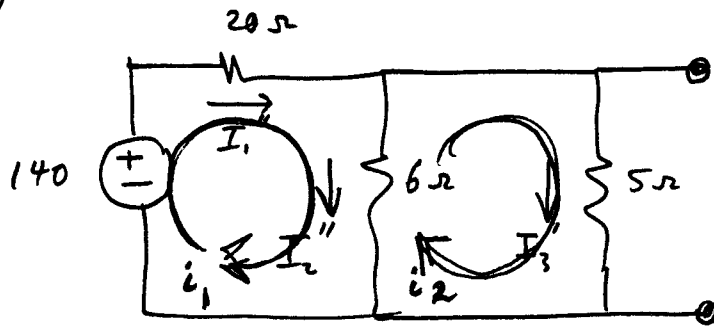
KCL A : (node voltage)

$$\frac{I_2'}{6} + \frac{I_3'}{5} = \frac{I_1'}{20} + 18$$

$$V_A = 43.2 \text{ V}$$

$$I_1' = \underline{-2.16 \text{ A}}, \quad I_2' = \underline{7.20 \text{ A}}, \quad I_3' = \underline{8.64 \text{ A}}$$

(2)



$$\begin{cases} -140 + 20i_1 + 6(i_1 - i_2) = 0 \\ 6(i_2 - i_1) + 5i_2 = 0 \end{cases}$$

$$i_1 = 6.16 \text{ A} \quad , \quad i_2 = 3.36 \text{ A}$$

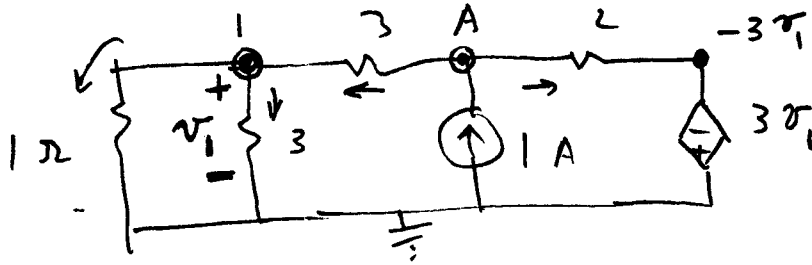
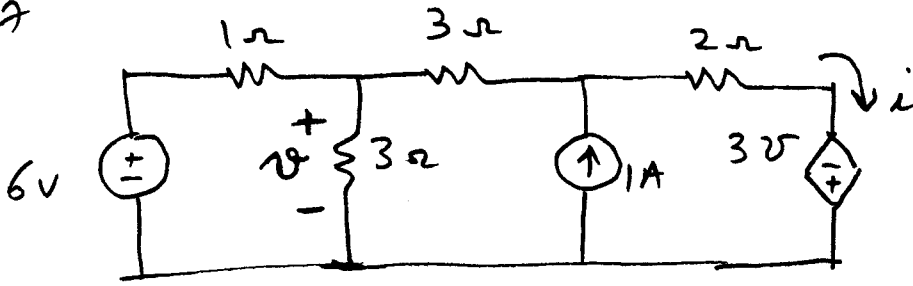
$$\therefore \begin{matrix} \rightarrow \\ i_1 \end{matrix} I_1'' = \underline{6.16}$$

$$\begin{matrix} \leftarrow \\ i_1 - i_2 \end{matrix} I_2'' = i_1 - i_2 = \underline{2.80 \text{ A}}$$

$$\begin{matrix} \leftarrow \\ i_2 \end{matrix} I_3'' = \underline{3.36 \text{ A}}$$

$$\therefore \begin{cases} I_1 = 4 \text{ A} \\ I_2 = 10 \text{ A} \\ I_3 = 12 \text{ A} \end{cases}$$

2.49 :
97

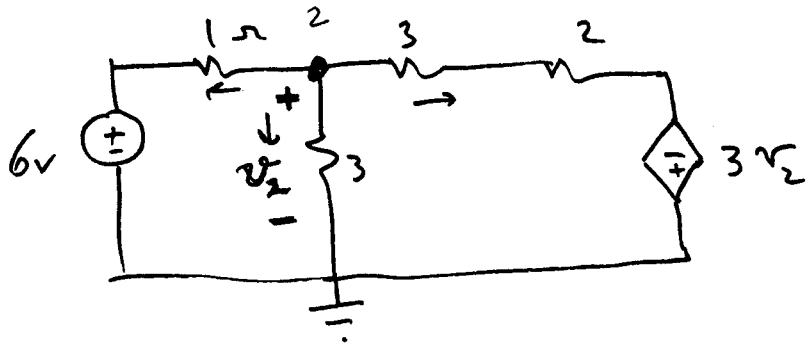


$$\text{KCL 1: } \frac{v_1}{1} + \frac{v_1}{3} = \frac{v_A - v_1}{3} \quad \Rightarrow \quad 5v_1 = v_A.$$

$$\text{KCL A: } \frac{v_A - v_1}{3} + \frac{v_A - (-3v_1)}{2} = 1$$

$$\cancel{2v_A} - 2v_1 + \cancel{3v_A} + 9v_1 = 6 \quad \Rightarrow \quad 5v_A + 7v_1 = 6.$$

$$\therefore v_1 = \frac{3}{16} \text{ V.}$$

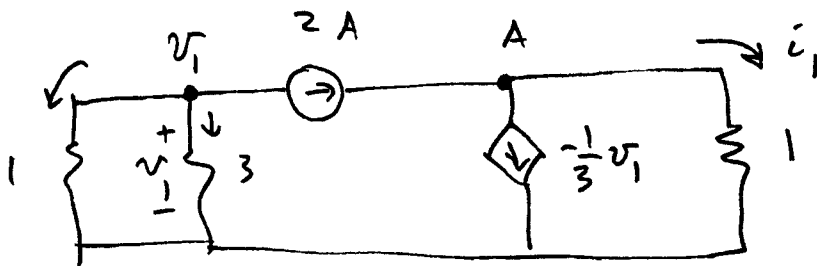
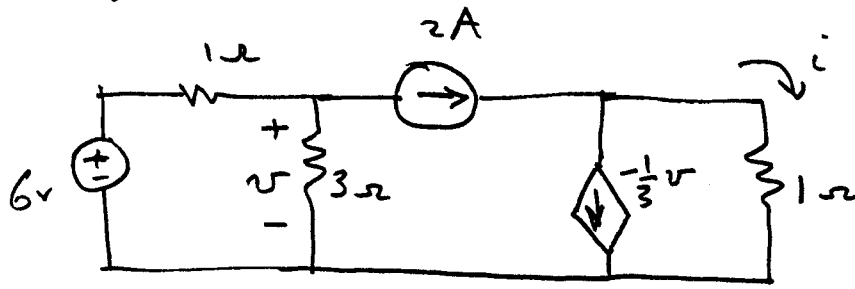


$$\frac{v_2 - 6}{1} + \frac{v_2}{3} + \frac{v_2 - (-3v_2)}{5} = 0$$

$$\Rightarrow v_2 = \frac{45}{16} \cdot V$$

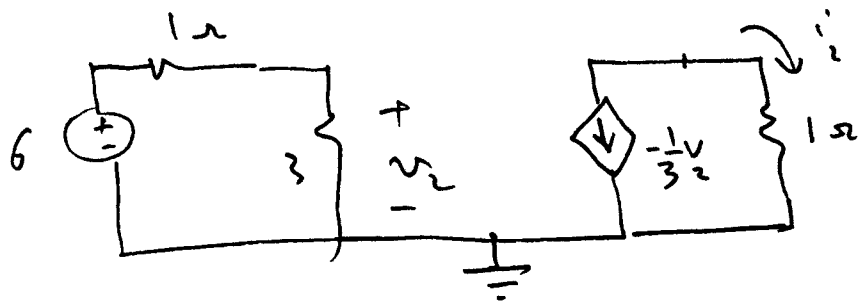
$$\therefore v = v_1 + v_2 = \frac{3}{16} + \frac{45}{16} = \underline{\underline{3V}}$$

2.50
98



$$\frac{v_1}{1} + \frac{v_1}{3} + 2 = 0 \quad \Rightarrow \quad v_1 = -\frac{3}{2} \text{ V.}$$

KCL A: $2 = -\frac{1}{3} \cancel{\frac{-3}{2}} + i_1 \Rightarrow i_1 = 2 + \frac{1}{2} = \frac{3}{2} \text{ A.}$



voltage division :

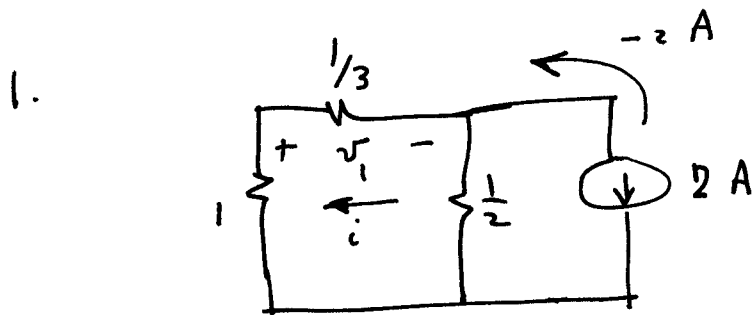
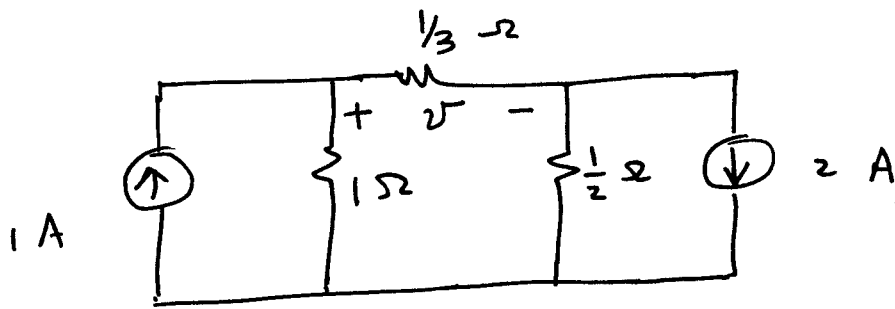
$$v_2 = \frac{3}{1+3} 6 = \frac{9}{2} \text{ V.}$$

$$\text{also: } i_2 = -\left(-\frac{1}{3} v_2\right) = \frac{v_2}{3} = \frac{9}{2} \cdot \frac{1}{3} = \frac{3}{2} \text{ A.}$$

$$\therefore v = v_1 + v_2 = -\frac{3}{2} + \frac{9}{2} = 3 \text{ V.}$$

$$i = i_1 + i_2 = \frac{3}{2} + \frac{3}{2} = 3 \text{ A.}$$

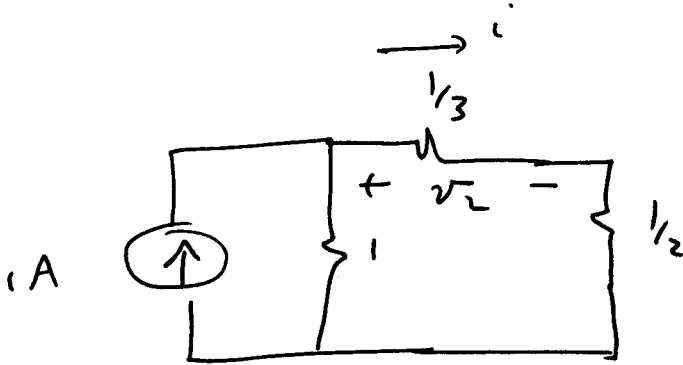
Exa.: Find v using superposition.



$$i = \frac{\frac{1}{2}}{1 + \frac{1}{3} + \frac{1}{2}} (-2) = -\frac{6}{11} \text{ A}$$

$$v_1 = \left(-\frac{1}{3}\right) \left(-\frac{6}{11}\right) = \frac{2}{11} \text{ V}$$

2.



$$i = \frac{1}{1 + \frac{1}{2} + \frac{1}{3}} \quad (1) = \frac{6}{11} \text{ A}$$

$$\therefore v_2 = \left(\frac{1}{3}\right) \left(\frac{6}{11}\right) = \frac{2}{11} \text{ V}$$

$$\therefore V = v_1 + v_2 = \frac{2}{11} + \frac{2}{11} = \frac{4}{11} \text{ V}$$

