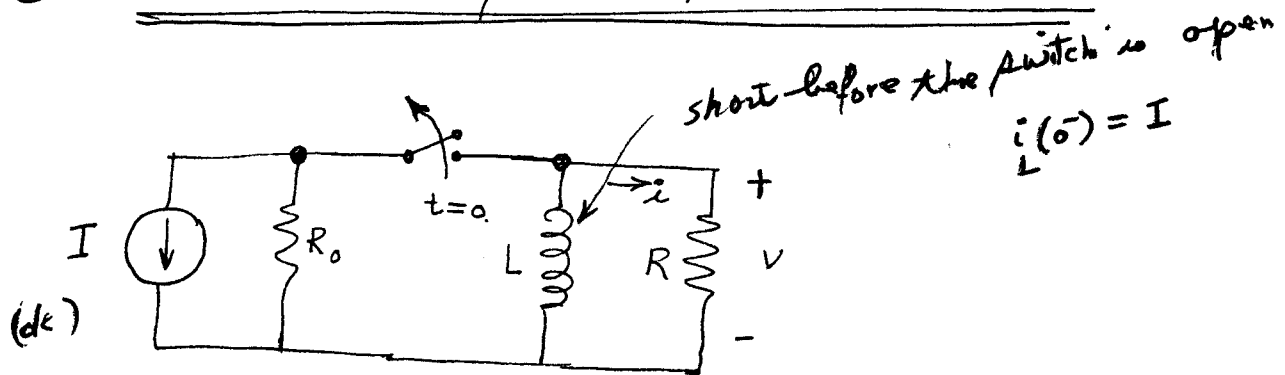


chap 7

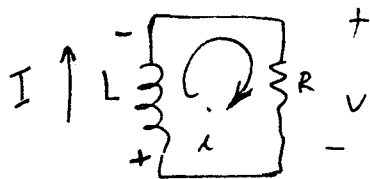
the natural response of RL and RC circuits

① Natural response of RL circuit:



let the switch be closed for $t < 0$.

for $t \geq 0$ we have



$$L \frac{di}{dt} + Ri = 0 \quad \text{1st-order linear DE}$$
$$\int_{i(t_0)}^{i(t)} \frac{di}{i} = -\frac{R}{L} \int_{t_0=0}^t dt \quad \Rightarrow \quad \ln \frac{i(t)}{i(0)} = -\frac{R}{L} t$$

$$i(t) = i(0) e^{-\frac{R}{L} t} \quad ; \quad t \geq 0.$$

$i(0^-) = i(0^+) = I = I_0$ (there cannot be an instantaneous change of current in an inductor). \Downarrow

$\therefore i(t) = I_0 e^{-\frac{R}{L} t}$, $t \geq 0$

$v = L \frac{di}{dt}$
 if i change
 - instantaneous const. $v \rightarrow \infty$
impossible!

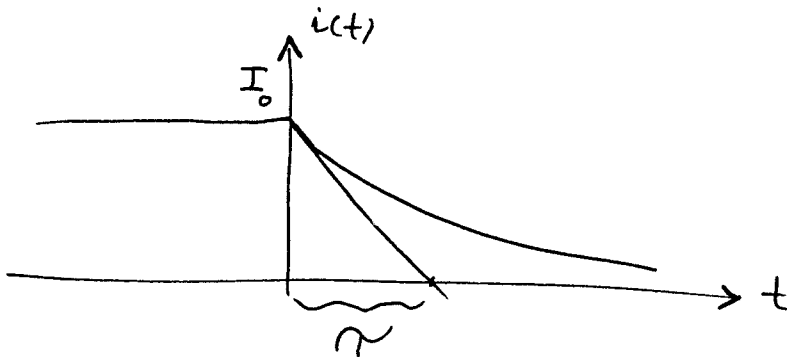
$\tau \triangleq \frac{L}{R} = \text{time constant (sec)}$

Time constant = the rate at which the current approaches zero

$i(t) = I_0 e^{-\frac{t}{\tau}}$; $t \geq 0$

note:

$\left. \frac{di(t)}{dt} \right|_{t=0} = -\frac{1}{\tau} I_0 e^{-t/\tau} \Big|_{t=0} = -\frac{I_0}{\tau}$



note: After five time constants the current is a negligible fraction of the initial value. (table 6.1)

$$v = iR = I_0 R e^{-t/\tau}, \quad t \geq 0.$$

Power dissipated in R is

$$P = vi = i^2 R = \frac{v^2}{R} = I_0^2 R e^{-2t/\tau}, \quad t \geq 0.$$

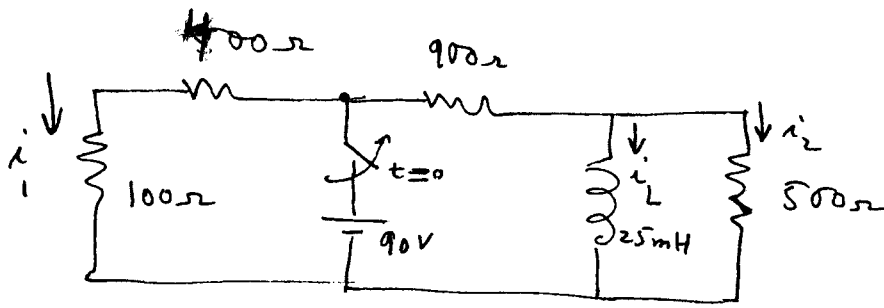
the energy delivered to R is

$$W = \int_0^t P(t') dt' = \int_0^t I_0^2 R e^{-2t'/\tau} dt'$$

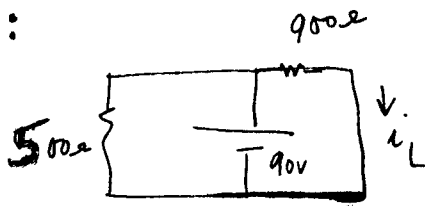
$$= (I_0^2 R) \frac{L/R}{2} (1 - e^{-2t/\tau}) = \frac{1}{2} L I_0^2 (1 - e^{-2t/\tau})$$

; $t \geq 0$.

Exa: Find a) $i_L(t)$; b) $i_1(t), t \geq 0$; $i_2(t), t \geq 0$.

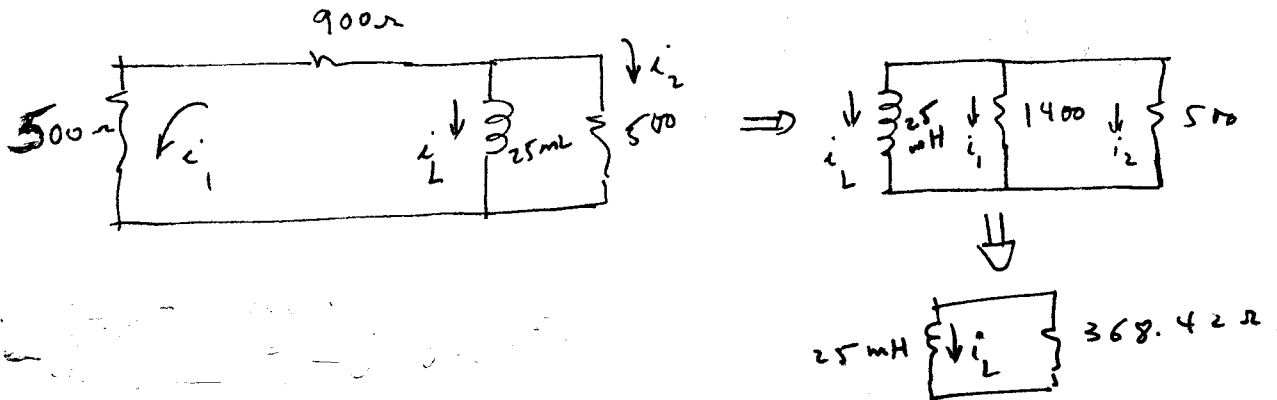


$t < 0$:



$$i_L(0^-) = i_L(0^+) = i_L(0) = \frac{90}{900} = 0.1 \quad A$$

$t > 0$:



$$R_{eq} = (500 + 900) \parallel 500 = \frac{(1400)(500)}{1900} = 368.42 \Omega$$

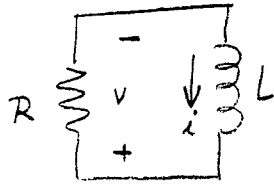
$$\tau = \frac{L}{R_{eq}} = 0.0679 \text{ ms. } t \text{ in ms}$$

$$i_L(t) = 100 e^{-14.72 t} \text{ mA}, \quad t \geq 0$$

$$i_1(t) = \frac{500}{1400 + 500} (-i_L(t)) = -26.32 e^{-14.72 t} \text{ mA}; \quad t \geq 0$$

$$i_2(t) = \frac{1400}{1400 + 500} (-i_L(t)) = -73.68 e^{-14.72 t} \text{ mA}; \quad t \geq 0$$

#6.1
197:



$$i = 8 e^{-20t}, \quad v = 240 e^{-20t} \quad \text{and} \quad \tau = \frac{1}{20}$$

(a) $R = ?$

$$v = i R \quad \therefore \quad 240 = 8 R \quad \Rightarrow \quad R = \underline{\underline{30 \Omega}}$$

(b) $L = ?$

$$\tau = \frac{L}{R} \quad \Rightarrow \quad L = \tau R = \frac{1}{20} \cdot 30 = \underline{\underline{1.5 \text{ H}}}$$

(c) $\tau = \underline{\underline{\frac{1}{20} \text{ Sec.}}}$

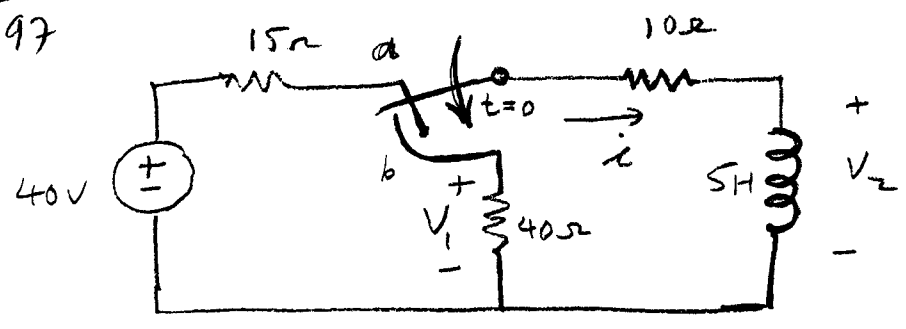
(d) $W = \frac{1}{2} L I_0^2 = \frac{1}{2} (1.5) (8^2) = \underline{\underline{48 \text{ J}}}$

(e) $P = v i = 1920 e^{-40t}$

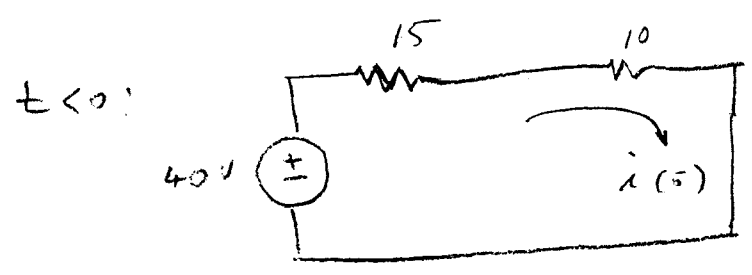
$$W = \int_0^{0.1} 1920 e^{-40t} dt = \frac{1920}{40} (1 - e^{(-40)(0.1)})$$

$$= 47.12 \text{ J.}$$

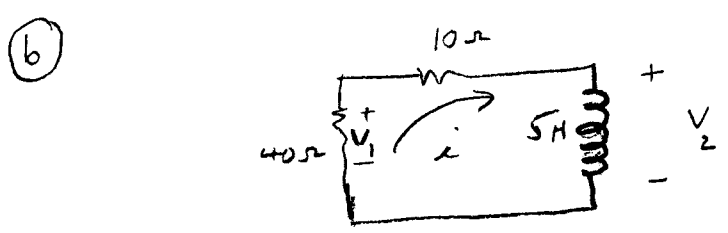
#6.2 :
197



(a) $i(0^-) = i(0^+) = ?$



$$i(0^-) = \frac{40}{25} = 1.6 \text{ A}$$



$$\tau = \frac{L}{R} = \frac{5}{50} = 0.1 \text{ sec}$$

(c) $i = 1.6 e^{-t/0.1} = 1.6 e^{-10t} \text{ A}$

$$V_1 = \frac{40}{50} \quad V_2 = \frac{40}{50} \left\{ 5 \frac{d}{dt} i \right\} = 4 \left(-16 e^{-10t} \right)$$

$$= -64 e^{-10t}$$

or $V_{15} - 40i = -40(1.6 e^{-10t}) =$

$$V_2 = -\frac{80}{5} e^{-10t}$$

$$\text{--- } \textcircled{d} \quad W_0 = \frac{1}{2} \frac{K}{5} (1.6)^2 = 6.4 \text{ J}$$

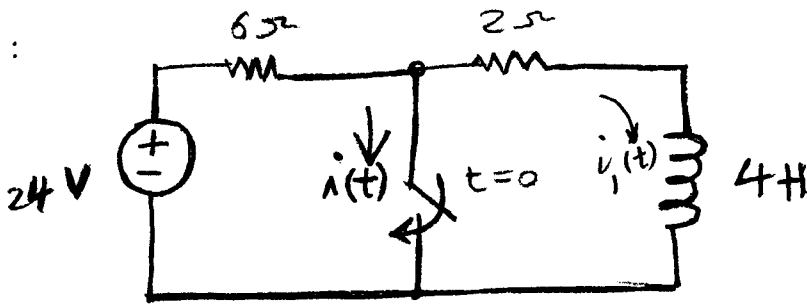
$$P_{10} = 10 i(t)^2 = 25.6 e^{-20t}$$

$$W_{10} = 25.6 \int_0^{0.05} e^{-20t} dt = (25.6) \left[\frac{1}{20} (1 - e^{(-20)(0.05)}) \right]$$

$$= 0.8091 \text{ J}$$

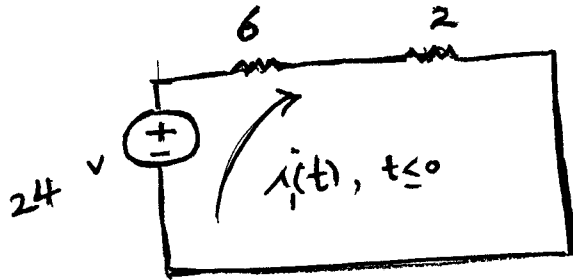
$$\frac{0.8091}{6.4} \times 100\% = 12.64\%$$

6.3
1.97



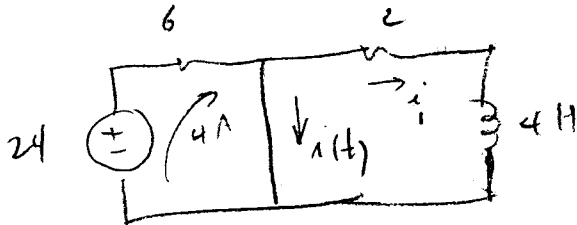
*
important
Find $i(t)$.

(a)



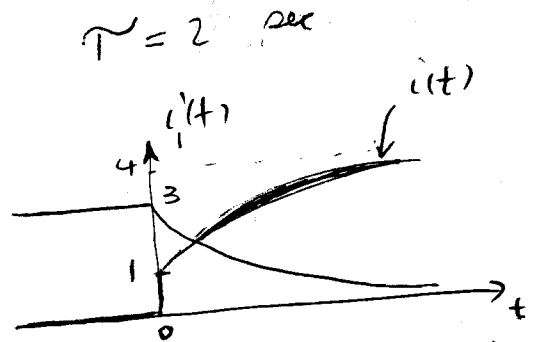
$$i_1(0^-) = \frac{24}{8} = 3 \text{ A}$$

$t > 0$



$$i_1(t) = 3e^{-t/2} \text{ A}, \quad t > 0$$

$$i(t) = 4 - i_1(t) = (4 - 3e^{-t/2}) \text{ u(t)}$$



$i(t) = 2 \text{ A}$
Find t ?

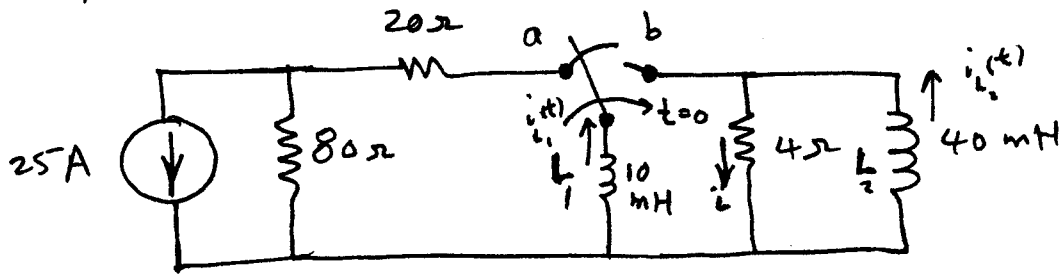
b)

$$4 - 3e^{-t/2} = 2$$

$$3e^{-t/2} = 2 \Rightarrow e^{-t/2} = \frac{2}{3}$$

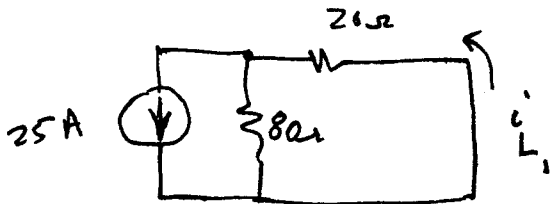
$$\ln\left(\frac{2}{3}\right) = -\frac{t}{2} \Rightarrow \underline{\underline{t = 0.81 \text{ sec}}}$$

$$\frac{6.8}{199} :$$



a) $i(t) = ? ; t \geq 0$

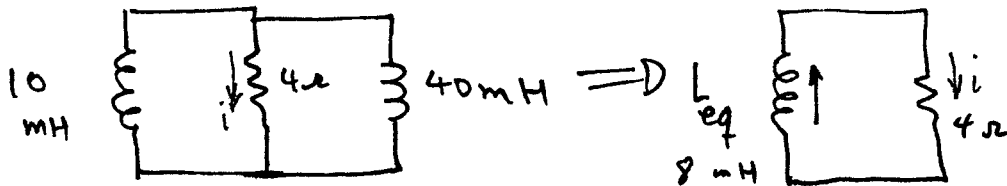
$t < 0 :$



$$-i_{L_1}(0^-) = \frac{80}{20+80} (-25) = -20 \text{ A} \Rightarrow i_{L_1}(0^-) = 20 \text{ A}$$

$$i_{L_2}(0^-) = 0$$

$t > 0 :$



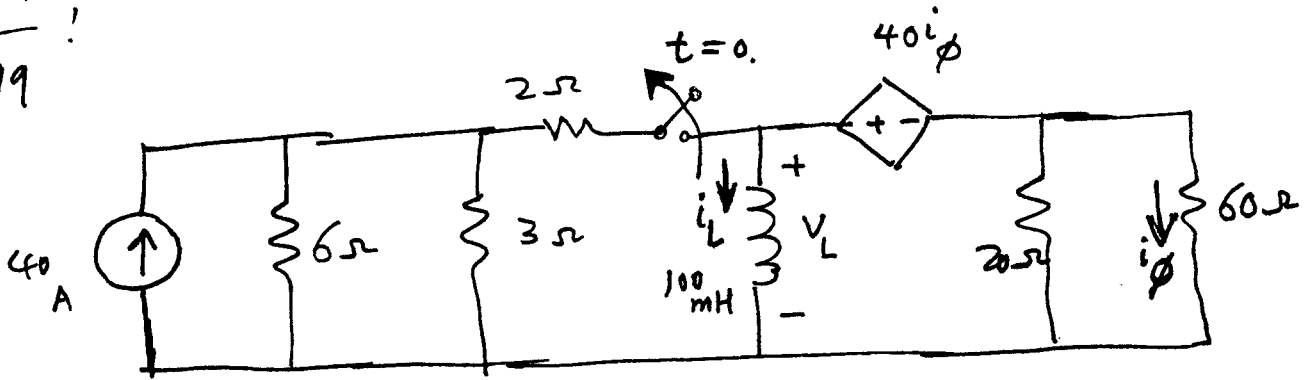
$$L_{eq} = \frac{(10)(40)}{10+40} = 8 \text{ mH}$$

$$\tau = \frac{L_{eq}}{R} = \frac{(8)(10^{-3})}{4} = 2 \text{ ms} , \quad i_L(0) = i_{L_1}(0) + i_{L_2}(0) = 20 \text{ A}$$

$$i(t) = 20 e^{-t/2\text{ms}} = 20 e^{-500t} \text{ A} \quad t \geq 0$$

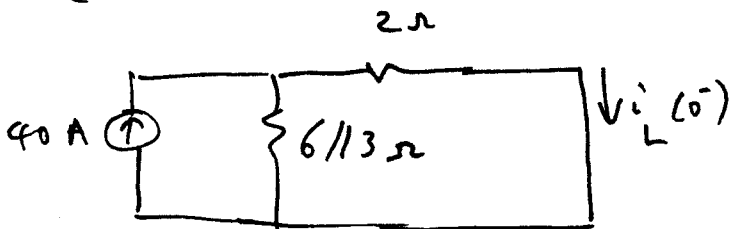
$$b) \quad W_{\text{diss}} = \frac{1}{2} L_{\text{eq}} I_0^2 = \frac{1}{2} (8 \times 10^{-3}) (20)^2 = \underline{1.6 \text{ J}}$$

$$\frac{6.9}{199}!$$



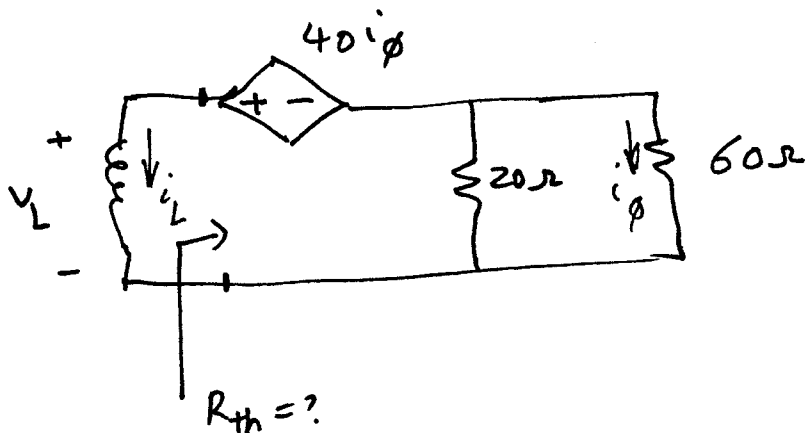
$$a) \quad i_L(t) = ? ; t \geq 0$$

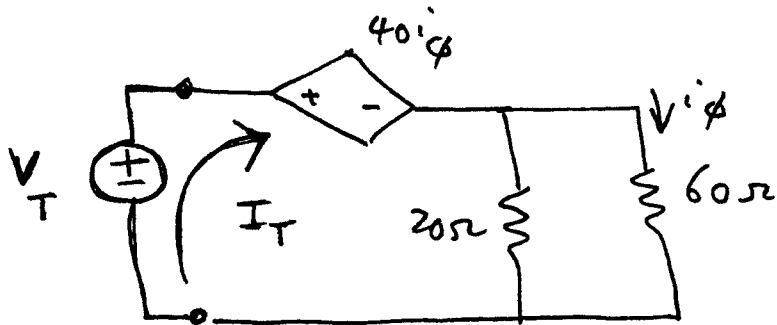
$t < 0$



$$i_L(0^-) = \frac{6 \parallel 3}{2 + 6 \parallel 3} (40) = 20 \text{ A}$$

$t > 0$





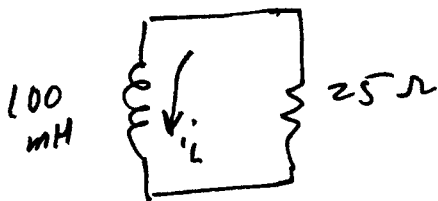
$$-V_T + 40 i_\phi + (20 \parallel 60) I_T = 0.$$

$$i_\phi = \frac{20}{20+60} I_T = \frac{1}{4} I_T$$

$$-V_T + 40 \left(\frac{1}{4} \right) I_T + (20 \parallel 60) I_T = 0.$$

$$\frac{V_T}{I_T} = 10 + \frac{(20 \parallel 60)}{80} = 25 \Omega$$

$$\begin{cases} R_{Th} = 25 \Omega \\ V_{Th} = 0 \end{cases}$$



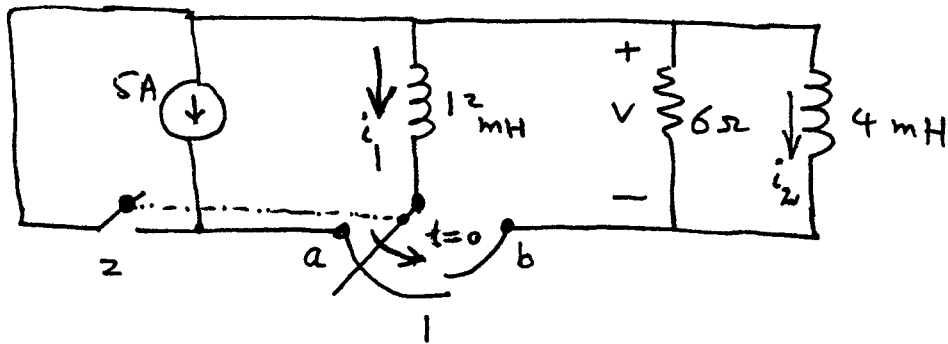
$$\tau = \frac{L}{R} = \frac{100 \text{ mH}}{25} = 4 \text{ ms}$$

$$i_L(t) = 20 e^{-t/4 \text{ ms}} = 20 e^{-250t} \text{ A}, \quad t \geq 0$$

$$b) \quad v_L = L \frac{di_L}{dt} = (100)(10^{-3})(20)(-250) e^{-250t} ; A \quad t \geq 0$$
$$= -500 e^{-250t} ; v \quad t \geq 0.$$

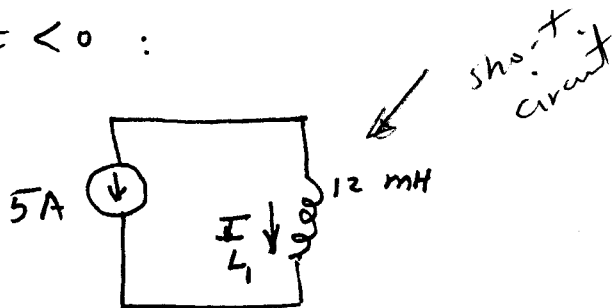
$$c) \quad i_{\phi} = \frac{20}{20+60} (-i_L) = \frac{-1}{4} (20 e^{-250t})$$
$$= -5 e^{-250t} ; A \quad t \geq 0.$$

6.11 :
199



a) $v, i_1, i_2 \Rightarrow ? \quad t \geq 0.$

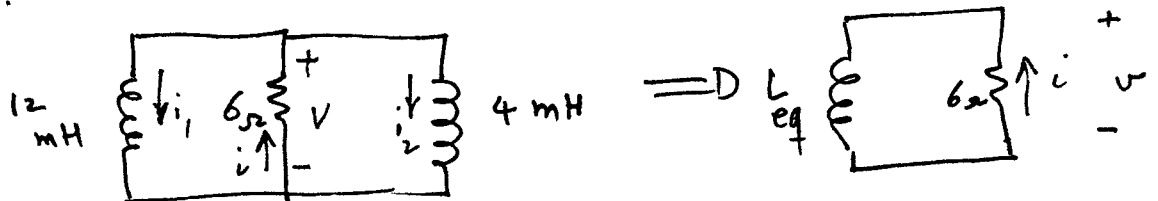
$t < 0 :$



$$I_{L_1} = i_{L_1}(0^-) = -5 \text{ A}$$

$$I_{L_2} = 0 \text{ A}$$

$t \geq 0 :$



$$i(0^-) = i_1(0^-) + i_2(0^-) = -5 \text{ A}, \quad L_{eq} = \frac{(4)(12)}{4+12} = 3 \text{ mH}$$

$$\tau = \frac{L_{eq}}{R} = \frac{(3)(10^{-3})}{6} = .5 \text{ ms}$$

$$i(t) = -5 e^{-t/(.5 \times 10^{-3})} = -5 e^{-2000t} \text{ A, } t \geq 0$$

$$v = -i(b) = 30 e^{-2000t} \text{ V, } t \geq 0$$

$$i_1 = \frac{1}{L_1} \int_0^t v dt - 5 = -1.25 e^{-2000t} - 3.75$$

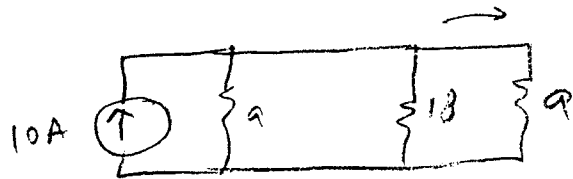
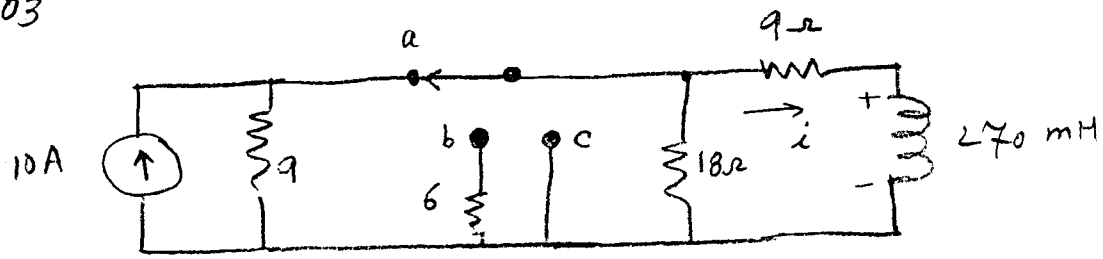
$$i_2 = \frac{1}{L_2} \int_0^t v dt = -3.75 e^{-2000t} + 3.75$$

Final values

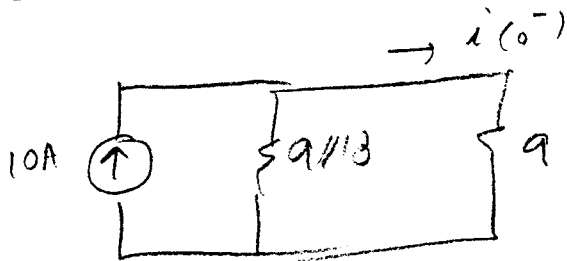
$$b) W(\text{Trapped}) = \frac{1}{2} L_1 (-3.75)^2 + \frac{1}{2} L_2 (3.75)^2 = 112.5 \text{ mJ.}$$

$$c) W(\text{diss}) = \int_0^{\infty} \left(\frac{v^2}{4R} \right) dt = 37.5 \text{ mJ.}$$

6.21 :
203

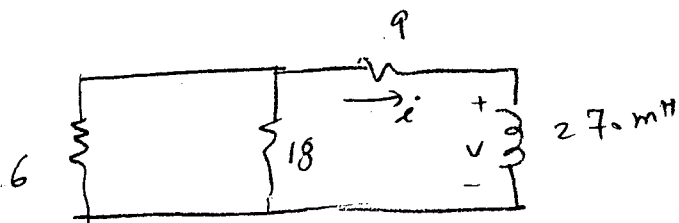


(a) $i(0^+) = ?$



$$i(0^-) = \frac{\frac{9 \parallel 18}{6} \cdot 10}{9 + \frac{9 \parallel 18}{6}} = \frac{60}{15} = 4 \text{ A}$$

(b)



$0 \leq t \leq 10 \text{ msec.}$

$$R_{eq} = 6 \parallel 18 + 9 = 4.5 + 9 = 13.5 \ \Omega$$

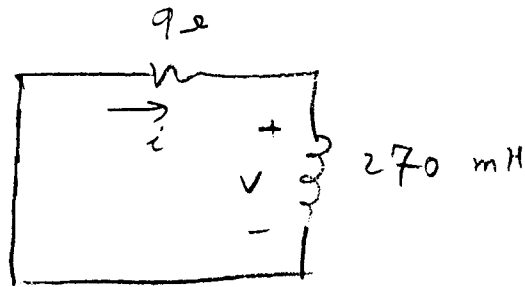
$$\tau = \frac{L}{R_{eq}} = \frac{270}{13.5} = 20 \text{ ms.}$$

$$i(t) = 4 e^{-t/0.02} \Rightarrow i(8\text{ms}) = 4 e^{-8/20} = 2.68 \text{ A.}$$

©

$$i(16 \text{ ms}) = ?$$

$$10 \leq t$$



$$T_2 = \frac{270}{9} = 30 \text{ ms} \quad \checkmark$$

$$i(10 \text{ ms}) = 4e^{-10/20} = 4e^{-1/2} = 2.4261 \quad \checkmark$$

$$i(t) = i(10) e^{-(t-t_0)/\tau} = 2.4261 e^{-(t-0.01)/0.03}$$

$$\text{for } 10 \leq t$$

$$i(16 \text{ ms}) = 2.4261 e^{-(0.016-0.01)/0.03} = 1.99 \text{ A}$$

Recall:

$$L \frac{di}{dt} + Ri = 0$$

$$\int_{i(t_0)}^{i(t)} \frac{di}{i} = -\frac{R}{L} \int_{t_0}^t dt \Rightarrow \ln \frac{i(t)}{i(t_0)} = -\frac{R}{L} (t-t_0)$$

$$\therefore i(t) = i(t_0) e^{-(t-t_0)/\tau} \quad \checkmark$$

(d)

$$v(10^{-} \text{ ms}) = ?$$

From (b) \rightarrow

$$v \Big|_{10^{-}} = L \frac{di}{dt} \Big|_{10^{-}} = (270) \frac{d}{dt} \left\{ 4e^{-t/0.02} \right\} \Big|_{10^{-}} = -32.75 \text{ V}$$

(e)

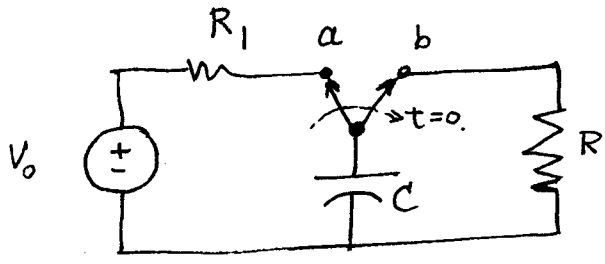
$$v(10^{+} \text{ ms}) = ?$$

From (c) \Rightarrow

$$v \Big|_{10^{+}} = L \frac{di}{dt} \Big|_{10^{+}} = 5(.175) \frac{d}{dt} \left\{ 2.429 e^{-(t-0.01)/0.02} \right\} \Big|_{10^{+}}$$

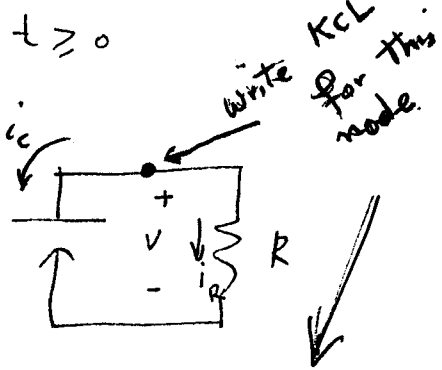
$$= -26.84 \text{ V}$$

Natural Response of RC circuit :



$$v_c(0^-) = v_c(0^+) = V_0$$

(there cannot be an instantaneous change of voltage in a capacitor)



$$i_c + i_R = 0$$

$$i = C \frac{dv}{dt}$$

if v changes instantaneously, $i \rightarrow \infty$ which is impossible!

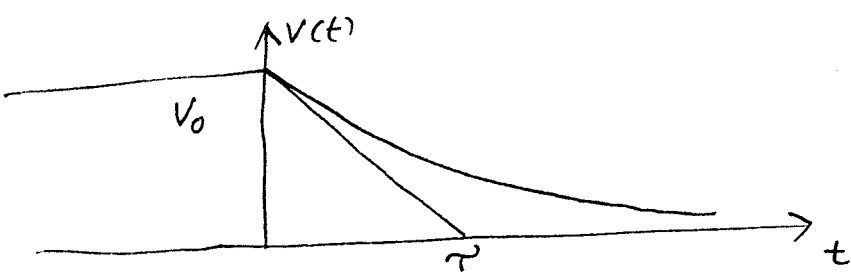
$$C \frac{dv}{dt} + \frac{v}{R} = 0$$

$$\Rightarrow \int_{v_0}^{v(t)} \frac{dv}{v} = -\frac{1}{RC} \int_0^t dt$$

$$\ln \frac{v}{v_0} = -\frac{1}{RC} t \Rightarrow v(t) = v_0 e^{-t/RC}$$

$$\tau \triangleq RC$$

$$\therefore v(t) = V_0 e^{-t/\tau}$$



$$\left. \frac{dv}{dt} \right|_{t=0} = -\frac{V_0}{\tau} e^{-t/\tau} \Big|_{t=0} = -\frac{V_0}{\tau}$$

$$i(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}, \quad t \geq 0$$

$$p = vi = \frac{V_0^2}{R} e^{-2t/\tau}, \quad t \geq 0$$

$$W = \int_0^t p(t) dt = \int_0^t \frac{V_0^2}{R} e^{-2t/\tau} dt$$

$$= \frac{V_0^2}{R} \frac{\tau}{2} (1 - e^{-2t/\tau})$$

$$= \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau}); \quad t \geq 0.$$

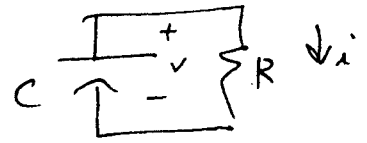
Exa.

6.13 :

200

$$v = 8e^{-5t} \text{ V}$$

$$i = 20e^{-5t} \mu\text{A}$$



(a)

$$v = iR \Rightarrow$$

$$8 = 20 \times 10^{-6} R \Rightarrow R = 400 \text{ k}\Omega$$

(b)

$$RC = \frac{1}{5} \Rightarrow C = \frac{1}{5R} = \frac{1}{(5 \times 400) \times 10^3}$$

2000

$$C = 0.5 \mu\text{F}$$

(c)

$$\tau = 5.2 \mu\text{s}$$

(d)

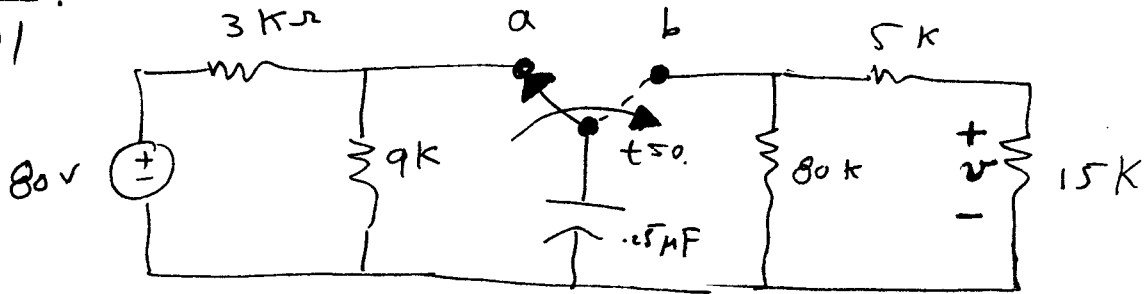
$$W_0 = \frac{1}{2} C V_0^2 = \left(\frac{1}{2}\right)(.5)(8^2) = 16 \mu\text{J}$$

(e)

$$W = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau})$$

$$= 16 (1 - e^{-\frac{2}{5.2}}) \approx 10.11 \mu\text{J}$$

6.14.
201



$v(t) = ?$

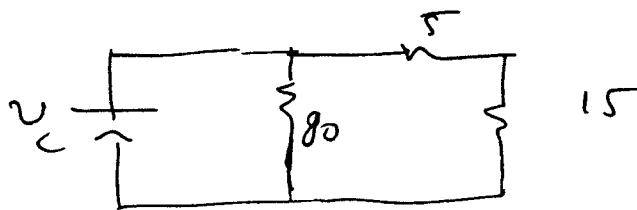
a) $v_c(0^-) = v_c(0^+) = \frac{9}{12} 80 = 60 \text{ V}$

$t \geq 0$: $R_{eq} = (5+15) // 80 = 16 \text{ k}\Omega$

$\tau = R_{eq} C = (16 \times 10^3) \cdot (0.25 \times 10^{-6}) = 4 \text{ ms}$

$v_c(t) = 60 e^{-t/0.004} = 60 e^{-250t} \text{ V}$

b) $v(t) = \frac{15}{20} v_c = 45 e^{-250t} \text{ V}$



$W(0) = \frac{1}{2} C V_e^2(0) = \frac{1}{2} (0.25 \times 10^{-6}) (60)^2 = 450 \mu\text{J}$

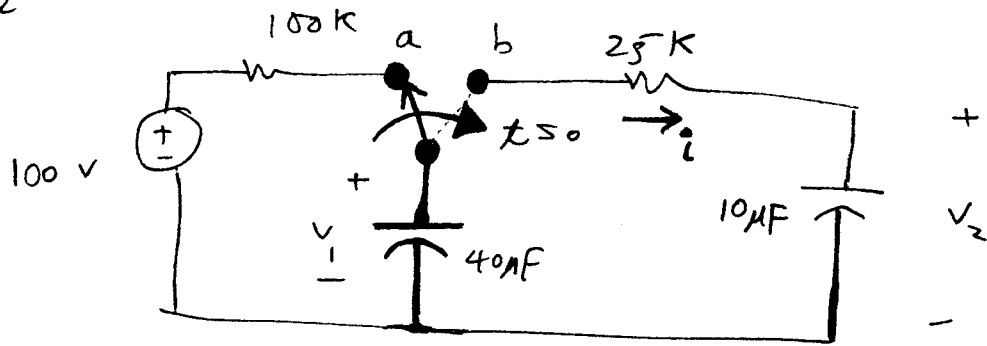
$W(\text{diss}) = \int_0^{0.004} \left(\frac{v_c^2}{80 \text{ k}\Omega} \right) dt = 77.82 \mu\text{J}$

$\therefore \dots = 17.29\%$ ← Power of 80 k.

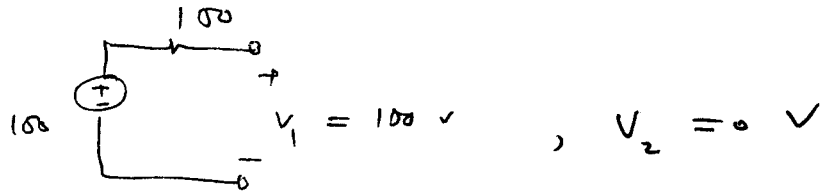
6.20 :

202

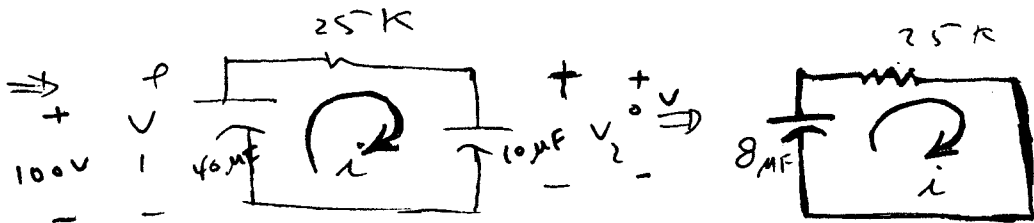
(a)



$t < 0 \Rightarrow$



$t > 0^+$



$$C_{eq} = 10 \parallel 40 = \frac{400}{50} = 8 \mu F$$

$$\tau = RC = (25 \times 10^3) (8 \times 10^{-6}) = 0.2 \text{ s}$$

$$v_1(t) = 100 e^{-5t} \text{ V}; \quad t > 0$$

$$i(t) = \frac{4}{25} e^{-5t} \text{ mA}; \quad t > 0$$

$$v_1(t) = \frac{-1}{c_1} \int_0^t i(t) dt + 100 = 20 e^{-5t} + 80 \text{ V}$$

$$v_2(t) = \frac{1}{c_2} \int_0^t i(t) dt + 0 = -80 e^{-5t} + 80 \text{ V}$$

$$(b) \quad W(0) = \frac{1}{2} C_1 V_1(0)^2 = \underline{\underline{200 \text{ mJ}}}$$

$$(c) \quad W(\text{Trapped}) = \frac{1}{2} C_1 V_1(\infty)^2 + \frac{1}{2} C_2 V_2(\infty)^2$$

$$= \left[\frac{1}{2} (40)(80)^2 + \frac{1}{2} (10)(80)^2 \right] (10^{-6}) = \underline{\underline{160 \text{ mJ}}}$$

$$(d) \quad W(\text{dissipated}) = 200 - 160 = \underline{\underline{40 \text{ mJ}}}$$

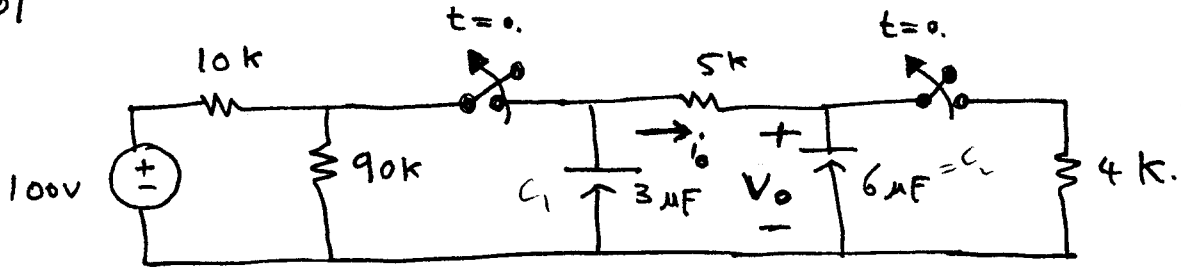
or

$$W(\text{dissipated}) = \int_0^{\infty} i^2(t) R dt$$

$$= \int_0^{\infty} \left(4e^{-5t} \times 10^{-3} \right)^2 (25 \times 10^3) dt$$

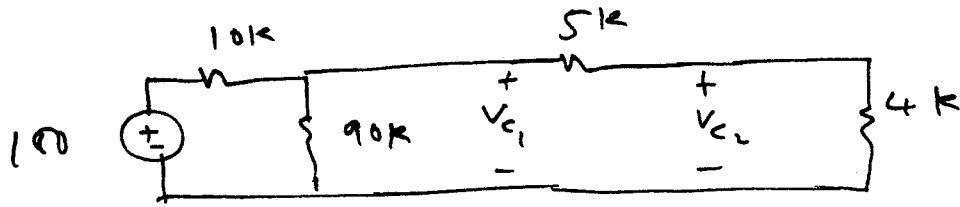
$$= 400 \times 10^{-3} \int_0^{\infty} e^{-10t} dt = 40 \text{ mJ.} \parallel$$

6.15 :
201



a)

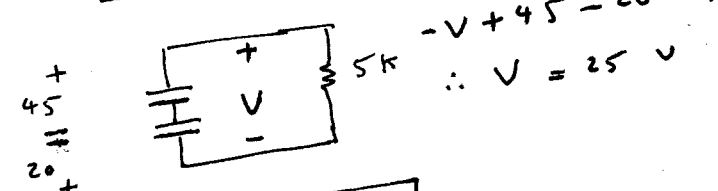
$t < 0$:



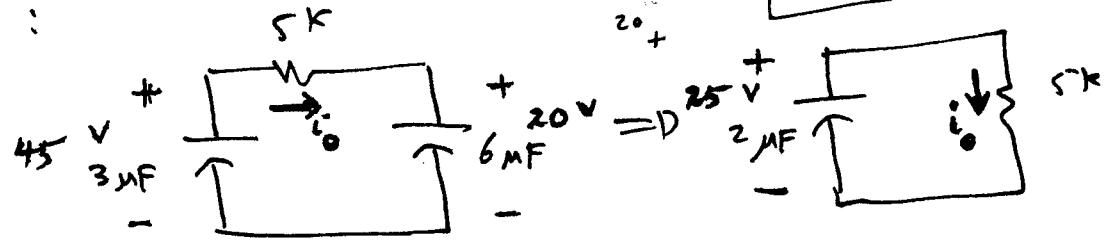
$$(4 + 5) \parallel 90 = \frac{(9)(90)}{90 + 9} = \frac{810}{99} = 8.18 \text{ k}\Omega$$

$$V_{C1} = \frac{8.18}{10 + 8.18} (100) \approx 45 \text{ V} ; t < 0$$

$$V_{C2} = \frac{4}{5 + 4} V_{C1} = \frac{(4)(45)}{9} = 20 \text{ V} ; t < 0$$



$t \geq 0$:



$$V_c(t) = 25 e^{-t/\tau} , \quad \tau = RC_{eq} = (5\text{k})(2 \times 10^{-6})$$

$$V_c(t) = 25 e^{-100t} \text{ V} , t \geq 0 \quad = 10 \text{ ms}$$

$$i_0 = \frac{v_c}{5k} = \frac{25}{5k} e^{-100t} = 5 e^{-100t} \text{ mA ; } t \geq 0$$

b)

$$v_{c_2}(t) = \frac{1}{6 \mu\text{F} \cdot \frac{1}{2}} \int_0^t i_0(\tau) d\tau + \underbrace{20}_{V_{c_2}}$$

$$= \frac{10^6}{6} \int_0^t (5 e^{-100t'}) (10^{-3}) dt' + 20$$

$$= -\frac{25}{3} e^{-100t} + \frac{85}{3} \text{ V}$$

c)

$$W(0) = \frac{1}{2} C_1 v_{c_1}^2(0) + \frac{1}{2} C_2 v_{c_2}^2(0) = 4237.5 \text{ } \mu\text{J.}$$

$$W_{\text{diss}} = \frac{1}{2} C_{\text{eq}} v_{c_{\text{eq}}}^2(0) = \frac{1}{2} (2)(25)^2 = 625 \text{ } \mu\text{J.}$$

$$W_{\text{Trapped}} = 4237.5 - 625 = 3612.5 \text{ } \mu\text{J.}$$

OR,

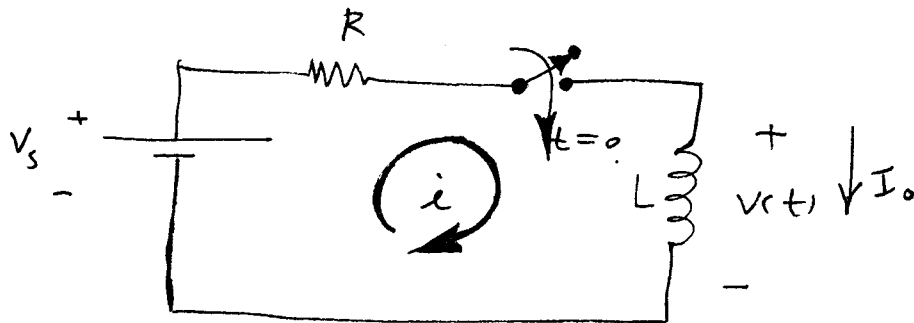
$$W_{\text{diss}} = \int_0^{\infty} i_0^2 (4)(5k) dt$$

$$= \int_0^{\infty} (5 e^{-100t} \times 10^{-3})^2 (5 \times 10^3) dt$$

$$= 625 \text{ } \mu\text{J.}$$

Chap 7.

Step Response of RL and RC circuit.



$$V_s = Ri + L \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{V_s - Ri}{L} = \frac{V_s}{L} - \frac{R}{L} i$$

$$= -\frac{R}{L} \left(i - \frac{V_s}{R} \right)$$

$$\int_{I_0}^{i(t)} \frac{di}{i - \frac{V_s}{R}} = \int_0^t -\frac{R}{L} dt$$

$$\ln \left(i - \frac{V_s}{R} \right) \Big|_{I_0}^{i(t)} = -\frac{R}{L} t$$

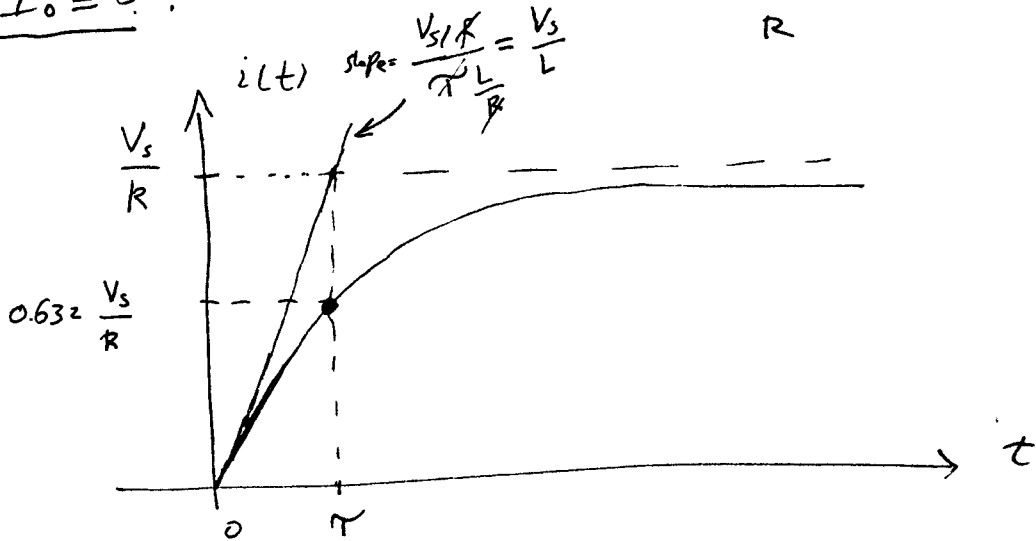
$$\ln \frac{i(t) - \frac{V_s}{R}}{I_0 - \frac{V_s}{R}} = -\frac{R}{L} t \Rightarrow i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R} \right) e^{-\frac{R}{L} t}$$

$t > 0$

$$i(t) = \text{Transient Res} + \text{Final value}$$

↑
 $\frac{V_s}{R}$

let $I_0 = 0$:



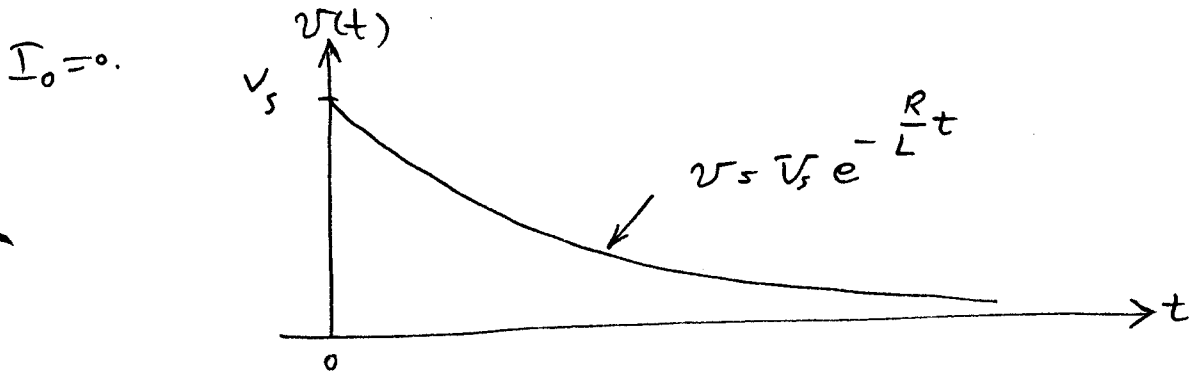
$$\left. \frac{di}{dt} \right|_{t=0} = -\frac{R}{L} \left(I_0 - \frac{V_s}{R} \right) e^{-\frac{R}{L}t} \Big|_{t=0} = \frac{V_s}{L}$$

if the current continued to increase at this rate, then

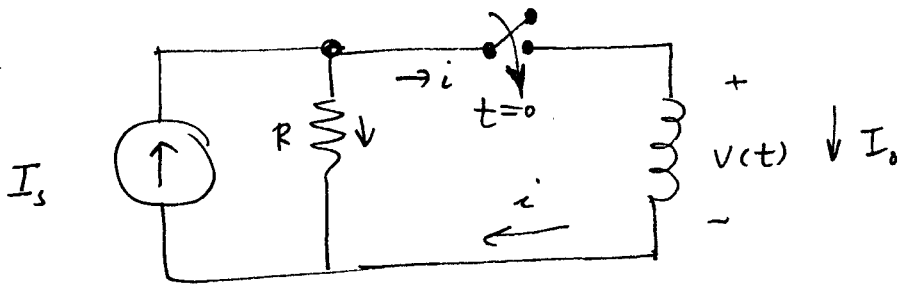
$$i(t) \sim \frac{V_s}{L} t$$

From which we see that at $t = \frac{L}{R}$ we get $i(t) \Big|_{t=\tau} = \frac{V_s \tau}{L} = \frac{V_s}{R}$

$$\text{Also, } v = L \frac{di}{dt} = L \left(-\frac{R}{L} \right) \left(I_0 - \frac{V_s}{R} \right) e^{-\frac{R}{L}t} = (V_s - I_0 R) e^{-\frac{R}{L}t}$$



Now assume that the series combination of V_s and R is converted via a source transformation to a parallel combination as shown:



KCL: $t > 0$

$$I_s = \frac{v}{R} + \underbrace{\frac{1}{L} \int_0^t v(t) dt + I_0}_{i(t)}$$

$$0 = \frac{1}{R} v'(t) + \frac{v(t)}{L}$$

$$\frac{dv}{dt} = -\frac{R}{L} v$$

$$\int_{v(0)}^{v(t)} \frac{dv}{v} = -\frac{R}{L} \int_0^t dt$$

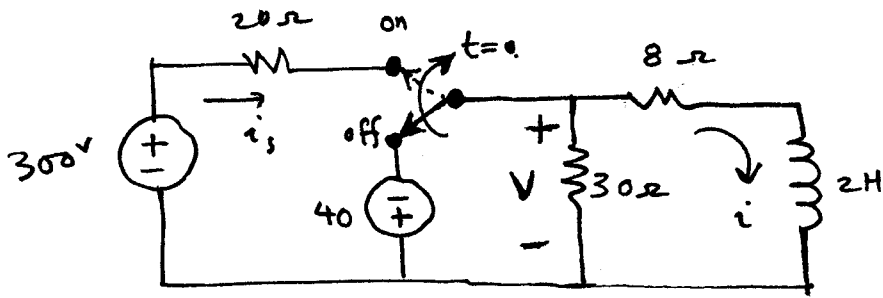
$$v(t) = v(0) e^{-\frac{R}{L}t} = R(I_s - I_0) e^{-\frac{R}{L}t} \quad \rightarrow \quad i(t) = I_s - \frac{v(t)}{R}$$

$$i(t) = \frac{1}{L} \int_0^t v(t) dt + I_0 \stackrel{\text{or}}{=} I_s - \frac{v(t)}{R} = I_s - \frac{v(0)}{R} e^{-\frac{R}{L}t}$$

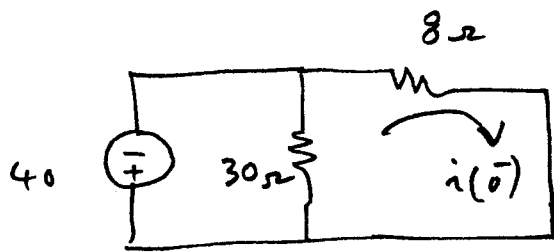
$$= I_s - \frac{RI_s - RI_0}{R} e^{-\frac{R}{L}t} = \cancel{I_s} + \left(\frac{I_0}{\cancel{I_s}} - \cancel{I_s} \right) e^{-\frac{R}{L}t}$$

note that the current of the inductor can not change instantaneously at $t=0$. Therefore, $v(0) = R(I_s - I_0)$.

7.2 :
227



a) $t < 0$: $i(t) = ? \quad t \geq 0$.

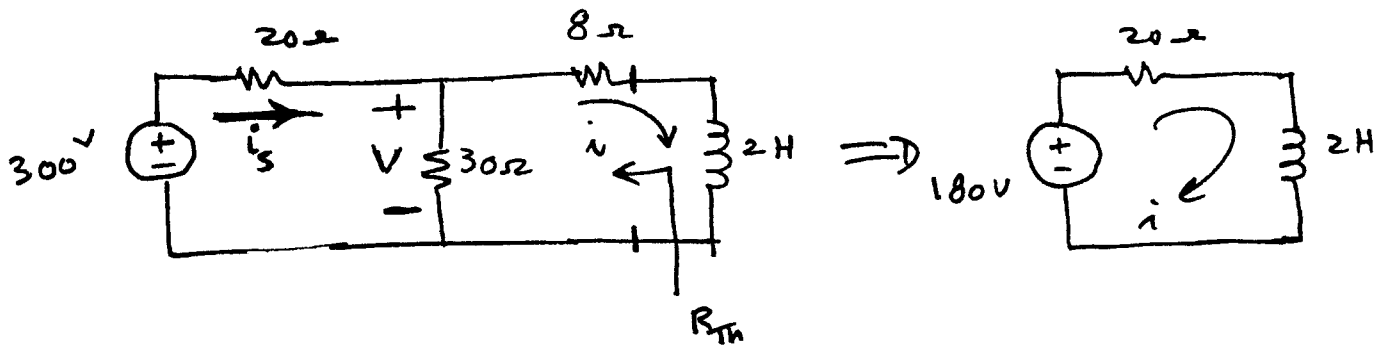


$$+40 + 8 i(0^-) = 0$$

$$\therefore i(0^-) = -\frac{40}{8}$$

$$i(0^-) = \frac{-40}{8} = -5 \text{ A}$$

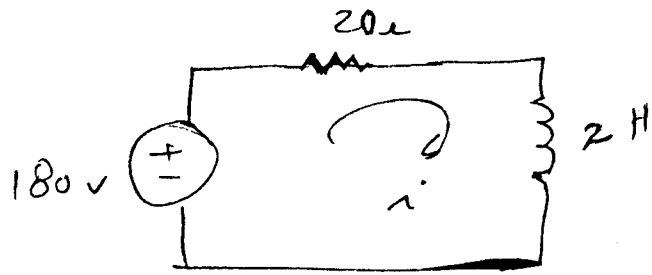
$t > 0$:



$$V_T = \frac{30}{20+30} (300) = 180 \text{ V}$$

$$\tau = \frac{L}{R_T} = \frac{2}{20} = 0.1 \text{ s}$$

$$R_T = 20 \parallel 30 + 8 = 20 \Omega$$



$$i'(t) = \frac{180}{20} + \left(-5 - \frac{180}{20}\right) e^{-10t} \quad ; \quad t \geq 0$$

$$i(t) = 9 - 14e^{-10t} \quad ; \quad t \geq 0$$

(b)

$$v(t) = v_8 + \overset{2}{L} \frac{di}{dt}$$

$$= 8i + 2 \frac{di}{dt}$$

$$= 8(9 - 14e^{-10t}) + 2(140e^{-10t})$$

$$= 72 + 168e^{-10t} \quad v \quad ; \quad t \geq 0$$

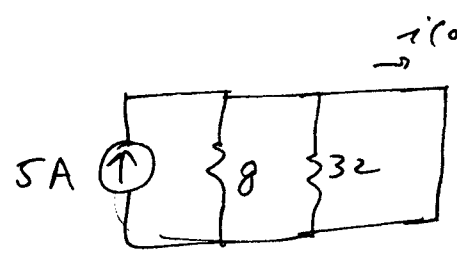
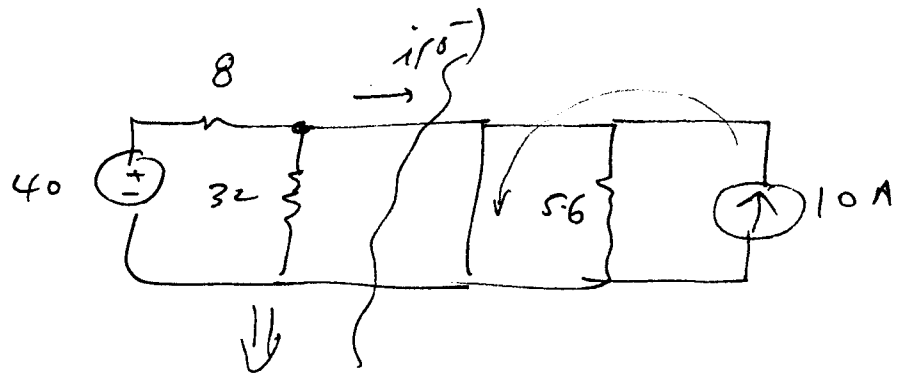
(c)

$$i_s = \frac{v(t)}{30} + i = \frac{72 + 168e^{-10t}}{30} + (9 - 14e^{-10t})$$

$$= 11.4 - 8.4e^{-10t} \quad A \quad ; \quad t \geq 0$$

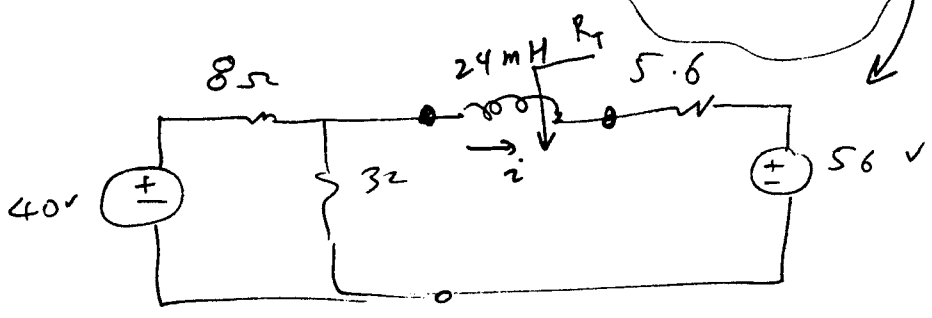
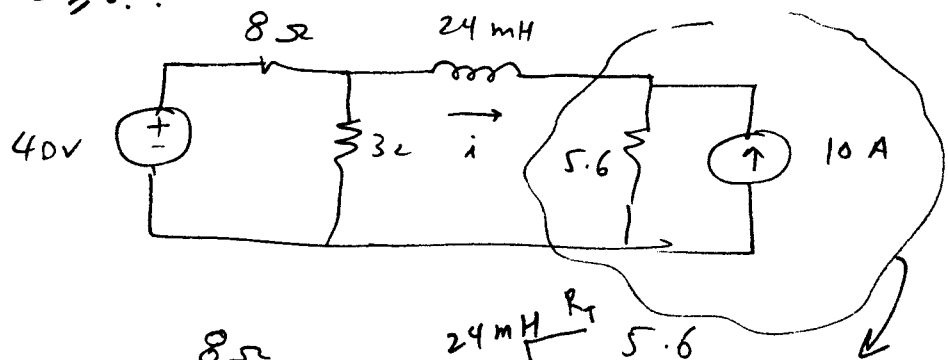
7.3
227

$t < 0$:



$i(0^-) = 5 \text{ A}$

$t \geq 0$:

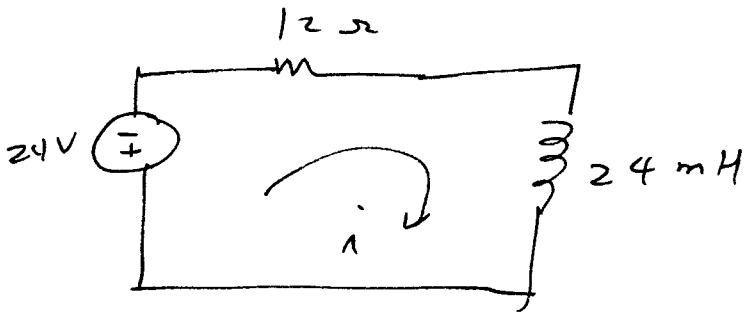


$$R_T = 8 // 32 + 5.6 = 12 \Omega$$

$$V_T = -24 \text{ V} \quad (\text{Verify}) \rightarrow$$

$$V_{32\Omega} = \frac{32}{8+32} \cdot 40 = 32$$

$$V_T = 32 - 56 = -24$$

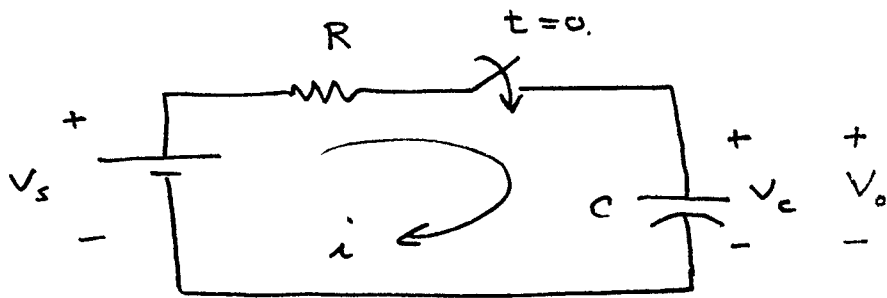


$$i(\infty) = \frac{V_T}{R_T} = \frac{-24}{12} = -2 \text{ A}$$

$$\tau = \frac{L}{R_T} = \frac{24}{12} = 2 \text{ ms}$$

$$i(t) = -2 + [5 - (-2)] e^{-500t} \quad ; t \geq 0$$

The step response of An RC Circuit



$t \geq 0$:

$$V_s = Ri + \frac{1}{C} \int_0^t i(\tau) d\tau + V_c(0)$$

$$0 = R \frac{di}{dt} + \frac{1}{C}$$

$$\frac{di}{dt} = -\frac{i}{RC} \Rightarrow \frac{di}{i} = -\frac{1}{RC} dt$$

$$\therefore \ln \left| \frac{i}{I_0} \right| = -\frac{1}{RC} t \Rightarrow i(t) = I_0 e^{-t/RC}; \quad t \geq 0$$

since there cannot be an instantaneous change in the voltage across the capacitor, we get

$$\underbrace{i(0)}_{I_0} = \frac{V_s - V_0}{R}$$

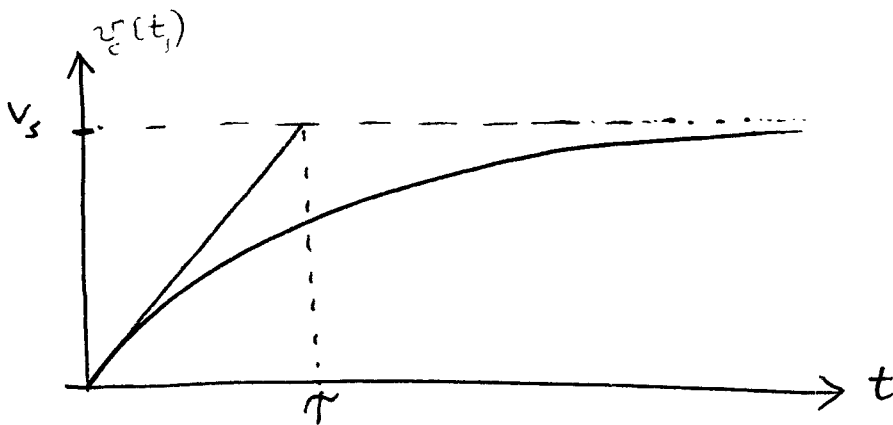
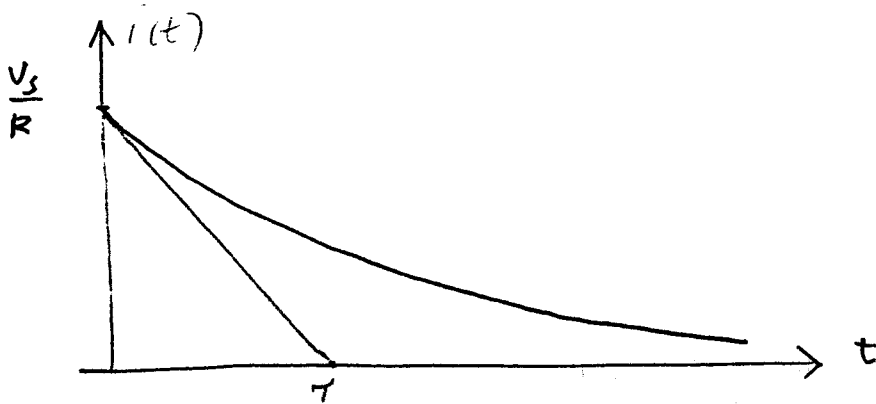
$$\therefore i(t) = \frac{V_s - V_0}{R} e^{-t/RC} \quad t \geq 0.$$

$$v_c(t) = V_s - iR$$

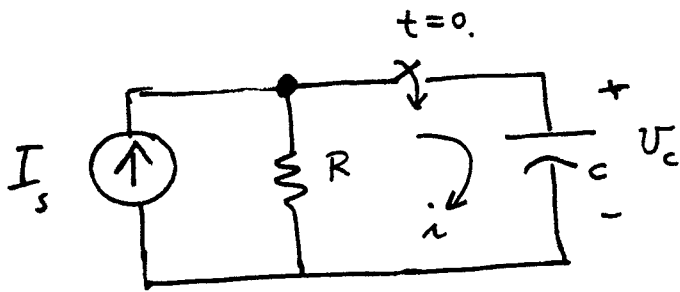
$$= V_s - R \left(\frac{V_s - V_0}{R} \right) e^{-t/RC}$$

$$= V_s + (V_0 - V_s) e^{-t/RC} ; t \geq 0$$

let $V_0 = 0$.



It is also informative to derive the current and voltage by starting the following circuit:



RCL: ($t \geq 0$)

$$I_s = \frac{V_c}{R} + C \frac{dV_c}{dt}$$

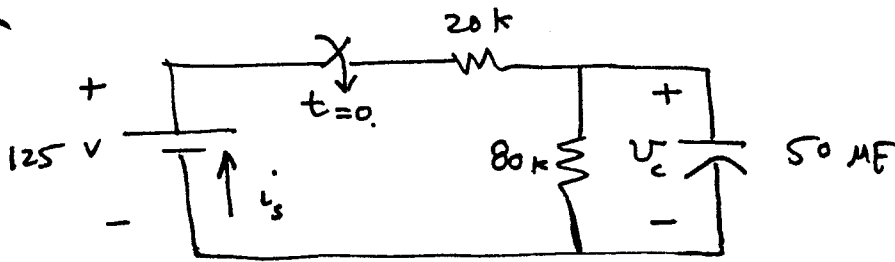
the form of this DE is similar to DE of the current of the inductor.

$$\therefore V_c = I_s R + (V_0 - I_s R) e^{-t/RC}$$

$$i(t) = C \frac{dV_c}{dt} = \cancel{C} \left(-\frac{1}{RC} \right) (V_0 - I_s R) e^{-t/RC}$$

$$= \frac{V_s}{R} \left(\cancel{I_s} - \frac{V_0}{R} \right) e^{-t/RC}, \quad t \geq 0.$$

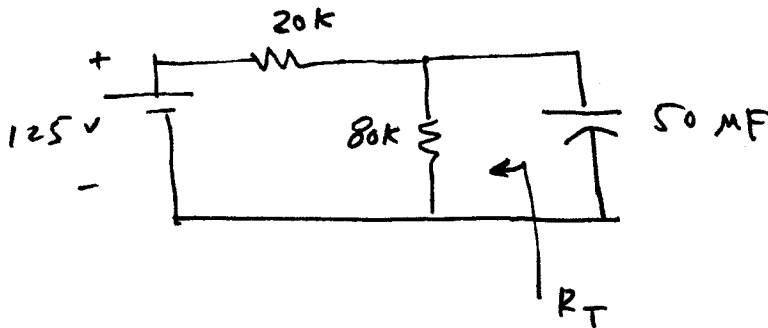
$$\frac{7.9}{229} :$$



a) $v_c(t) = ? ; t \geq 0$

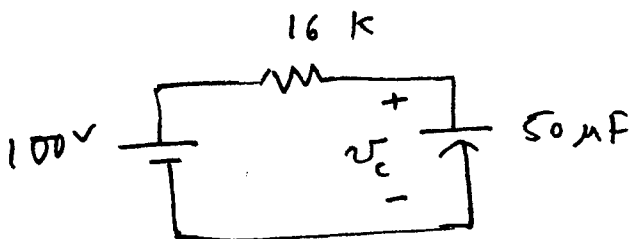
$t < 0 \Rightarrow v_c(t) = 0.$

$t \geq 0 :$



$$R_T = 20 \parallel 80 = \frac{(20)(80)}{20+80} = 16 \text{ k}$$

$$V_T = \frac{80}{20+80} (125) = 100 \text{ V.}$$



know

$$v_c(t) = V_s + (V_0 - V_s) e^{-t/RC}, \quad t \geq 0$$
$$= 100 + (0 - 100) e^{-t/RC}$$

$$\tau = RC = (16 \times 10^3)(50)(10^{-6}) = 800 \times 10^{-3} = .8$$

$$\therefore v_c(t) = 100 - 100 e^{-1.25t} \quad ; \quad t \geq 0.$$

b) $i_s(t) = ? \quad t \geq 0.$

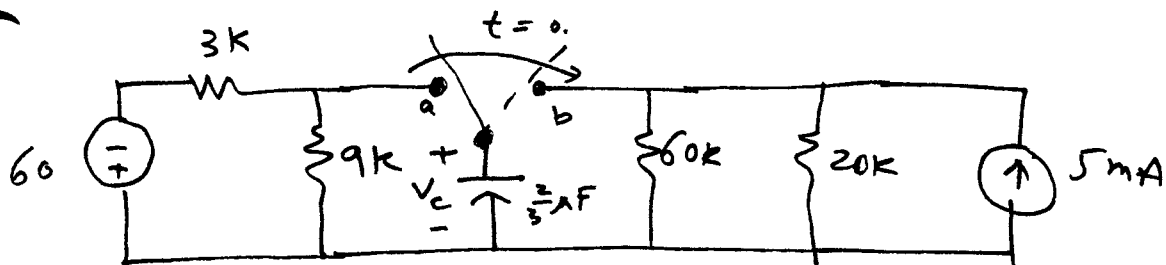
$$i_s(t) = i_{R_0} + i_c(t)$$

$$= \frac{v_c}{80k} + C \frac{dv_c}{dt}$$

$$= \frac{100 - 100 e^{-1.25t}}{(80)(10^3)} + (50 \times 10^{-6})(125 e^{-1.25t})$$

=

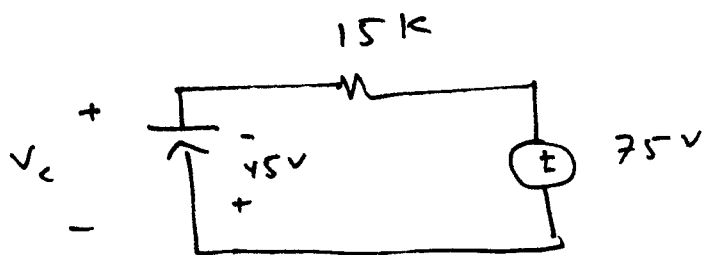
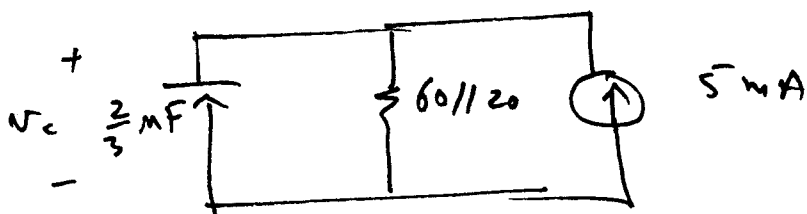
$$\frac{7.10}{229} :$$



$$v_c(t) = ? ; t \geq 0$$

$$t < 0 : \Rightarrow v_c(t^-) = v_c(t^+) = \frac{9}{3+9} (-60) = -45 \text{ V}$$

$$t > 0^+ :$$



$$\tau = RC = 10 \text{ ms}$$

$$v_c(\infty) = 75 \text{ V}$$

$$\therefore v_c(t) = 75 + (-45 - 75) e^{-10^4 t} = (75 - 120 e^{-10^4 t}) \text{ V}$$

$$t \geq 0^+$$