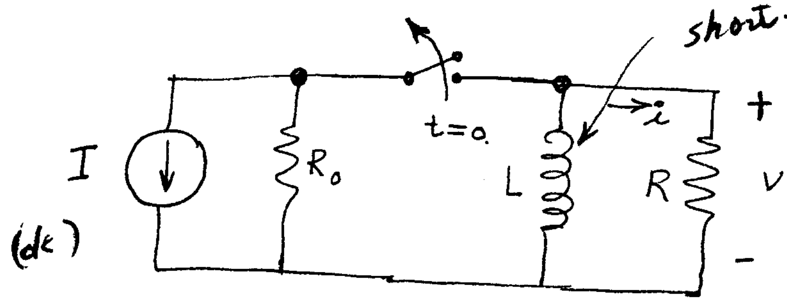
chap 7

the natural response of RL and RC circuits

① Natural response of RL Circuit :

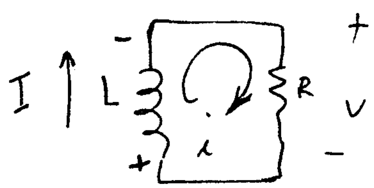
short before the switch is open

$i_L(0^-) = I$



let the switch be closed for $t < 0$.

for $t \geq 0$ we have



$$L \frac{di}{dt} + Ri = 0 \qquad \text{1st-order linear DE}$$

$$\int_{i(t_0)}^{i(t)} \frac{di}{i} = -\frac{R}{L} \int_{t_0=0}^{t} dt \Rightarrow \ln \frac{i(t)}{i(0)} = -\frac{R}{L} t$$

$$i(t) = i(0)\, e^{-\frac{R}{L} t} \quad ; \quad t \geq 0.$$

$i(0^-) = i(0^+) = I = I_0$ (there cannot be an instantaneous change of current in an inductor). $\Downarrow$ $v = L\frac{di}{dt}$

if i changes instantaneously, $v \to \infty$

impossible!

$$\therefore \quad i(t) = I_0 e^{-\frac{R}{L}t}, \quad t \geq 0$$

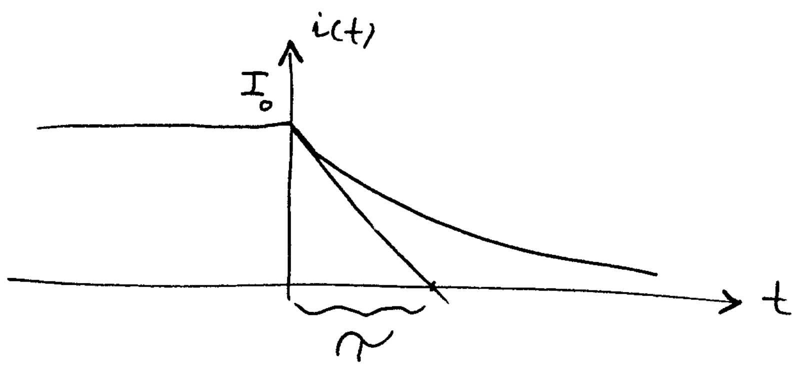
$$\tau \triangleq \frac{L}{R} = \text{time constant (sec)}$$

Time Constant = the rate at which the current approaches zero

$$i(t) = I_0 e^{-\frac{t}{\tau}} \quad ; \quad t \geqslant 0$$

note:

$$\left.\frac{di(t)}{dt}\right|_{t=0} = -\frac{1}{\tau} I_0 e^{-t/\tau}\Big|_{t=0} = -\frac{I_0}{\tau}$$



note: After five time constants the current is a negligible fraction of the initial value. (table 6.1)

$$v = iR = I_0 R e^{-t/\tau} \quad , \quad t \geq 0$$

Power dissipated in R is

$$P = vi = i^2 R = \frac{v^2}{R} = I_0^2 R e^{-2t/\tau} \quad , \quad t \geq 0.$$

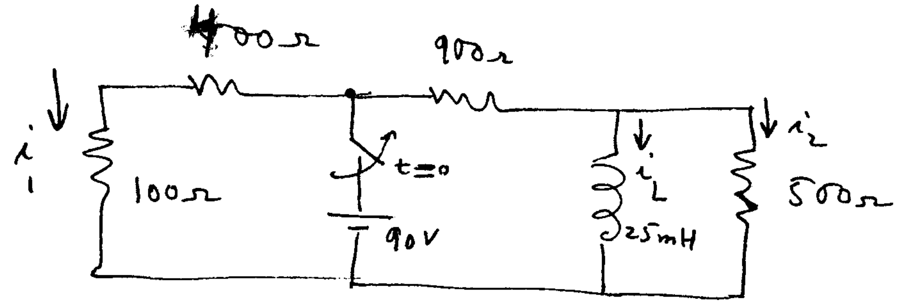
the energy <u>delivered</u> to R is

$$W = \int_0^t P(t')\, dt' = \int_0^t I_0^2 R e^{-2t'/\tau}\, dt'$$

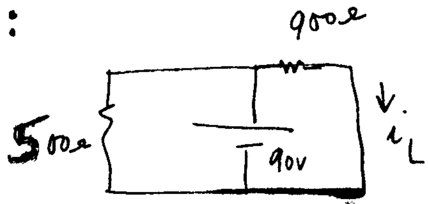
$$= (I_0^2 R) \frac{\overset{L/R}{\cancel{\tau}}}{2} \left(1 - e^{-2t/\tau}\right) = \frac{1}{2} L I_0^2 \left(1 - e^{-2t/\tau}\right)$$

$$; \quad t \geq 0$$

Exa: Find a) $i_L(0^+)$; b) $i_1(t), t \geq 0$; $i_2(t), t \geq 0$

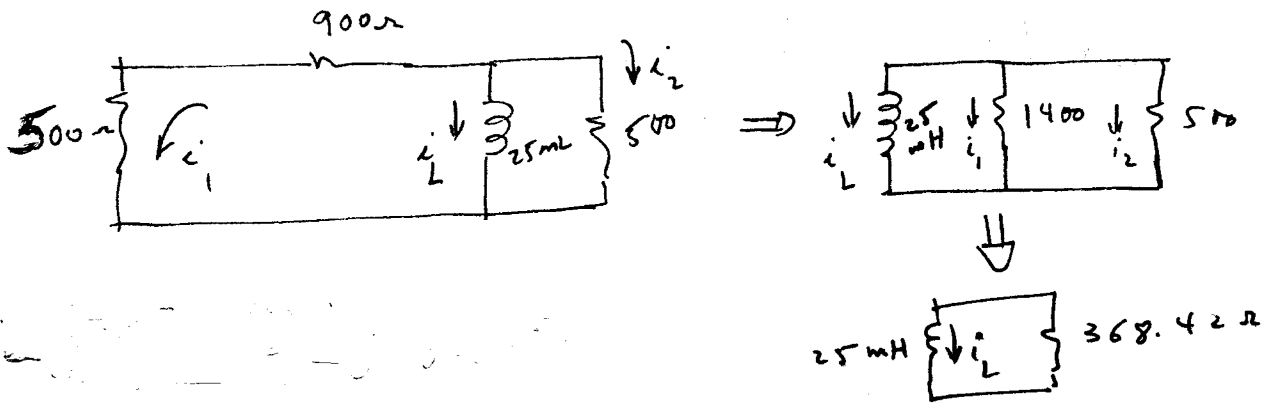


t<0:



$$i_L(0^-) = i_L(0^+) = i_L(0) = \frac{90}{900} = 0.1 \quad A$$

t>0:



$$R_{eq} = (500+900) \,//\, 500 = \frac{(1400)(500)}{1900} = 368.42\ \Omega$$

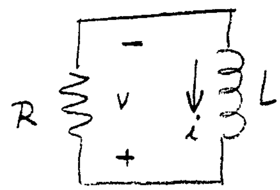
$$\tau = \frac{L}{R_{eq}} = 0.0679 \text{ ms.} \qquad t \text{ in ms}$$

$$i_L(t) = \underset{mA}{100}\, e^{-14.72t} \text{ mA} \quad ; \quad t \geq 0$$

$$i_1(t) = \frac{500}{1400+500}\left(-i_L(t)\right) = -26.32\, e^{-14.72t} \text{ mA} ; \quad t \geq 0$$

$$i_2(t) = \frac{1400}{1400+500}\left(-i_L(t)\right) = -73.68\, e^{-14.72t} \text{ mA} ; \quad t \geq 0$$

$\#\frac{6.1}{197}$:



$i = 8\,e^{-20t}$, $v = 240\,e^{-20t}$; $\tau = \frac{1}{20}$

(a) $R = ?$

$v = i\,R$ ∴ $240 = 8R \Rightarrow$ $\underline{\underline{R = 30\ \Omega}}$

(b) $L = ?$

$\tau = \frac{L}{R} \Rightarrow$ $L = \tau R = \frac{1}{20} \cdot 30 = \underline{\underline{1.5\ H}}$

(c) $\underline{\underline{\tau = \frac{1}{20}\ \text{sec.}}}$

(d) $W = \frac{1}{2} L I_o^2 = \frac{1}{2}(1.5)(8^2) = \underline{\underline{48\ J}}$

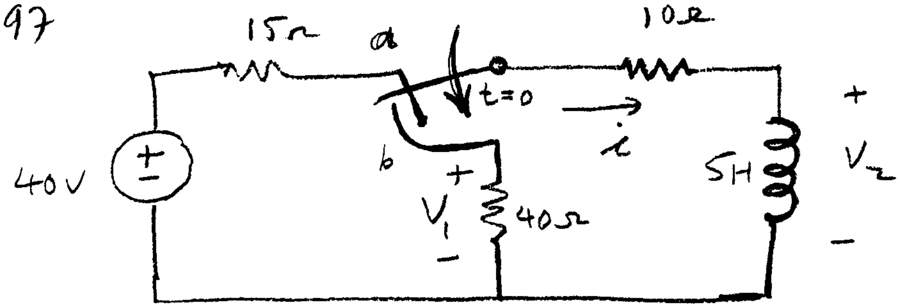
(e) $P = vi = 1920\,e^{-40t}$

$$W = \int_0^{0.1} 1920\,e^{-40t} = \frac{1920}{40}\left(1 - e^{(-40)(0.1)}\right)$$

$= 47.12\ J.$

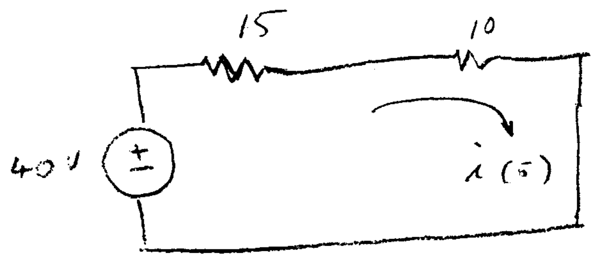
*

#6.2 / 197 :



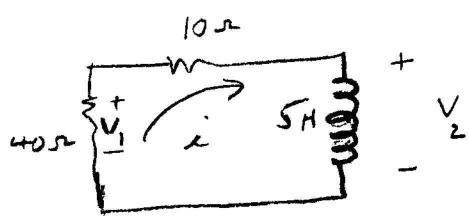
(a) $i(0^-) = i(0^+) = ?$

t<0:



$$i(0^-) = \frac{40}{25} = 1.6 \text{ A}$$

(b)



$$\tau = \frac{L}{R} = \frac{5}{50} = .1 \text{ sec}$$

(c) $$i = 1.6e^{-t/.1} = 1.6e^{-10t} \text{ A}$$

$$\boxed{V_1 = \frac{40}{50} V_2 = \frac{40}{50}\left\{5\frac{d}{dt} i\right\} = 4\left(-16e^{-10t}\right) = -64e^{-10t}}$$

or $V_1 = -40i = -40(1.6e^{-10t}) =$

$$V_2 = -\overset{80}{\cancel{16}}\, e^{-10t}$$

(d) $$W_0 = \frac{1}{2}\,\frac{\cancel{1}}{5}\,(1.6)^2 = 6.4 \text{ J}$$

$$P_{10} = 10\, i(t)^2 = 25.6\, e^{-20t}$$

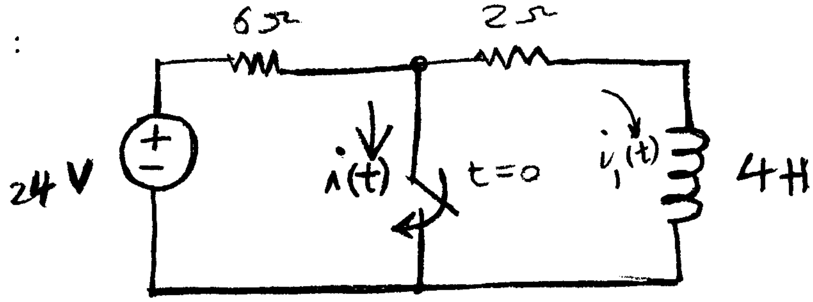
$$W_{10} = 25.6 \int_0^{.05} e^{-20t} = (25.6)\left[\frac{1}{20}\left(1 - e^{(-20)(.05)}\right)\right]$$

$$= 0.8091 \text{ J}$$

$$\frac{.8091}{6.4} \times 100\% = 12.64\%$$

*

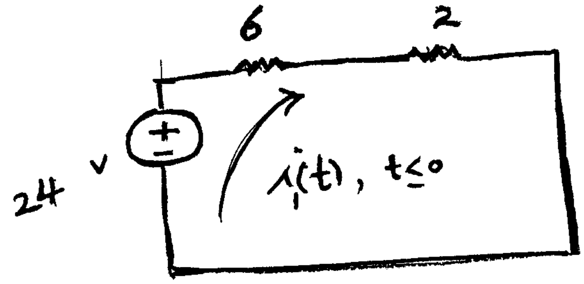
$\frac{6.3}{197}$:



important

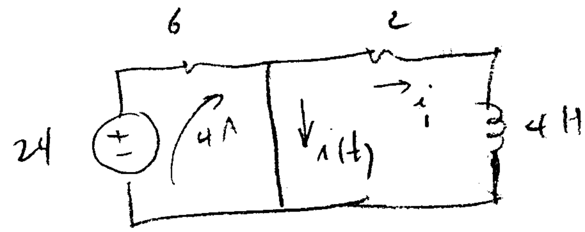
Find i(t).

(a)

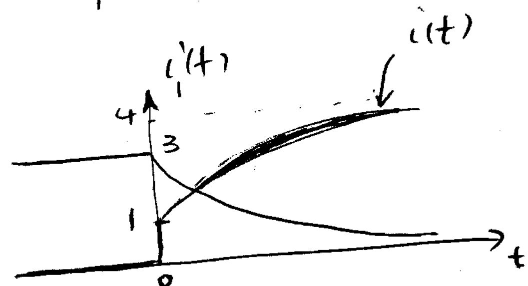


$$i_1(0^-) = \frac{24}{8} = 3 \text{ A.}$$

$t \geq 0$.



$\tau = 2$ sec



$$i_1(t) = 3e^{-t/2} \text{ A}, \quad t \geq 0$$

$$i(t) = 4 - i_1(t) = (4 - 3e^{-t/2})\, u(t)$$

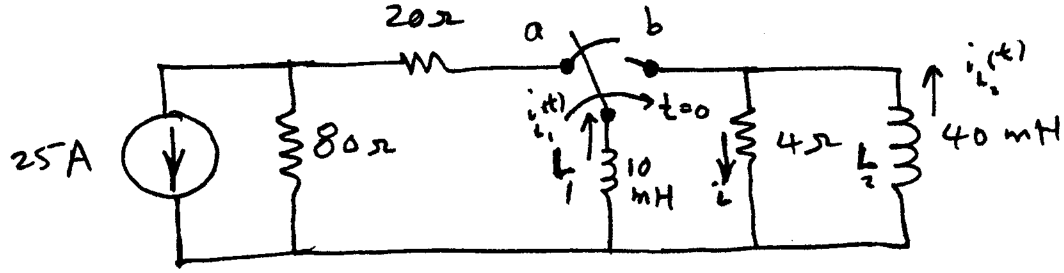
$i(t) = 2$ A
Find t ?

b) $4 - 3e^{-t/2} = 2$

$3e^{-t/2} = 2 \Rightarrow e^{-t/2} = \frac{2}{3}$

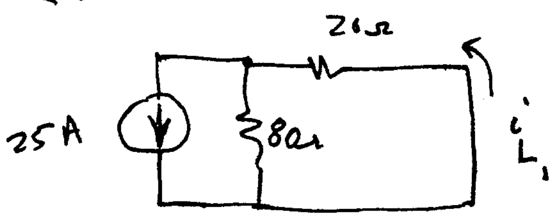
$\ln\left(\frac{2}{3}\right) = -\frac{t}{2} \Rightarrow t = .81$ sec

$\frac{6.8}{199}$:



a) $i(t) = ?$; $t \geq 0$

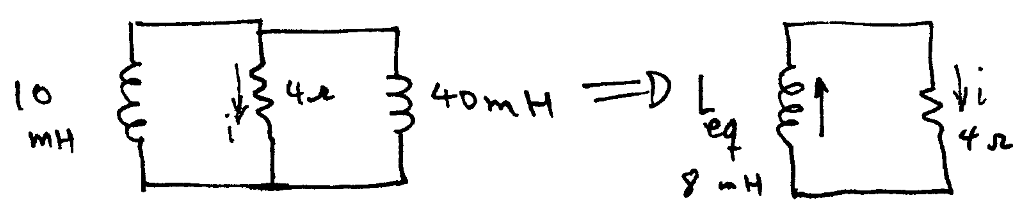
$t < 0$:



$$i_{L_1}(0^-) = \frac{80}{\underset{100}{20+80}}(-25) = -20 \text{ A} \Rightarrow i_{L_1}(0^-) = \underline{20 \text{ A}}$$

$$i_{L_2}(0^-) = \underline{0.}$$

$t > 0$:



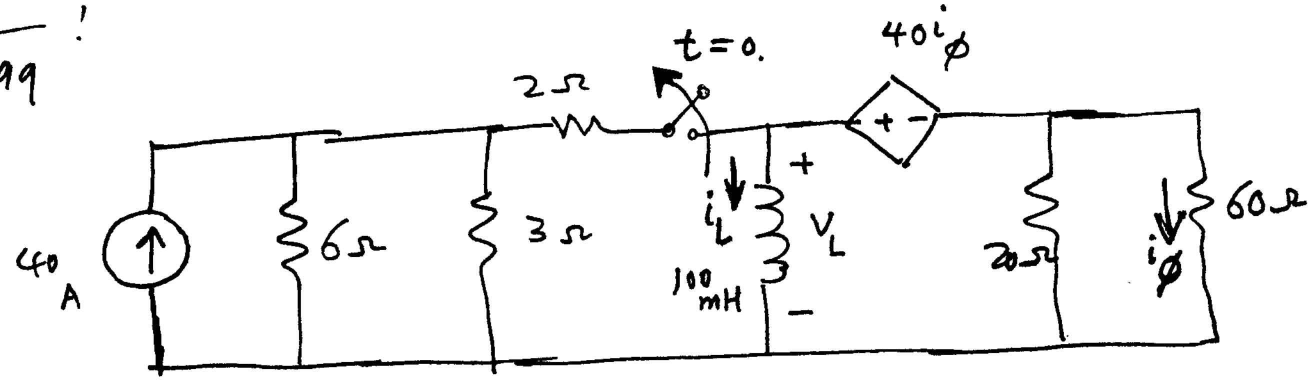
$$L_{eq} = \frac{(10)(40)}{10+40} = 8 \text{ mH}$$

$$\tau = \frac{L_{eq}}{R} = \frac{(8)(10^{-3})}{4} = \underline{2 \text{ ms}} \quad , \quad i_L(0) = i_{L_1}(0) + i_{L_2}(0) = 20 \text{ A}$$

$$i(t) = 20\, e^{-t/2\text{ms}} = 20\, e^{-500t} \text{ A} \qquad t \geq 0.$$

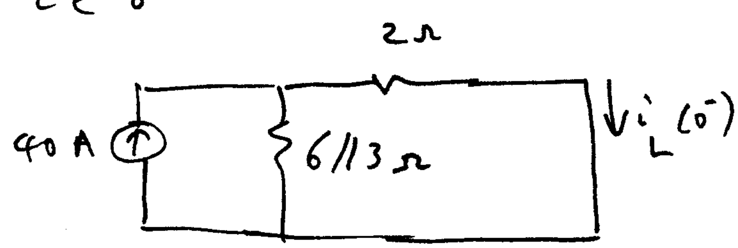
b) $W_{diss} = \frac{1}{2} L_{eq} I_o^2 = \frac{1}{2}(8\times10^{-3})(20)^2 = \underline{1.6\ J}$

$\frac{6.9}{199}$!



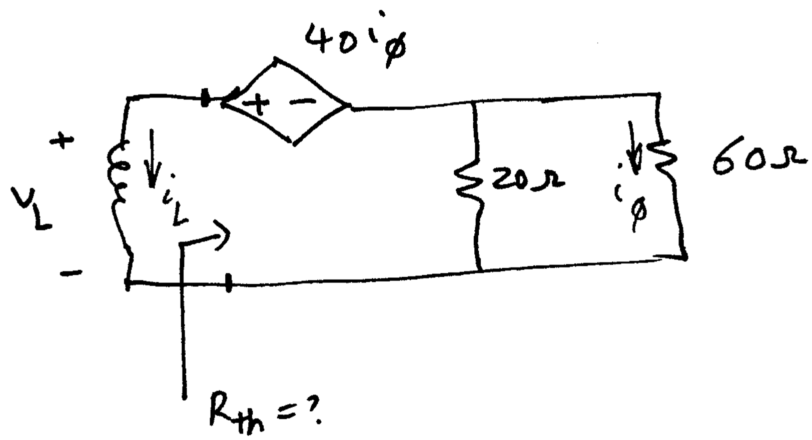
a) $i_L(t) = ?\quad ;\ t \geq 0$

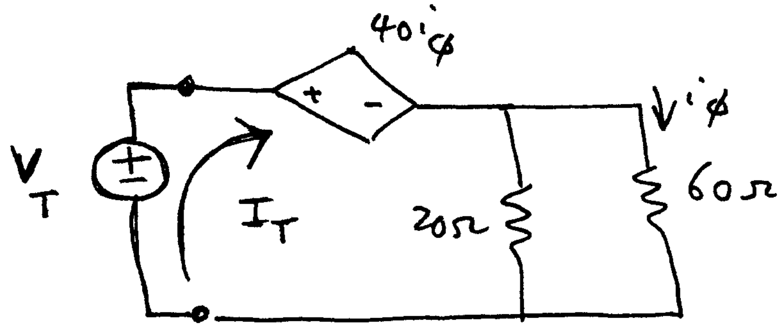
$t < 0$



$$i_L(0^-) = \frac{\cancel{6/\!/3}^{\,2}}{2 + \cancel{6/\!/3}_{\,2}}(40) = 20\ A$$

$t > 0$





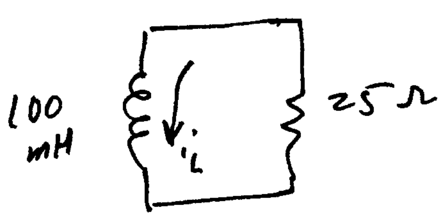
$$-V_T + 40\, i_\phi + (20 /\!/ 60)\, I_T = 0.$$

$$i_\phi = \frac{20}{20+60}\, I_T = \frac{1}{4} I_T$$

$$-V_T + \overset{10}{\cancel{40}}\left(\frac{1}{\cancel{4}}\right) I_T + (20 /\!/ 60)\, I_T = 0.$$

$$\frac{V_T}{I_T} = 10 + \frac{\overset{\cancel{1200}\ 15}{(20)(60)}}{\underset{8\cancel{0}}{20+60}} = 25\ \Omega$$

$$\begin{cases} R_{Th} = \underline{25\ \Omega} \\ V_{Th} = 0 \end{cases}$$



$$\tau = \frac{L}{R} = \frac{100\text{ mH}}{25} = 4\text{ ms}$$

$$i_L(t) = 20 e^{-t/4\text{ms}} = 20 e^{-250t}\ ;\ A.$$
$$t \geq 0$$

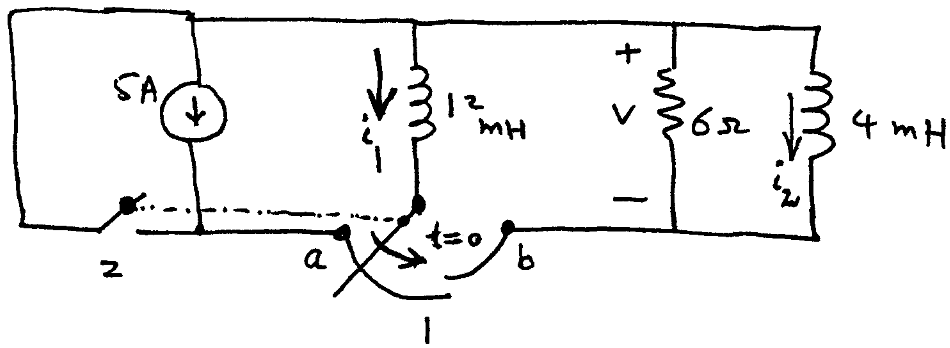
b) $$v_L = L\frac{di_L}{dt} = (100)(10^{-3})(20)(-250)\,e^{-250t} \;;\; A \quad t \geq 0$$

$$= -500\,e^{-250t} \;;\; V \quad t \geq 0.$$

c) $$i_\phi = \frac{20}{20+60}(-i_L) = \frac{-1}{4}\left(20\,e^{-250t}\right)$$

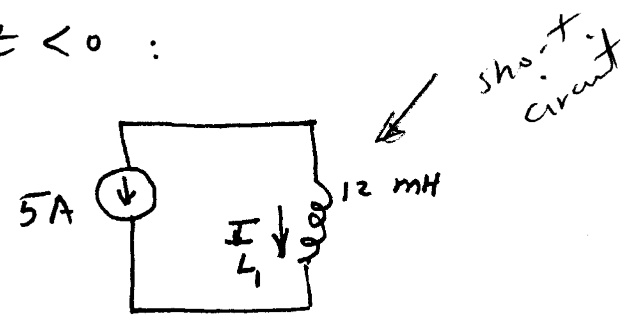
$$= -5\,e^{-250t} \;;\; A \quad t \geq 0 \;.$$

$\frac{6.11}{199}$:



a) $v,\ i_1,\ i_2 \Rightarrow ?$ $t \geq 0.$

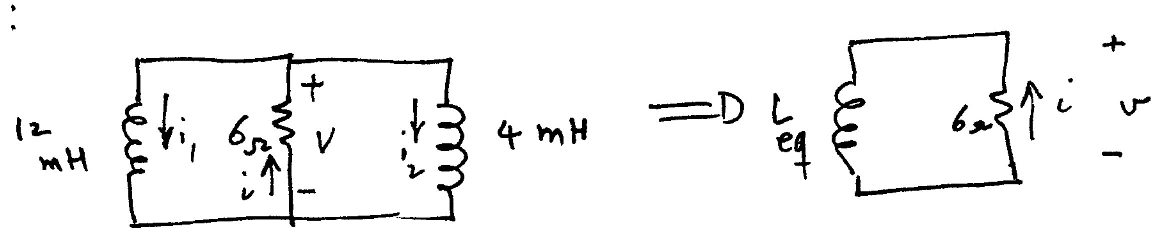
$t < 0$:



$I_{L_1} = i_{L_1}(0^-) = -5\ A$

$I_{L_2} = 0\ A$

$t \geq 0$:



$i(0^-) = i_1(0^-) + i_2(0^-) = \underline{-5\ A}$, $L_{eq} = \frac{(4)(12)}{4+12} = \underline{3\ mH}$

$\tau = \frac{L_{eq}}{R} = \frac{(3)(10^{-3})}{6} = .5\ \underline{ms}$

$$i(t) = -5\,e^{-t/(.5)(10^{-3})} = -5\,e^{-2000t} \quad A, \quad t \geq 0$$

$$v = -i\,(6) = 30\,e^{-2000t} \quad V, \quad t \geq 0$$

$$i_1 = \frac{1}{L_1}\int_0^t v\,dt - 5 = -1.25\,e^{-2000t} - 3.75$$

$$i_2 = \frac{1}{L_2}\int_0^t v\,dt = -3.75\,e^{-2000t} + 3.75$$

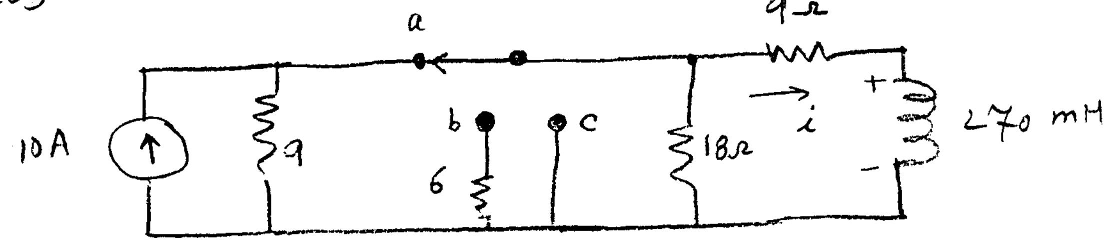
Final values

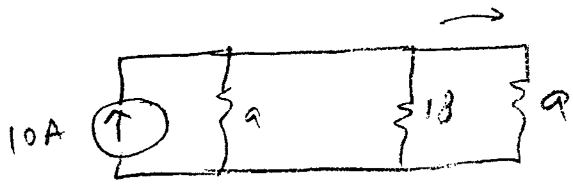
b) $W(\text{Trapped}) = \frac{1}{2} L_1 (-3.75)^2 + \frac{1}{2} L_2 (3.75)^2 = 112.5$ mJ.

c) $W(\text{diss}) = \int_0^\infty \left(\frac{v^2}{4\,\Omega}\right) dt = 37.5$ mJ.

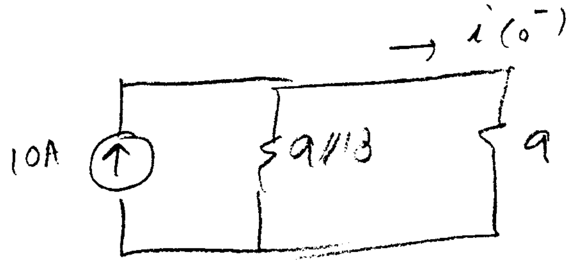
*

$\frac{6.21}{203}$:



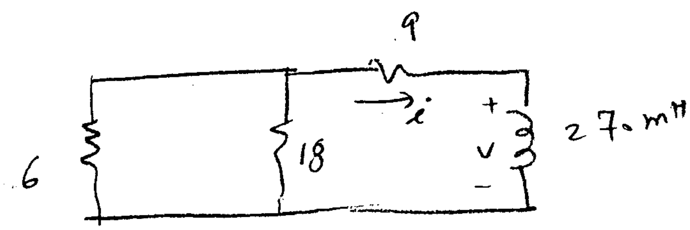


(a) $i(0^+) = ?$



$$i(0^-) = \frac{\cancel{9//18}\ 6}{9 + \cancel{9//18}\ 6}\ 10 = \frac{60}{15} = 4 \text{ A}$$

(b) $0 \le t \le 10$ msec.

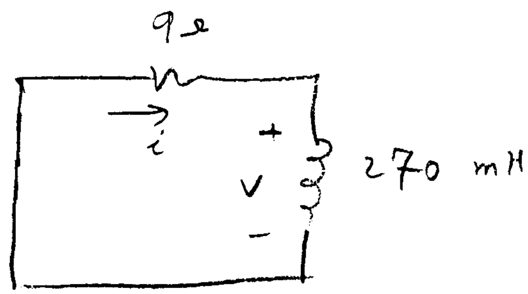


$$R_{eq} = 6//18 + 9 = 4.5 + 9 = 13.5\ \Omega$$

$$\tau_1 = \frac{L}{R_{eq}} = \frac{270}{13.5} = 20 \text{ ms.}$$

$$i(t) = 4e^{-t/0.02} \Rightarrow i(8\text{ms}) = 4e^{-8/20} = 2.68 \text{ A.}$$

(c) $i(16\,ms) = ?$ $\qquad 10 \le t$



$$T_2 = \frac{270}{9} = 30 \text{ ms}$$

$$i(10\,ms) = 4e^{-10/20} = 4e^{-1/2} = 2.4261$$

$$i(t) = i(10)\,e^{-(t-t_{10})/\tau} = 2.4261\,e^{-(t-0.01)/0.03}$$

for $10 \le t$

$$i(16\,ms) = 2.4261\,e^{-(.016-0.01)/0.03} = 1.99 \text{ A.}$$

Recall:

$$L\frac{di}{dt} + Ri = 0$$

$$\int_{i(t_0)}^{i(t)} \frac{di}{i} = -\frac{R}{L}\int_{t_0}^{t} dt \Rightarrow \ln\frac{i(t)}{i(t_0)} = -\frac{R}{L}(t-t_0)$$

$$\therefore\; i(t) = i(t_0)\,e^{-(t-t_0)/\tau}$$

(d) $v(10^- \text{ ms}) = ?$

From (b) $\longrightarrow$

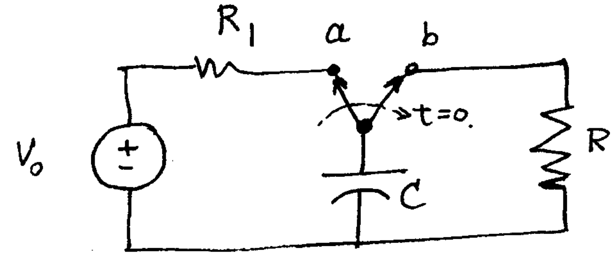
$$v\Big|_{10^-} = L\frac{di}{dt}\Big|_{10^-} = (.270)\frac{d}{dt}\left\{4e^{-t/0.02}\right\}\Big|_{10^-} = -32.75\text{ V}$$

(e) $v(10^+ \text{ ms}) = ?$ From (c) $\Rightarrow$

$$v\Big|_{10^+} = L\frac{di}{dt}\Big|_{10^+} = (.270)\frac{d}{dt}\left\{2.4261\, e^{-(t-0.01)/0.02}\right\}\Big|_{10^+}$$

$= -26.84\text{ V}$

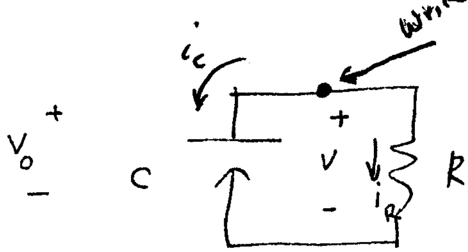
Natural Response of RC circuit :



$v_c(0^-) = v_c(0^+) = V_0$

(there cannot be an instantaneous change of voltage in a capacitor)

$t \geq 0$



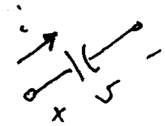
write KCL for this node.

$i_c + i_R = 0$

$i = C\dfrac{dv}{dt}$

if v changes instantly, $i \to \infty$ which is impossible!

$$C\frac{dv}{dt} + \frac{v}{R} = 0 \Rightarrow \int_{v_0}^{v(t)} \frac{dv}{v} = -\frac{1}{RC}\int_0^t dt$$



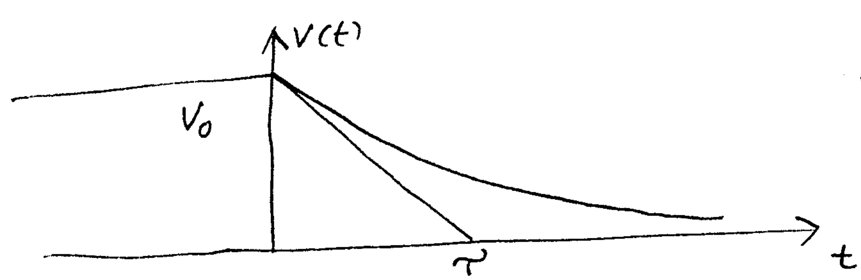
$i = C\dfrac{dv}{dt}$

v constant $\Rightarrow i \to 0$ open-circuit

$$\ln\frac{v}{v_0} = -\frac{1}{RC}t \Rightarrow v(t) = v_0 e^{-t/RC}$$

$\tau \triangleq RC$

$$\therefore \quad v(t) = V_0 e^{-t/\tau}, \quad t \geq 0$$



$$\left.\frac{dv}{dt}\right|_{t=0} = -\frac{V_0}{\tau}e^{-t/\tau}\Big|_{t=0} = -\frac{V_0}{\tau}$$

$$i(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau} \quad , \qquad t \geq 0$$

$$p = vi = \frac{V_0^2}{R} e^{-2t/\tau} \quad , \qquad t \geq 0$$

$$w = \int_0^t p(t)\,dt = \int_0^t \frac{V_0^2}{R} e^{-2t/\tau}\,dt$$

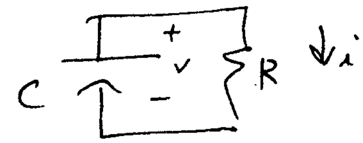
$$= \frac{V_0^2}{\cancel{R}} \frac{\overset{\cancel{R}C}{\cancel{\tau}}}{2} \left(1 - e^{-2t/\tau}\right)$$

$$= \frac{1}{2} C V_0^2 \left(1 - e^{-2t/\tau}\right) \quad ; \qquad t \geq 0 .$$

Exa :

6.13 : $v = 8e^{-5t}$ V

200 $i = 20\, e^{-5t}$ μA



(a) $v = iR \Rightarrow$

$8 = 20\times10^{-6} R \Rightarrow R = 400$ kΩ

(b) $RC = \frac{1}{5} \Rightarrow C = \frac{1}{5R} = \frac{1}{\underbrace{(5\times400)}_{2000}\times10^{3}}$

$C = .5$ μF

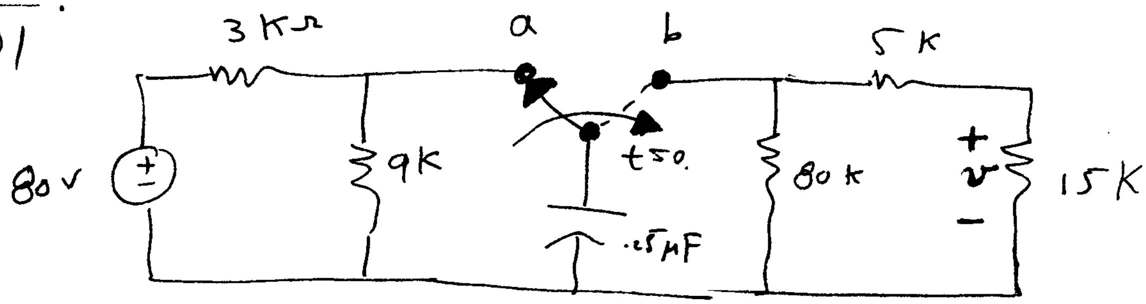
(c) $\tau = .2$ s.

(d) $W_0 = \frac{1}{2} C V_0^2 = (\frac{1}{2})(.5)(8^2) = 16$ μJ

(e) $W = \frac{1}{2} C V_0^2 (1 - e^{-2t/\tau})$

$= 16\,(1 - e^{-\frac{.2}{.2}}) \simeq 10.11$ μJ

6.14.
201



$v(t) = ?$

(a) $v_c(0^-) = v_c(0^+) = \frac{9}{12} 80 = 60 \text{ v}$

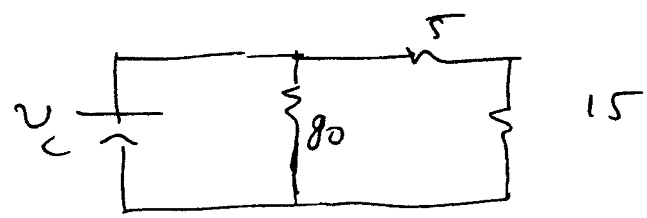
$t \geq 0$: $R_{eq} = (5+15) // 80 = 16 \text{ K}\Omega$

$\tau = R_{eq}C = (16 \times .25 \times 10^{-3}) = 4 \text{ ms.}$

$v_c(t) = 60 e^{-t/.004} = 60 e^{-250t}$, v.

$v(t) = \frac{15}{20} v_c = 45 e^{-250t}$, v. $t \geq 0$.

(b)



$W(0) = \frac{1}{2} C V_c^2(0) = \frac{1}{2}(.25 \times 10^{-6})(60)^2 = 450 \text{ μJ}$

$W(dis) = \int_0^{.004} \frac{v_c^2}{80 K} dt = 77.82 \text{ μJ}$

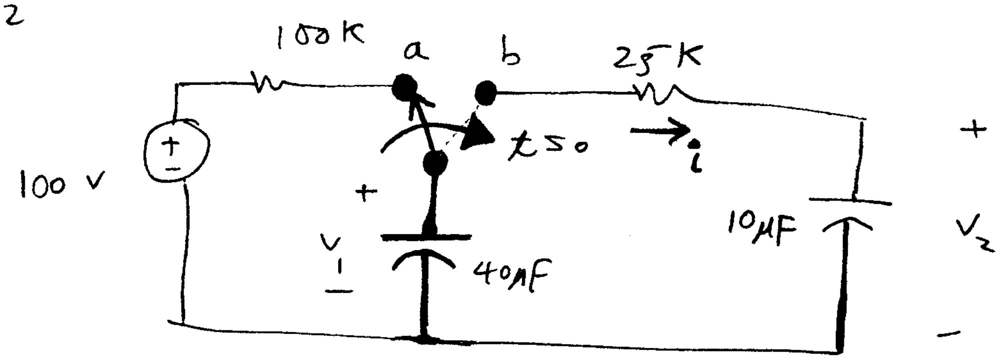
power of 80 K.

$\% \;-- = 17.29\%$.

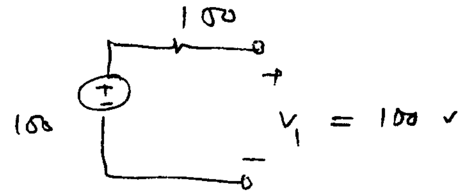
6.20 :

202

(a)

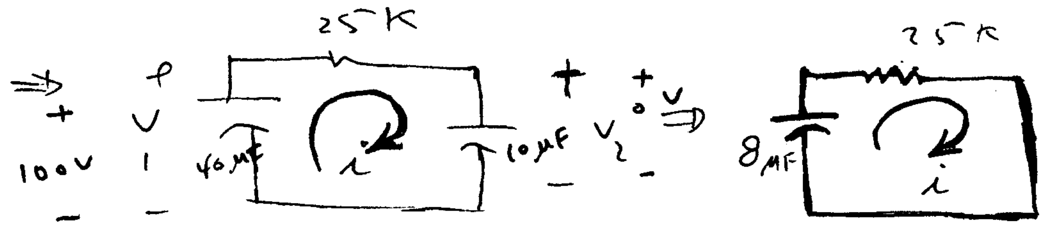


$t < 0 \Rightarrow$



$V_1 = 100$ V , $V_2 = 0$ V

$t > 0^+ \Rightarrow$



$C_{eq} = 10 \,//\, 40 = \frac{400}{50} = 8\,\mu F$

$\tau = RC_{eq} = (25)(8)(10^3)(10^{-6}) = 0.2$ s

$v_{8\mu}(t) = 100\, e^{-5t}$ v. ; $t \geq 0$

$i(t) = \frac{\overset{4}{\cancel{100}}}{\cancel{25}\,K}\, e^{-5t}$ mA ; $t \geq 0$

$v_1(t) = \frac{-1}{C_1}\int_0^t i(x)\,dx + 100 = 20e^{-5t} + 80$ V ;

$v_2(t) = \frac{1}{C_2}\int_0^t i(x)\,dx + 0 = -80e^{-5t} + 80$ V ;

(b) $W(0) = \frac{1}{2} C_1 V_1^2(0) = \underline{\underline{200 \text{ mJ}}}$

(c) $W(\text{Trapped}) = \frac{1}{2} C_1 V_1^2(\infty) + \frac{1}{2} C_2 V_2^2(\infty)$

$$= \left[\frac{1}{2}(40)(80)^2 + \frac{1}{2}(10)(80)^2\right] 10^{-6} = \underline{\underline{160 \text{ mJ}}}$$

(d) $W(\text{dissipated}) = 200 - 160 = \underline{\underline{40 \text{ mJ}}}$

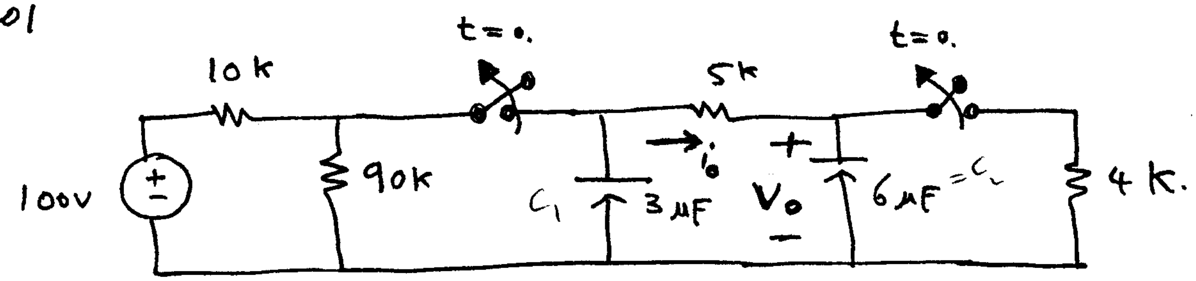
or

$$W(\text{dissipated}) = \int_0^\infty i^2(t) R \, dt$$

$$= \int_0^\infty \underbrace{\left(4e^{-5t} \times 10^{-3}\right)^2}_{16\times10^{-6} e^{-10t}} \underbrace{\left(25\times10^3\right)}_{25\times10^3} dt = 40 \text{ mJ.}$$

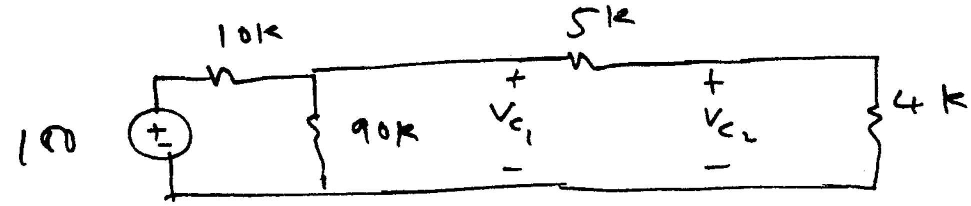
$$= 400\times10^{-3} \underbrace{\int_0^\infty e^{-10t} \, dt}_{\underbrace{-\frac{1}{10} e^{-10t} \Big|_0^\infty}_{1/10}}$$

$\frac{6.15}{201}$:



a)

$t < 0$:



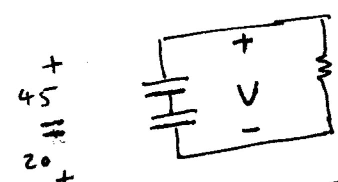
$$(4+5) \parallel 90 = \frac{(9)(90)}{90+9} = \frac{\cancel{9}(90)}{\cancel{99}\,11} = \underline{8.18\ k\Omega}$$

$$V_{c_1} = \frac{8.18}{10+8.18}(100) \cong \underline{45\ V.} \quad ; \quad t<0$$

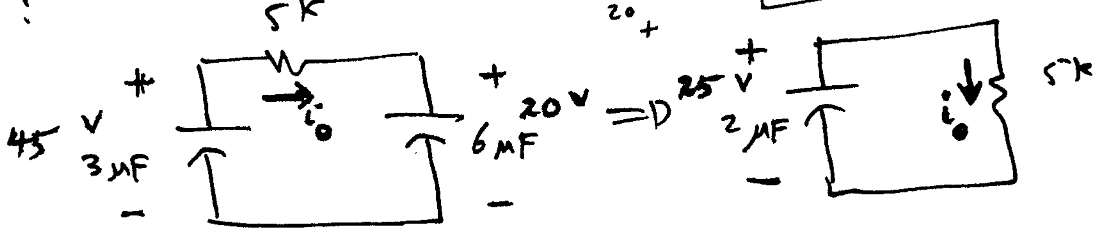
$$V_{c_2} = \frac{4}{5+4} V_{c_1} = \frac{(4)(\overset{5}{\cancel{45}})}{\cancel{9}} = \underline{20\ V} \quad ; \ t<0$$

$-V + 45 - 20 = 0.$

$\therefore V = 25\ V$



$t \geq 0$:



$$v_c(t) = 25\,e^{-t/\tau} \quad , \quad \tau = RC_{eq} = (5k)(2 \times 10^{-6})$$

$$= 10\ ms$$

$$v_c(t) = 25 e^{-100t}\ V, \ t \geq 0$$

$$i_o = \frac{v_C}{5K} = \frac{25}{5K} e^{-100t} = 5e^{-100t} \text{ mA} \; ; \; t \geq 0$$

b) $$\underbrace{v_o(t)}_{v_{c_2}(t)} = \frac{1}{\underset{6\mu F}{C_2}} \int_0^t i_o(\tau)\,d\tau + \underbrace{20}_{V_{c_2}}$$

$$= \frac{10^6}{6} \int_0^t (5e^{-100t'})(10^{-3})\,dt' + 20$$

$$= -\frac{25}{3} e^{-100t} + \frac{85}{3} \text{ V}$$

c) $$W(0) = \frac{1}{2} C_1 V_{c_1}^2(0) + \frac{1}{2} C_2 V_{c_2}^2(0) = 4237.5 \; \mu J.$$

$$W_{diss} = \frac{1}{2} C_{eq} V_{c_{eq}}^2(0) = \frac{1}{2}(2)(25)^2 = 625 \; \mu J.$$

$$W_{Trapped} = 4237.5 - 625 = 3612.5 \; \mu J.$$

or,

$$W_{diss} = \int_0^\infty i_o^2(t)(5K)\,dt$$

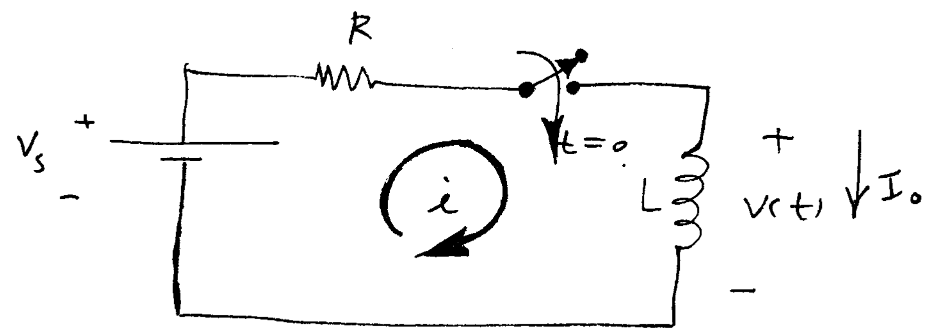
$$= \int_0^\infty \left(5e^{-100t} \times 10^{-3}\right)^2 (5 \times 10^3)\,dt$$

$$= 625 \; \mu J.$$

Chap 7.

Step Response of RL and RC circuit.

Driven RL



$$V_s = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{V_s - Ri}{L} = \frac{V_s}{L} - \frac{R}{L}i$$

$$= -\frac{R}{L}\left(i - \frac{V_s}{R}\right)$$

$$\int_{I_0}^{i(t)} \frac{di}{i - \frac{V_s}{R}} = \int_0^t -\frac{R}{L}\,dt$$

$$\ln\left(i - \frac{V_s}{R}\right)\Big|_{I_0}^{i(t)} = -\frac{R}{L}t$$

$$\ln\frac{i(t) - \frac{V_s}{R}}{I_0 - \frac{V_s}{R}} = -\frac{R}{L}t \Rightarrow i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-\left(\frac{R}{L}\right)t}$$

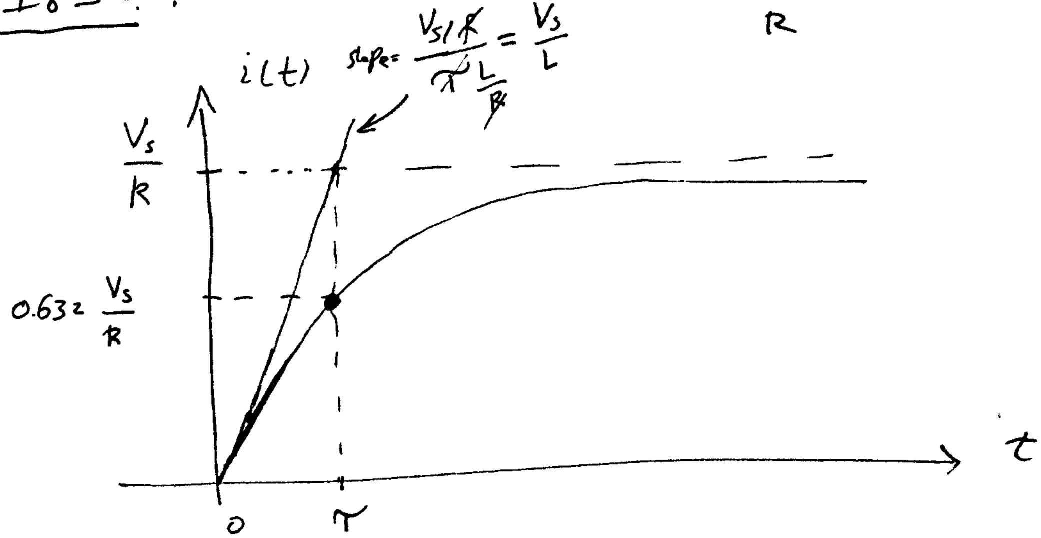
$t \geq 0.$

$$i(t) = \underset{\text{Res}}{\text{Transient}} + \underset{\text{value}}{\text{Final}}$$

$$\uparrow$$

$$\frac{V_s}{R}$$

<u>let $I_0 = 0$</u> :



$$\left.\frac{di}{dt}\right|_{t=0} = -\frac{\cancel{R}}{L}\left(\cancel{I_0}^{\,0} - \frac{V_s}{\cancel{R}}\right)e^{-\frac{R}{L}t}\Bigg|_{t=0} = \frac{V_s}{L}$$

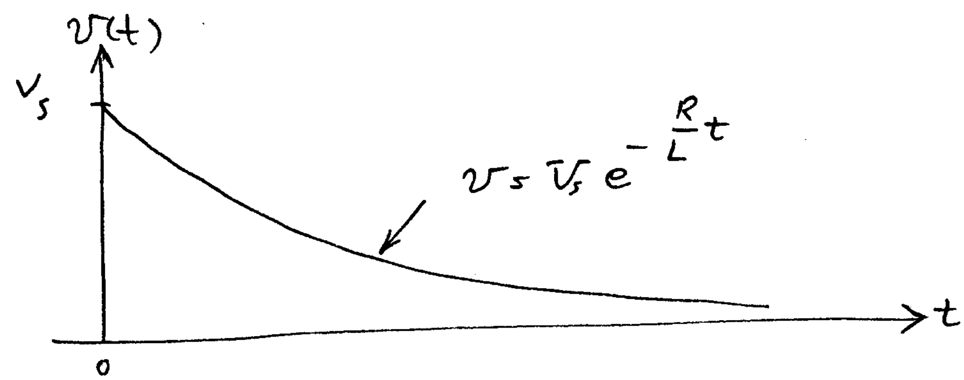
if the current continued to increase at this rate, then

$$i(t) = \frac{V_s}{L}t$$

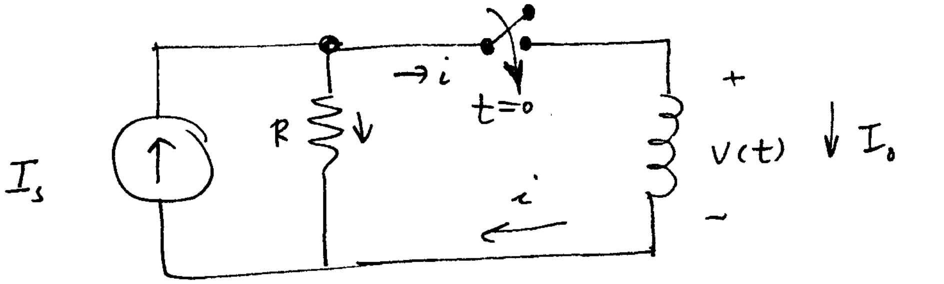
From which we see that at $t = \cancel{\tau}^{\frac{L}{R}}$ we get $\left. i(t)\right|_{t=\tau} = \frac{V_s\cancel{\tau}^{\frac{L}{R}}}{L} = \frac{V_s}{R}$

Also, $v = L\frac{di}{dt} = \cancel{L}\left(-\frac{R}{\cancel{L}}\right)\left(I_0 - \frac{V_s}{R}\right)e^{-\frac{R}{L}t} = (V_s - I_0R)e^{-\frac{R}{L}t}$

$I_0 = 0.$



Now assume that the series combination of V_s and R is converted via a source transformation to a parallel combination as shown:



KCL : $t > 0$

$$I_s = \frac{v}{R} + \underbrace{\frac{1}{L}\int_0^t v(t)\,dt + I_0}_{i(t)}$$

note that the current of the inductor can not change instantaneously at $t=0$. Therefore,

$v(0) = R(I_s - I_0)$.

$$0 = \frac{1}{R} v'(t) + \frac{v(t)}{L}$$

$$\frac{dv}{dt} = -\frac{R}{L} v$$

$$\int_{v(0)}^{v(t)} \frac{dv}{v} = -\frac{R}{L}\int_0^t dt$$

$$v(t) = v(0)\, e^{-\left(\frac{R}{L}\right)t} = \overbrace{R(I_s - I_0)}^{v(0)} e^{-\frac{R}{L}t}$$

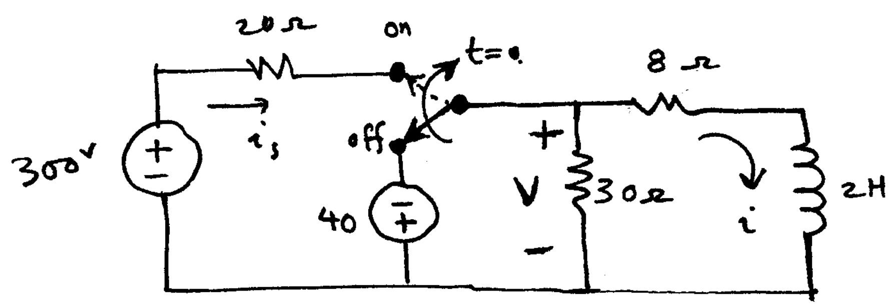
$$i(t) = I_s - \frac{V(t)}{R}$$

$$i(t) = \frac{1}{L}\int_0^t v(t)\,dt + I_0 \overset{\text{or}}{=} I_s - \frac{v(t)}{R} = I_s - \frac{v(0)}{R} e^{-\frac{R}{L}t}$$

$$= I_s - \frac{\cancel{R} I_s - \cancel{R} I_0}{\cancel{R}} e^{-\frac{R}{L}t} = \overset{\frac{V_s}{R}}{\cancel{I_s}} + \Big(I_0 - \overset{\frac{V_s}{R}}{\cancel{I_s}} \Big) e^{-\frac{R}{L}t}$$

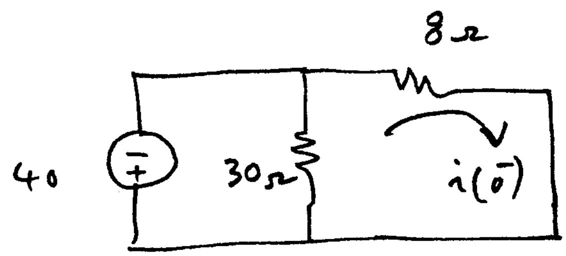
\# 7.2 :

227



$i(t) = ?$ $\quad t \geq 0$

a) $t < 0$:

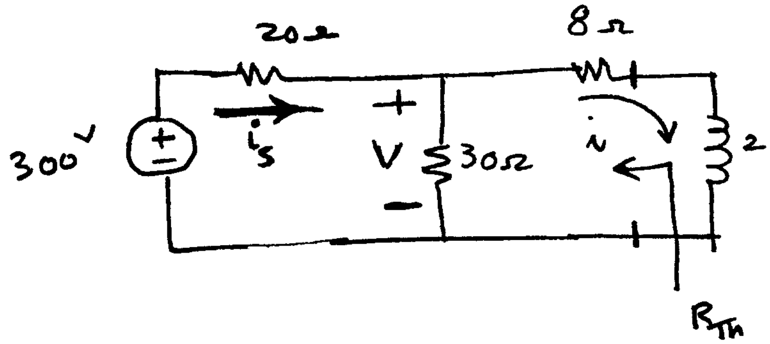


$+40 + 8\, i(0^-) = 0$

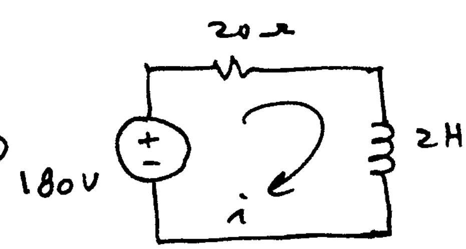
$\therefore \; i(0^-) = -\frac{40}{8}$

$$i(0^-) = \frac{-40}{8} = -5 \text{ A}$$

$t > 0$:

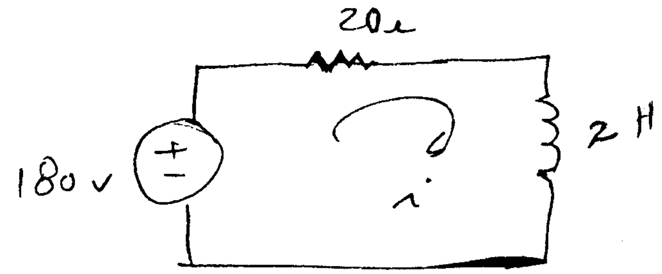


$\Rightarrow$



$$V_T = \frac{30}{20+30}(300) = 180 \text{ V} \quad \Rightarrow \quad \tau = \frac{L}{R_T} = \frac{2}{20} = .1 \text{ s}$$

$$R_T = 20 \parallel 30 + 8 = 20\ \Omega$$



$$i(t) = \frac{180}{20} + \left(-5 - \frac{180}{20}\right) e^{-10t} \quad ; \quad t \geq 0 .$$

$$i(t) = 9 - 14e^{-10t} \quad ; \quad t \geq 0 .$$

(b)

$$v(t) = v_8 + \overset{2}{(L)}\frac{di}{dt}$$

$$= 8i + 2\frac{di}{dt}$$

$$= 8(9 - 14e^{-10t}) + 2(140e^{-10t})$$

$$= 72 + 168e^{-10t} \text{ v} \quad ; \quad t \geq 0$$

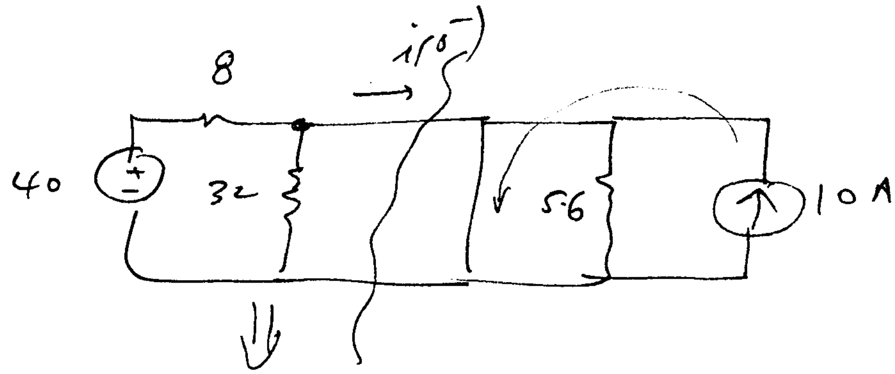
(c)

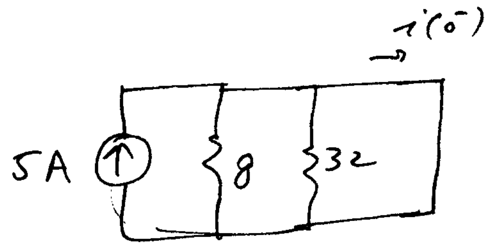
$$i_s = \overset{v(t)/30}{i_{30}} + i = \frac{72 + 168e^{-10t}}{30} + (9 - 14e^{-10t})$$

$$= 11.4 - 8.4e^{-10t} \text{ A} \quad ; \quad t \geq 0$$

$\frac{7.3}{227}$:

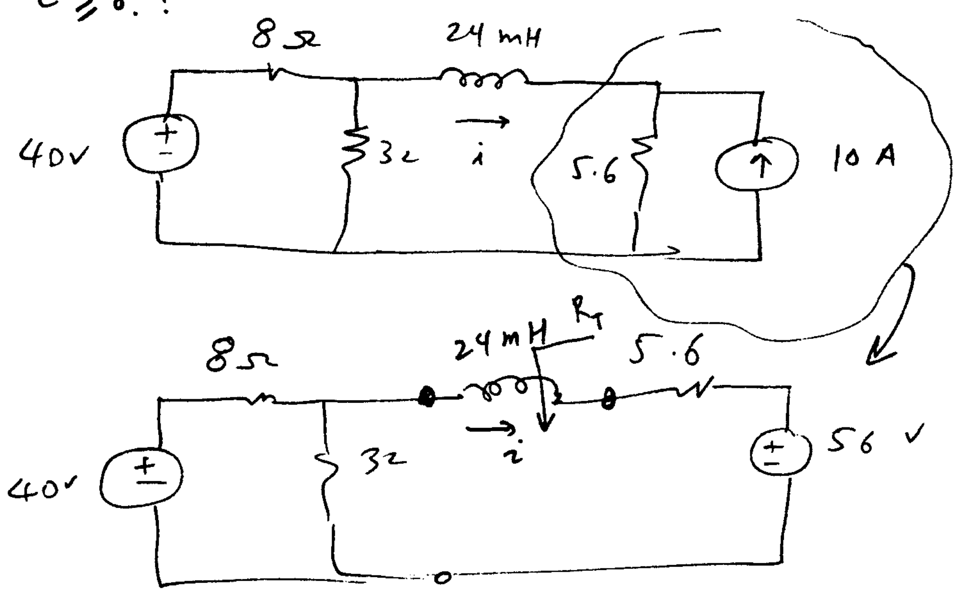
$t < 0$:





$i(0^-) = 5$ A

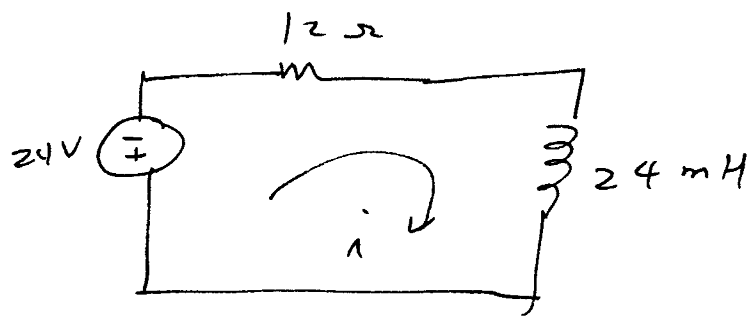
$t \geq 0$:



$R_T = 8 \,//\, 32 + 5.6 = 12 \ \Omega$

$V_T = -24 \ V$ (verify) $\rightarrow$ $V_{32\Omega} = \frac{32}{8+32} \, 40 = 32$

$V_T = 32 - 56 = -24$

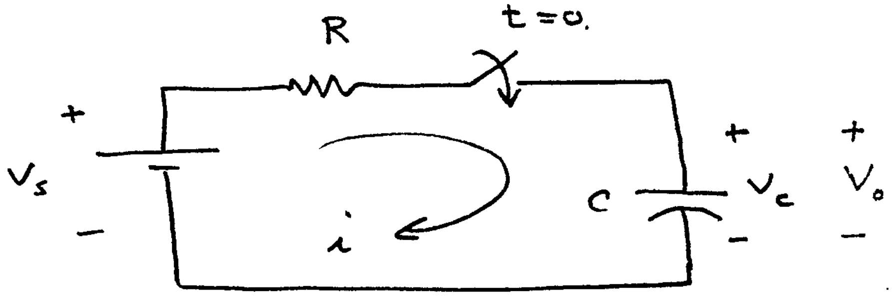


$i(\infty) = \frac{V_T}{R_T} = \frac{-24}{12} = -2 \ A$

$\tau = \frac{L}{R_T} = \frac{24}{12} = 2 \ ms$

$i(t) = -2 + [5 - (-2)] e^{-500t} \quad ; \ t \geq 0$

The step response of An RC Circuit



$t \geq 0$:

$$V_s = Ri + \frac{1}{C}\int_0^t i(\tau)\,d\tau + V_c(0)$$

$$0 = R\frac{di}{dt} + \frac{i}{C}$$

$$\frac{di}{dt} = -\frac{i}{RC} \Longrightarrow \frac{di}{i} = -\frac{1}{RC}\,dt$$

$$\therefore \ln i\Big|_{I_o}^{i(t)} = -\frac{1}{RC}\,t \Longrightarrow i(t) = I_o e^{-t/RC} \quad ; \quad t \geq 0$$

since there cannot be an instantaneous change in the voltage across the capacitor, we get

$$\underbrace{i(0)}_{I_o} = \frac{V_s - V_o}{R}$$

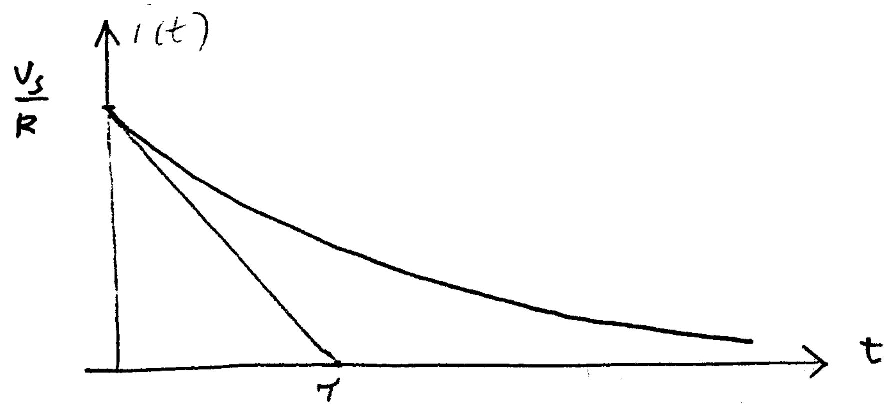
$$\therefore \quad i(t) = \frac{V_s - V_o}{R}\, e^{-t/RC}. \qquad t \geq 0.$$

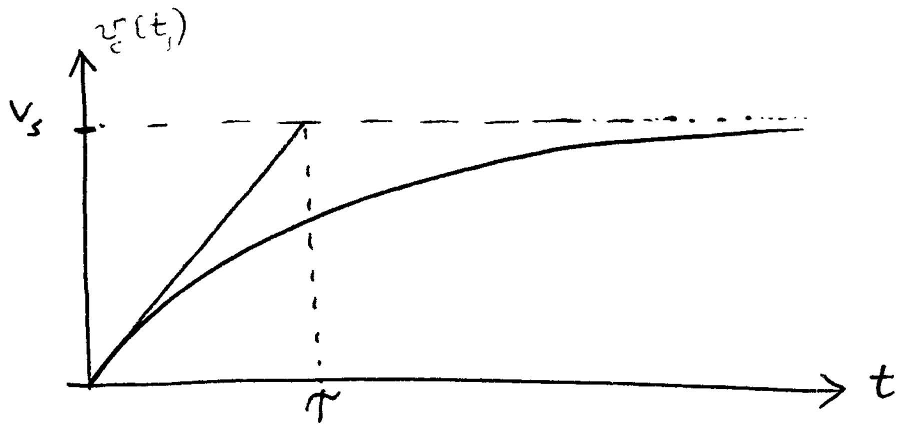
$$v_c(t) = V_s - iR$$

$$= V_s - \cancel{R}\left(\frac{V_s - V_0}{\cancel{R}}\right)e^{-t/RC}$$

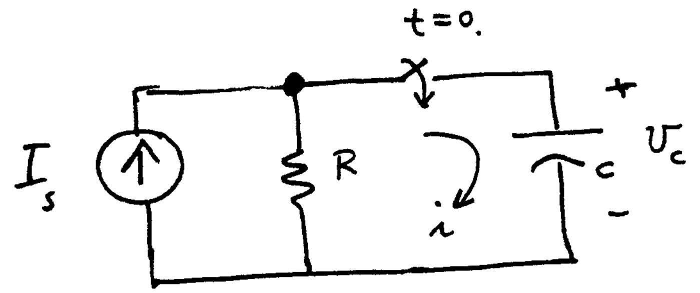
$$= V_s + (V_0 - V_s)\,e^{-t/RC} \quad ; \quad t \geq 0$$

Let $V_0 = 0$.





It is also informative to derive the current and voltage by starting the following circuit:



KCL: $(t \geq 0)$

$$I_s = \frac{v_c}{R} + C\frac{dv_c}{dt}$$

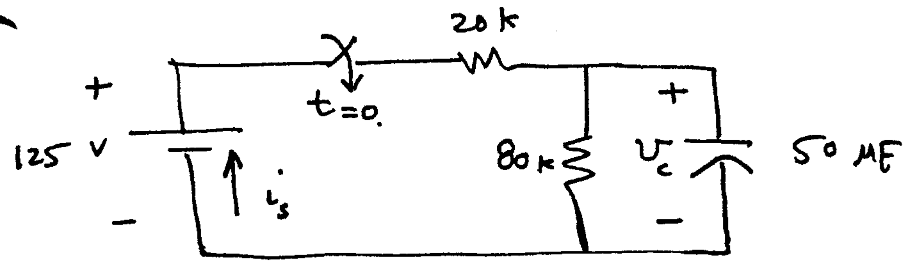
the form of this DE is similar to DE of the current of the inductor.

$$\therefore \quad v_c = I_s R + (V_0 - I_s R)e^{-t/RC}$$

$$i(t) = C\frac{dv_c}{dt} = \cancel{C}\left(-\frac{1}{R\cancel{C}}\right)(V_0 - I_s R)e^{-t/RC}$$

$$= \left(\cancel{I_s}^{\frac{V_s}{R}} - \frac{V_0}{R}\right)e^{-t/RC}, \quad t \geq 0.$$

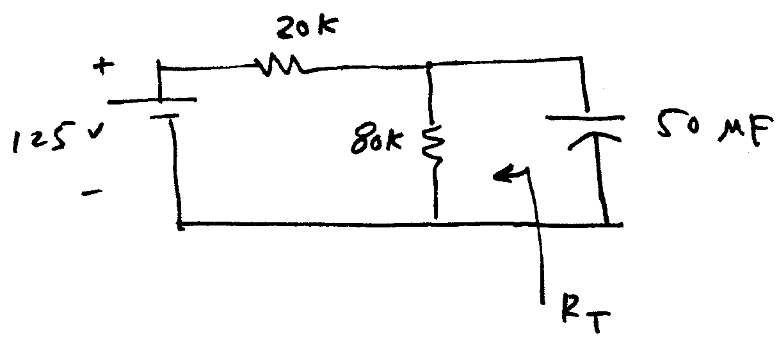
$\frac{7.9}{229}$:



a) $v_c(t) = ?$; $t \geq 0$

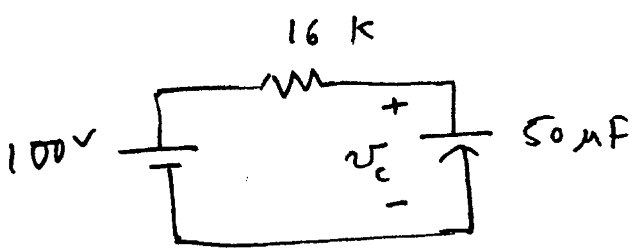
$t < 0 \Rightarrow v_c(t) = 0.$

$t \geq 0$:



$$R_T = 20 \,//\, 80 = \frac{(2\cancel{0})(8\cancel{0})}{\cancel{20+80}\ 1\cancel{00}} = 16 \text{ k}$$

$$V_T = \frac{80}{20+80}(125) = 100 \text{ V.}$$



Know

$$v_c(t) = V_s + (V_0 - V_s)\, e^{-t/RC} \quad , \quad t \geq 0$$

$$= 100 + (0 - 100)\, e^{-t/RC}$$

$$\tau = RC = (16 \times 10^{+3})(50)(10^{-6}) = 800 \times 10^{-3} = .8$$

$$\therefore \quad v_c(t) = 100 - 100\, e^{-1.25t} \quad ; \quad t \geq 0.$$

b) $i_s(t) = ?$ $\quad t \geq 0.$

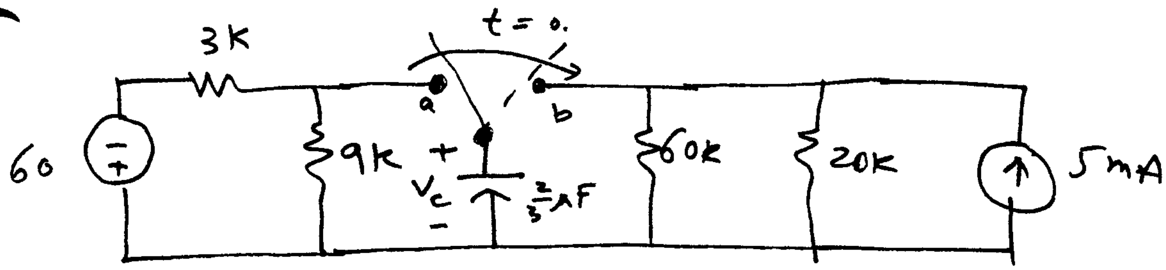
$$i_s(t) = i_{80} + i_c(t)$$

$$= \frac{v_c}{80k} + C \frac{dv_c}{dt}$$

$$= \frac{100 - 100e^{-1.25t}}{(80)(10^3)} + (50 \times 10^{-6})(125\, e^{-1.25t})$$

$$=$$

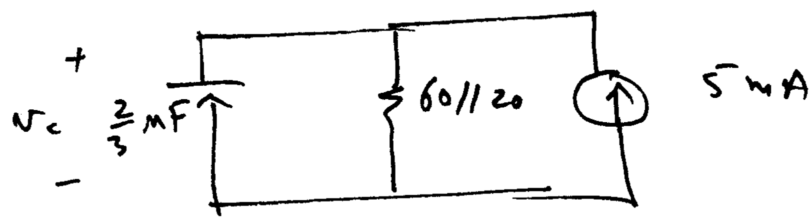
$\frac{7.10}{229}$:

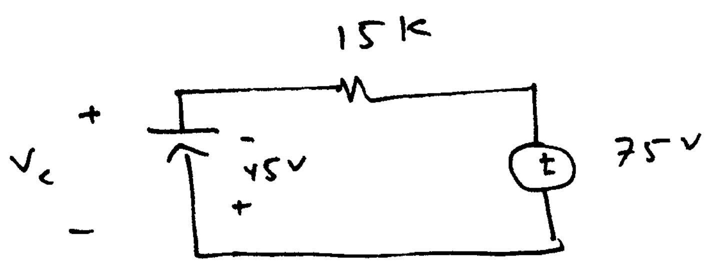


$v_c(t) = ?$; $t \geq 0$

$t < 0$: $\Rightarrow v_c(0^-) = v_c(0^+) = \frac{9}{3+9}(-60) = -45\,V$

$t > 0^+$:





$\tau = RC = 10$ ms

$V_c(\infty) = 75\ V$

$\therefore\ V_c(t) = 75 + (-45 - 75)e^{-100t} = (75 - 120e^{-100t})\,V$

$t \geq 0^+$