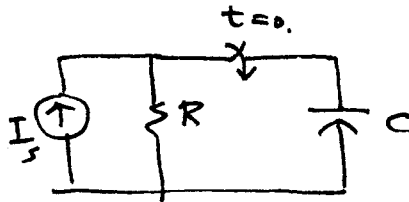
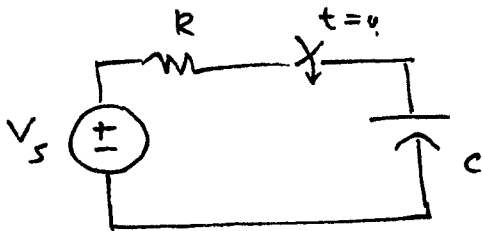
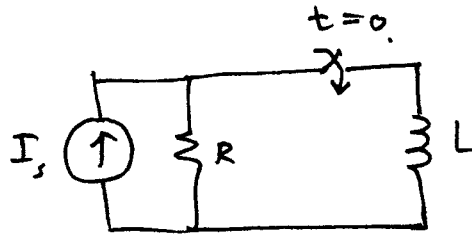
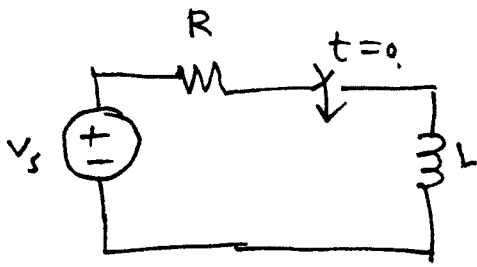


# General method for finding the

## Step Response of 1<sup>st</sup> order RL or

### RC circuit:

Basically, we deal with four different cases:



The DE describing the  $i$  or  $v$  is of the form:

$$\frac{dx}{dt} + \frac{x}{\tau} = K$$

where  $K$  can be zero.

When  $x$  has reached its final value,

$$\frac{dx}{dt} = 0 \Rightarrow x_f = K\tau$$

$$\frac{dx}{dt} = k - \frac{x}{\tau} = \frac{k\tau - x}{\tau} = \frac{-(x - k\tau)}{\tau}$$

$$\int_{x(0)}^{x(t)} \frac{dx}{x - x_f} = -\frac{1}{\tau} \int_0^t dt$$

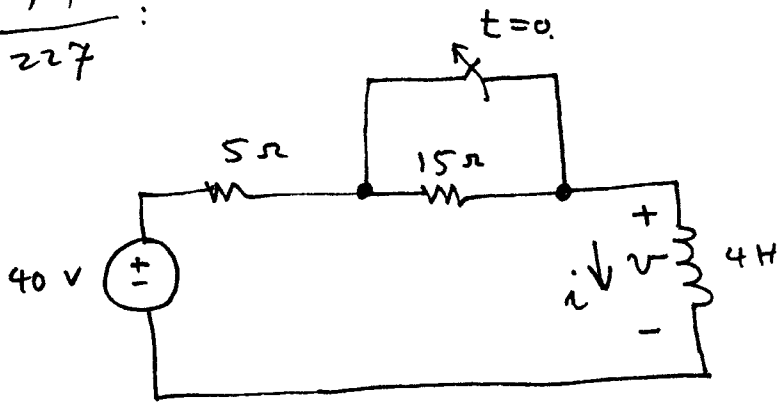
$$\ln(x - x_f) \Big|_{x(0)}^{x(t)} = -\frac{1}{\tau} t \Rightarrow \ln \frac{x - x_f}{x(0) - x_f} = -\frac{t}{\tau}$$

$$\therefore x(t) = x_f + [x(0) - x_f] e^{-t/\tau} ; t \geq 0$$

Need to find

1. Final value. ( $x_f$ )
2. initial value ( $x(0)$ )
3. time constant ( $\tau$ ).

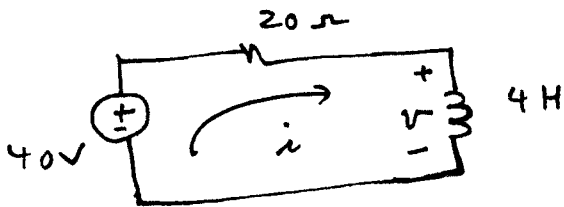
$$\frac{7.4}{227} :$$



$t < 0 :$

$$i(t) = \frac{40}{5} = 8 \text{ A} \quad \Rightarrow \quad i(0^+) = 8 \text{ A}$$

$t \geq 0 :$



$$i(t) = i_f(t) + (i(0) - i_f) e^{-t/\tau}$$

$$\tau = \frac{L}{R} = \frac{4}{20} = \frac{1}{5} \text{ s.}$$

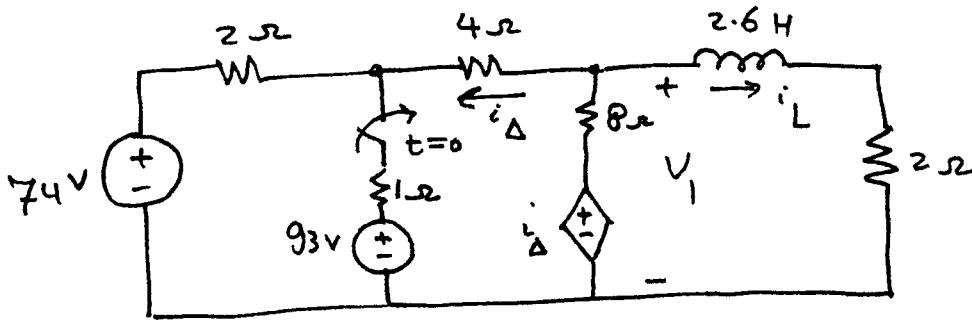
$$i_f(t) = \frac{40}{20} = 2 \text{ A.}$$

$$\therefore i(t) = 2 + (8 - 2)e^{-5t} = 2 + 6e^{-5t} \text{ A ; } t \geq 0^+$$

$$v(t) = 40 - 20i = 40 - (40 + 120e^{-5t})$$

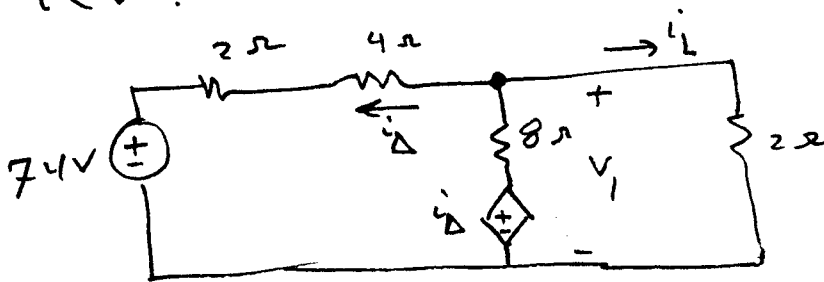
$$= -120e^{-5t} \text{ V ; } t \geq 0^+$$

$$\frac{7.7}{228} :$$



$$v_1(t) = ? ; t \geq 0.$$

$t < 0 :$



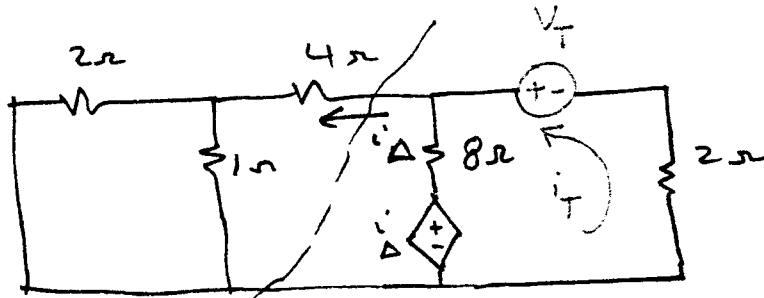
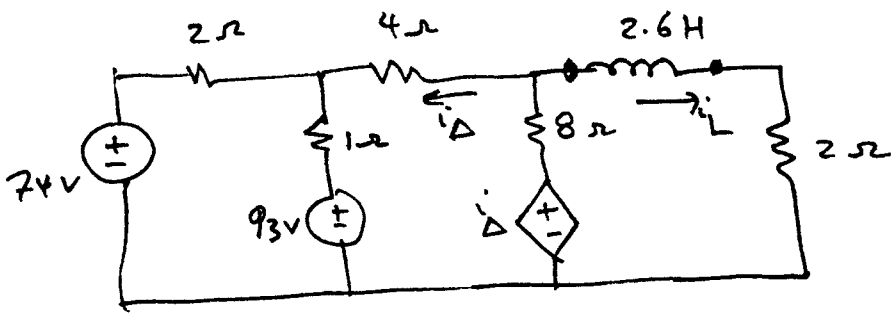
$$\text{KCL: } \left\{ \begin{aligned} & \frac{-74 + v_1}{6} + \frac{v_1 - i_\Delta}{8} + \frac{v_1}{2} = 0. \\ & i_\Delta = \frac{v_1 - 74}{6} \end{aligned} \right.$$

$$\therefore v_1 = 14 \text{ V}$$

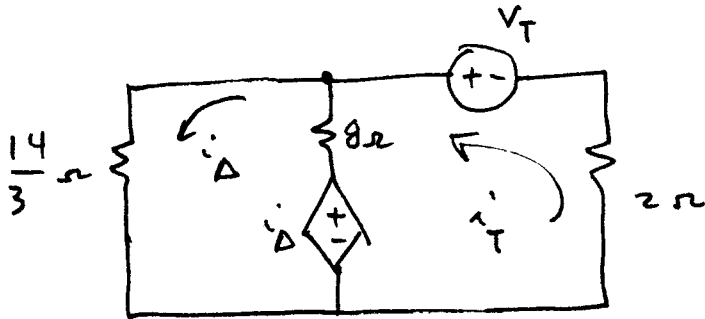
$$i_L(0^-) = \frac{v_1}{2} = \frac{14}{2} = \underline{7 \text{ A}}$$

$t > 0 :$

$R_T, V_T = ?$

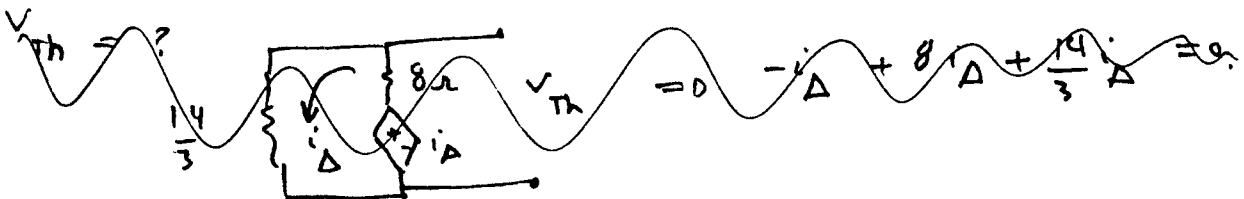


$$2 // 1 + 4 = \frac{2}{\frac{1}{2} + \frac{1}{1}} + 4 = \frac{2}{\frac{3}{2}} + 4 = \frac{2+12}{3} = \frac{14}{3} \Omega.$$

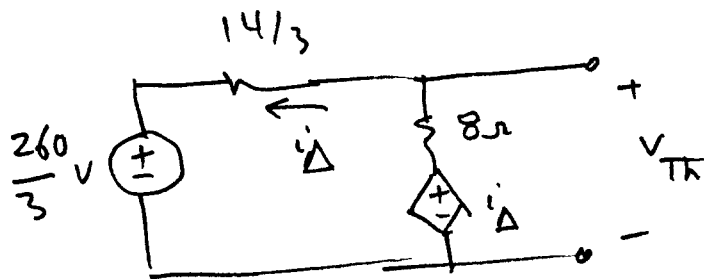
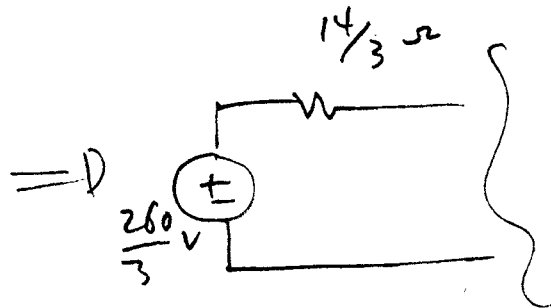
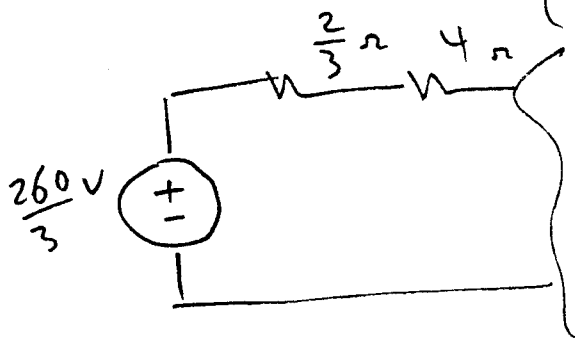
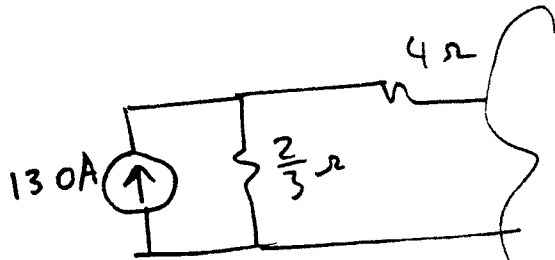
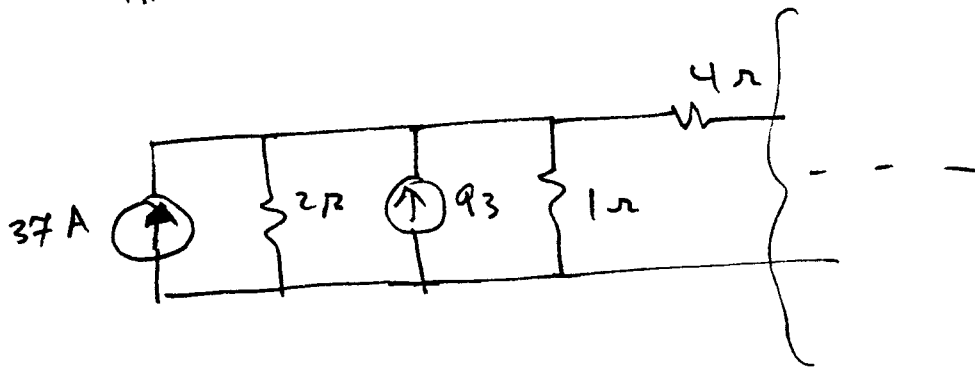


KVL:

$$\begin{cases} -i_{\Delta} + 8(i_{\Delta} - i_T) + \frac{14}{3}i_{\Delta} = 0 \Rightarrow i_{\Delta} = \frac{24}{35}i_T \\ 2i_T - V_T + \frac{14}{3}i_{\Delta} = 0 \Rightarrow V_T = \underbrace{\frac{26}{5}}_{R_T} i_T \end{cases}$$



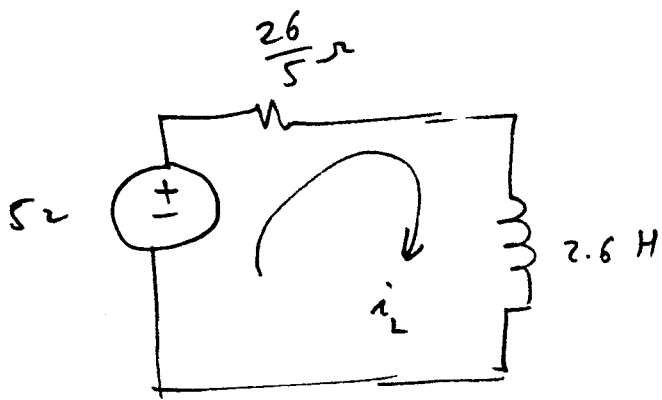
$$V_{th} = ?$$



$$\frac{14}{3} i_{\Delta} + \frac{260}{3} - i_{\Delta} + 8 i_{\Delta} = 0.$$

$$\frac{35}{3} i_{\Delta} + \frac{260}{3} = 0 \Rightarrow i_{\Delta} = -\frac{260}{35} = -\frac{52}{7} \text{ A}$$

$$V_{th} = -8 i_{\Delta} + i_{\Delta} = -7 i_{\Delta} = (-7) \left( -\frac{52}{7} \right) = \underline{52 \text{ V}}$$



$$i(t) = i_f + (i_0 - i_f) e^{-t/\tau}$$

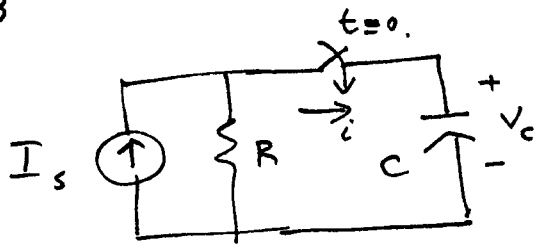
$$\tau = \frac{L}{R} = \frac{2.6}{26/5} = 0.5, \quad i_f = \frac{52}{26/5} = 10 \text{ A}$$

$$i_L(t) = 10 + (7 - 10) e^{-2t}$$

$$i_L(t) = 10 - 3e^{-2t} \text{ A} \quad t \geq 0$$

$$v_L(t) = 2.6 \frac{di_L}{dt} + 2i_L = 20 + 9.6e^{-2t} \text{ V}, \quad t \geq 0$$

$$\frac{7.8}{228} ;$$



$$i(t) = 5e^{-50t} \text{ mA} ; t \geq 0$$

$$v_c(t) = 200 - 200e^{-50t} \text{ V} ; t \geq 0$$

a)  $I_s, R, C, \tau = ?$

$$\begin{cases} V_s = 200 \\ V_c(0) - V_s = -200 \end{cases} \Rightarrow v_c(0) = 0$$

$$I_0 = 5 = I_s \Rightarrow \underline{\underline{I_s = 5}}$$

$$V_c(\infty) = \frac{I_s R}{5} = 200 \Rightarrow R = \frac{200}{5} = \underline{\underline{40 \text{ k}\Omega}}$$

$$\tau = \frac{R}{50} = \frac{1}{50} \Rightarrow \underline{\underline{C = 0.5 \mu\text{F}}}$$

b)  $P = v_i = (5e^{-50t} \times 10^{-3}) (200 - 200e^{-50t}) \text{ W}$   
 $= e^{-50t} - e^{-100t} \text{ W}$

$$W = \int_0^{t_1} (e^{-50t} - e^{-100t}) dt, \quad W(\infty) = 10 \text{ mJ}$$

$$= 10 [e^{-100t_1} - 2e^{-50t_1} + 1] \text{ mJ}$$



$$W(t_1) = .36 (10) = 3.6 \text{ mJ}$$

$$\therefore 10 \left[ e^{-100 t_1} - 2e^{-50 t_1} + 1 \right] = 3.6$$

$$\therefore e^{-100 t_1} - 2e^{-50 t_1} + .64 = 0.$$

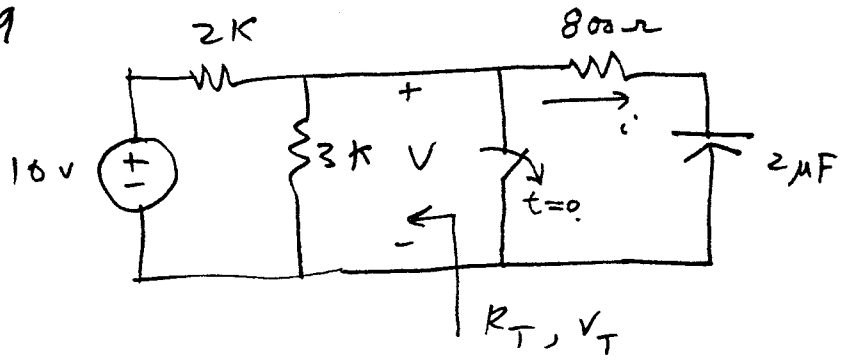
$$\text{or } x^2 - 2x + .64 = 0.$$

$$\textcircled{\otimes} \quad \cancel{x = 1.6}, \quad x = .4 \leftarrow (t_1 > 0)$$

$\uparrow$   $t_1 < 0$

$$\therefore e^{-50 t_1} = .4 \Rightarrow t_1 = \underline{\underline{18.33 \text{ ms}}}$$

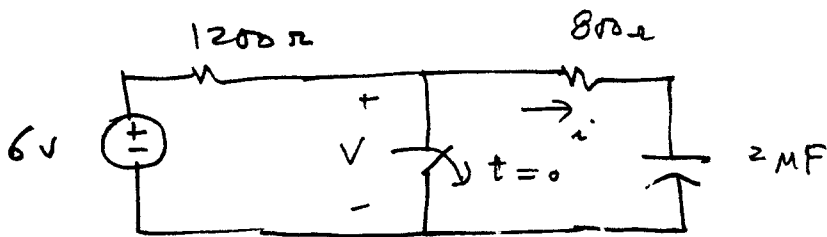
7.11 :  
229



a)

$$V_T = \frac{3}{2+3} 10 = 6 \text{ V}$$

$$R_T = 2 \parallel 3 = \frac{6}{5} \text{ k} = 1200 \Omega$$



$$t < 0 \Rightarrow i(0^-) = 0, \quad v_C(0^-) = 0.$$

$$t > 0 \Rightarrow v_C(0^+) = v_C(0^-) = 0 \Rightarrow i(0^+) = \frac{6}{1200 + 800} = \underline{\underline{3 \text{ mA}}}$$

b)  $i(\infty) = 0.$

c)  $\tau = (1200 + 800)(2 \times 10^{-6}) = 4 \times 10^{-3} \text{ ms.}$

d)  $i(t) = 0 + (3 - 0)e^{-250t} \text{ mA. ; } t \geq 0^+$

e)  $v(t) = 6 - 1200 i = 6 - (1.2)(3)e^{-250t}$