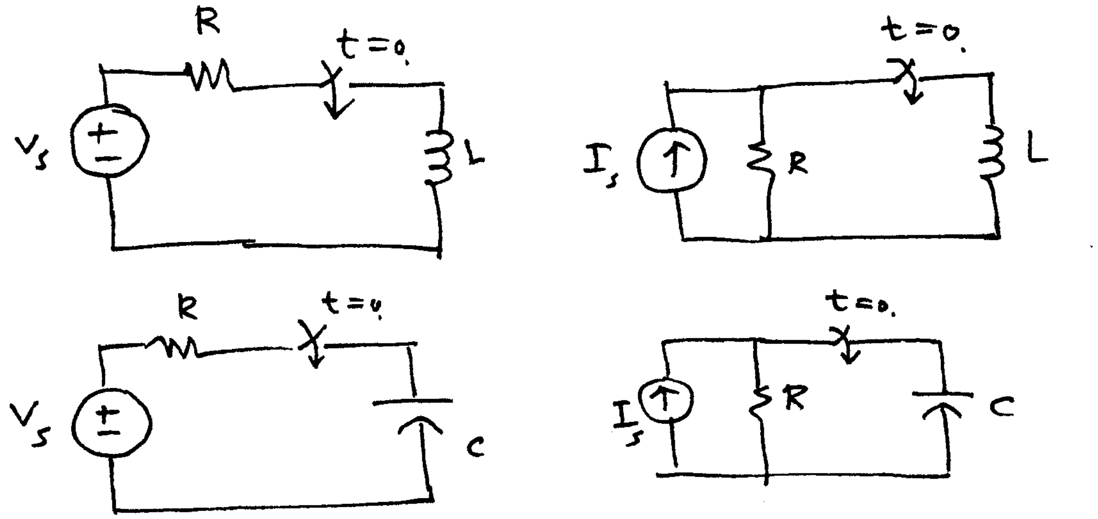
General method for finding the Step Response of 1st order RL or RC circuit:

Basically, we deal with four different cases:



The DE describing the i or v is of the form:

$$\frac{dx}{dt} + \frac{x}{\tau} = K$$

where K can be zero.

When x has reached its final value,

$$\frac{dx}{dt} = 0. \Rightarrow \quad x_f = K\tau.$$

$$\frac{dx}{dt} = k - \frac{x}{\tau} = \frac{k\tau - x}{\tau} = \frac{-(x - \overset{x_f}{\cancel{k\tau}})}{\tau}$$

$$\int_{x(0)}^{x(t)} \frac{dx}{x - x_f} = -\frac{1}{\tau}\int_0^t dt$$

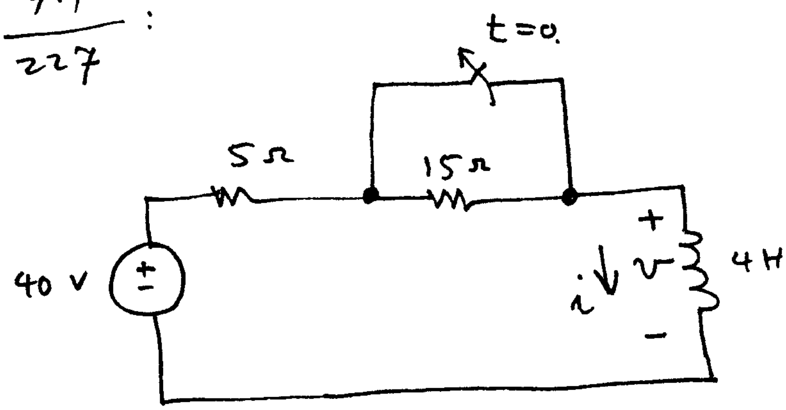
$$\ln(x - x_f)\Big|_{x(0)}^{x(t)} = -\frac{1}{\tau}t \Rightarrow \ln\frac{x - x_f}{x(0) - x_f} = -\frac{t}{\tau}$$

$$\therefore \quad x(t) = x_f + [x(0) - x_f]e^{-t/\tau} \quad ; \quad t \geq 0$$

Need to find

1. Final value. (x_f)
2. initial value ($x(0)$)
3. time Constant (τ).

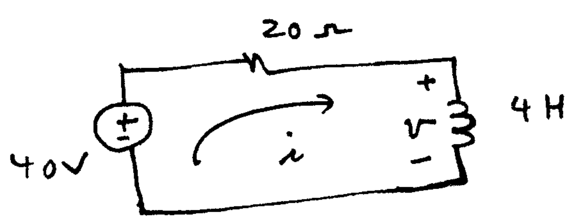
$\frac{7.4}{227}$:



$t < 0$:

$$i(t) = \frac{40}{5} = 8 \text{ A} . \Rightarrow i(0^+) = 8 \text{ A}$$

$t \geq 0.$:



$$i(t) = i_f(t) + (i(0) - i_f)e^{-t/\tau}$$

$$\tau = \frac{L}{R} = \frac{4}{20} = \frac{1}{5} \text{ s.}$$

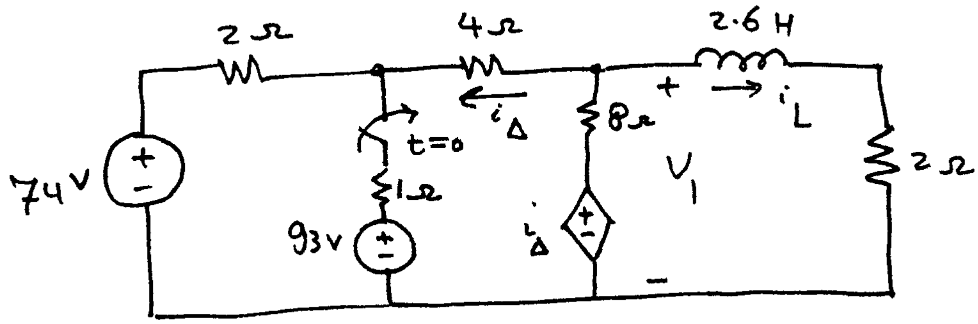
$$i_f(t) = \frac{40}{20} = 2 \text{ A} .$$

$$\therefore \quad i(t) = 2 + (8 - 2)e^{-5t} = 2 + 6e^{-5t} \text{ A} \; ; \; t \geq 0^+$$

$$v(t) = 40 - 20i = 40 - (40 + 120e^{-5t})$$

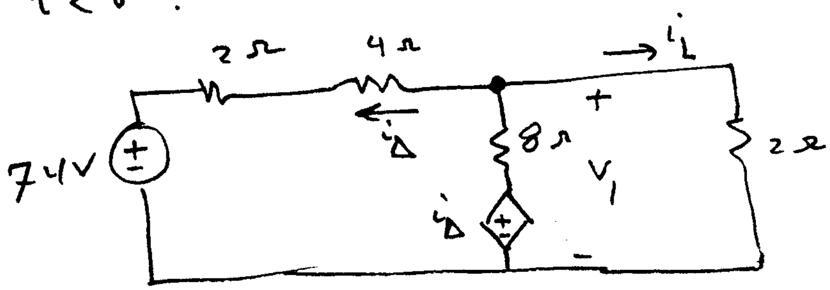
$$= -120e^{-5t} \text{ V} \; ; \; t \geq 0^+$$

$\frac{7.7}{228}$:



$v_1(t) = ?$; $t \geq 0$.

$t < 0$:



KCL :

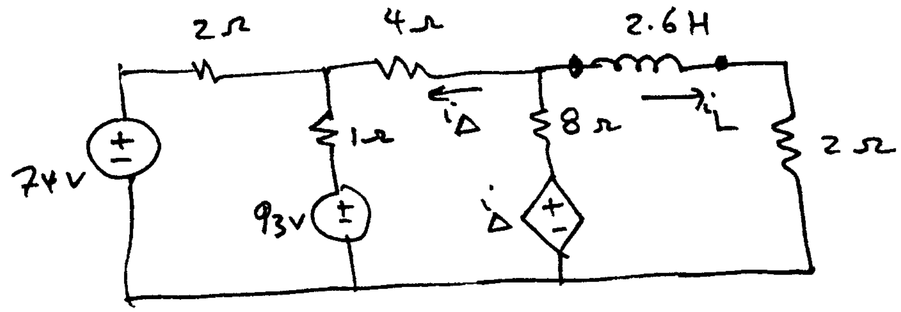
$$\begin{cases} \underbrace{\dfrac{-74+V_1}{6}}_{i_\Delta} + \dfrac{V_1 - i_\Delta}{8} + \dfrac{V_1}{2} = 0. \\ i_\Delta = \dfrac{V_1 - 74}{6} \end{cases}$$

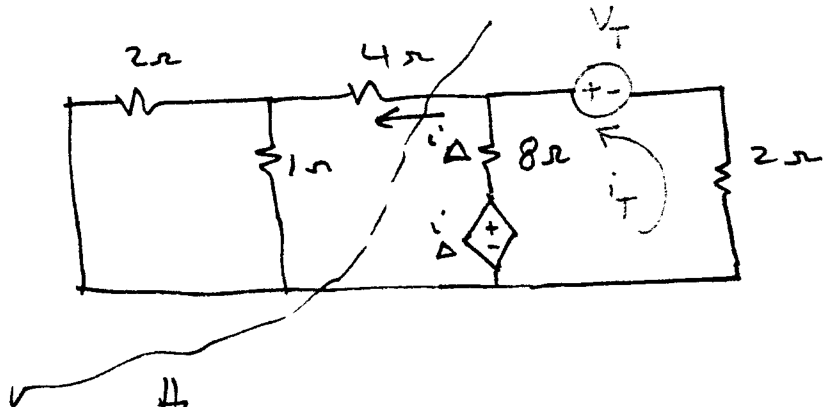
$\therefore \quad V_1 = 14\ V$

$i_L(0^-) = \frac{V_1}{2} = \frac{14}{2} = 7\ A$

$t \geq 0$:

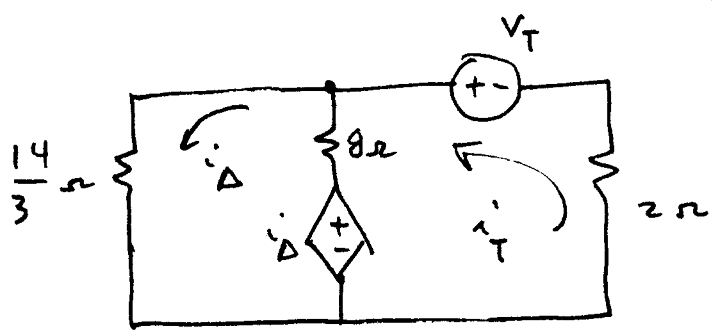
$R_T, V_T = ?$





$\Downarrow$

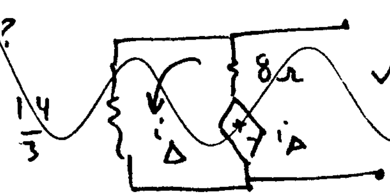
$$2 // 1 + 4 = \frac{2}{3} + 4 = \frac{2+12}{3} = \frac{14}{3}\ \Omega.$$



KVL :

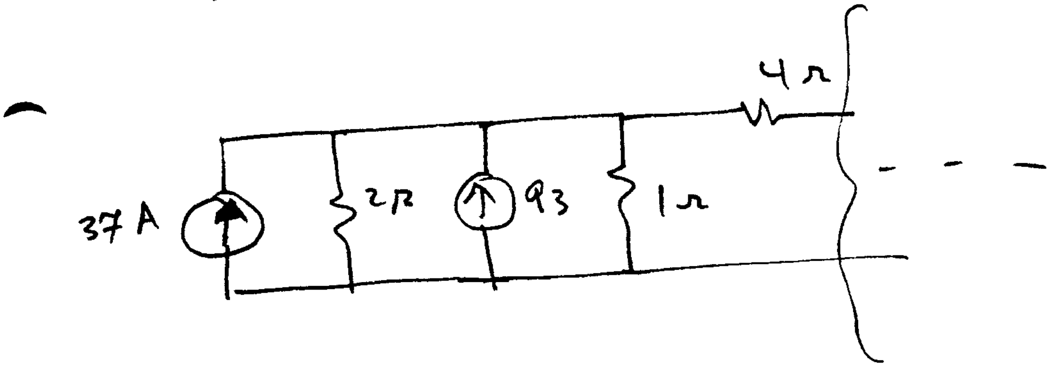
$$\begin{cases} -i_\Delta + 8(i_\Delta - i_T) + \frac{14}{3} i_\Delta = 0 \Rightarrow i_\Delta = \frac{24}{35} i_T \\ 2 i_T - V_T + \frac{14}{3} i_\Delta = 0 \Rightarrow V_T = \underbrace{\frac{26}{5}}_{R_T} i_T \end{cases}$$

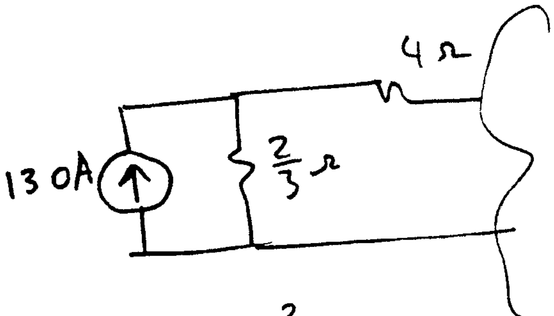
$V_{Th} = ?$

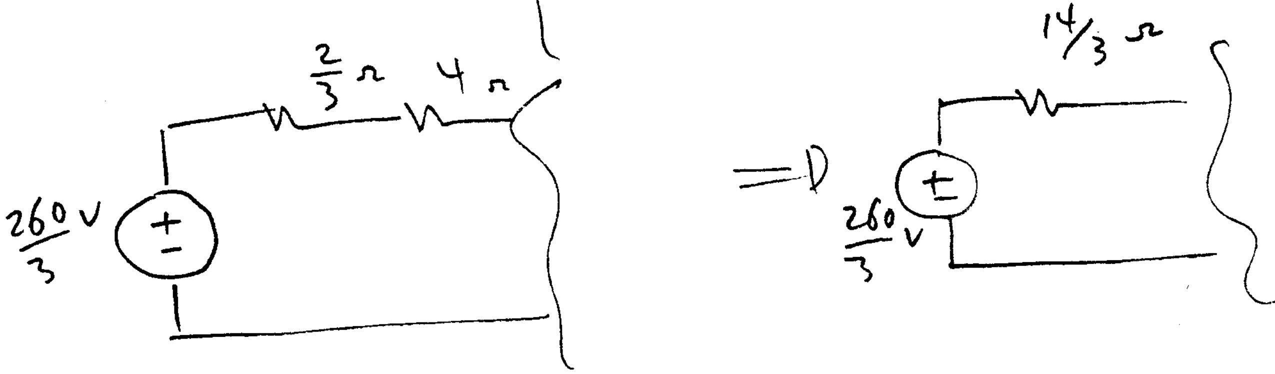


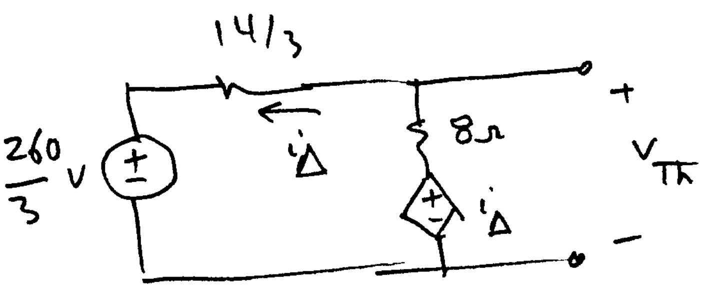
$V_{Th} = 0$ $\quad -i_\Delta + 8 i_\Delta + \frac{14}{3} i_\Delta = 0$

$V_{Th} = ?$





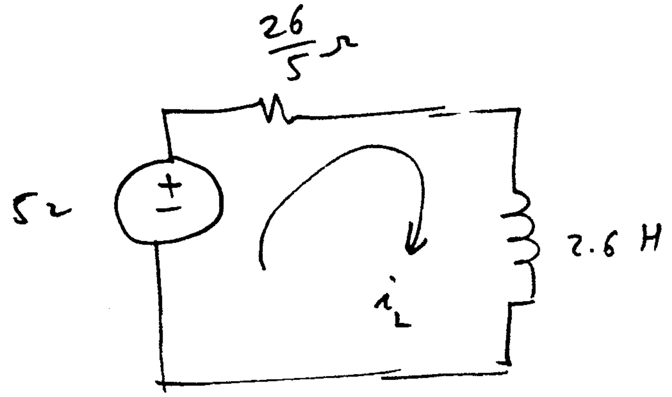




$$\frac{14}{3} i_\Delta + \frac{260}{3} - i_\Delta + 8 i_\Delta = 0.$$

$$\frac{35}{\cancel{3}} i_\Delta + \frac{260}{\cancel{3}} = 0. \implies i_\Delta = -\frac{260}{35} = -\frac{52}{7}\ \text{A}$$

$$V_{th} = -8 i_\Delta + i_\Delta = -7 i_\Delta = (-\cancel{7})\left(\frac{-52}{\cancel{7}}\right) = \underline{52\ \text{V}}$$



$$i(t) = i_f + (i_0 - i_f)e^{-t/\tau}$$

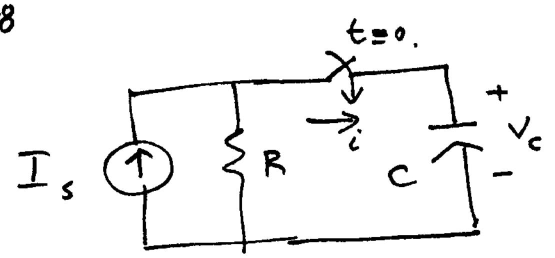
$$\tau = \frac{L}{R} = \frac{2.6}{26/5} = .5 \quad , \quad i_f = \frac{52}{26/5} = 10\ A$$

$$i_L(t) = 10 + (7-10)e^{-2t}$$

$$i_L(t) = 10 - 3e^{-2t}\ A \qquad t \geq 0$$

$$V_1(t) = 2.6\frac{di_L}{dt} + 2i_L = 20 + 9.6e^{-2t}\ V, \quad t \geq 0.$$

$\frac{7.8}{228}$:



$i(t) = 5e^{-50t}$ mA ; $t \geq 0$

$v_c(t) = 200 - 200e^{-50t}$ v ; $t \geq 0$

a) $I_s, R, C, \tau = ?$

$$\begin{cases} V_s = 200 \\ V_c(0) - V_s = -200 \end{cases} \Rightarrow v_c(0) = 0.$$

$I_0 = 5 = I_s \Rightarrow \underline{\underline{I_s = 5}}$

$V_c(\infty) = \underbrace{I_s}_{5} R = 200 \Rightarrow R = \frac{200}{5} = \underline{\underline{40 \text{ k}\Omega}}$

$\tau = \underbrace{R}_{40k} C = \frac{1}{50} \Rightarrow \underline{\underline{C = .5 \, \mu F}}$

b) $P = vi = (5e^{-50t} \times 10^{-3})(200 - 200e^{-50t})$ W

$= e^{-50t} - e^{-100t}$ W.

$W = \int_0^{t_1} (e^{-50t} - e^{-100t}) dt$, $W(\infty) = 10$ mJ

$= 10[e^{-100t_1} - 2e^{-50t_1} + 1]$ mJ

$W(t_1) = .36\,(10) = 3.6 \text{ mJ}$

$$\therefore \quad 10\left[e^{-100 t_1} - 2e^{-50 t_1} + 1\right] = 3.6$$

$$\therefore \quad e^{-100 t_1} - 2e^{-50 t_1} + .64 = 0.$$

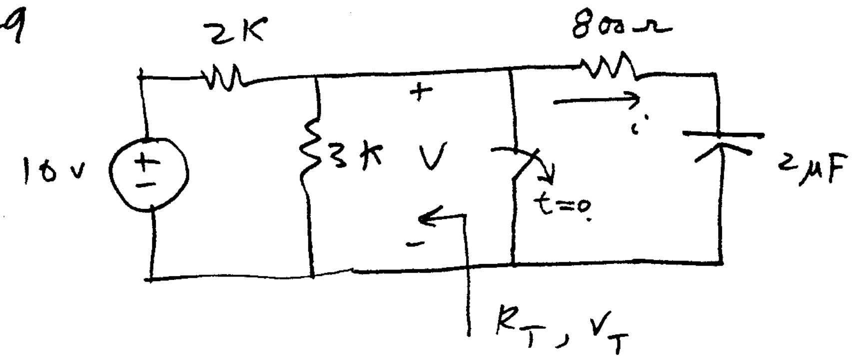
or $\quad x^2 - 2x + .64 = 0$

$\cancel{x = 1.6}$, $\quad x = .4 \leftarrow (t_1 > 0)$

$\uparrow t_1 < 0$

$$\therefore \quad e^{-50 t_1} = .4 \implies t_1 = 18.33 \text{ ms}$$

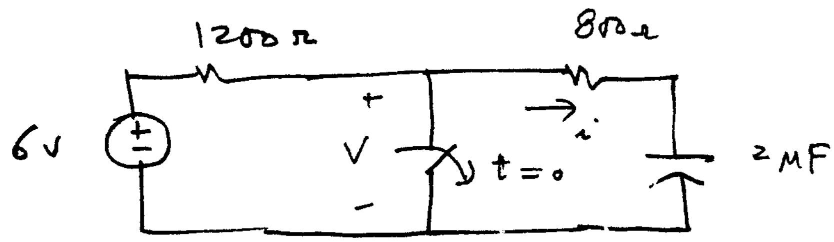
$\frac{7.11}{229}$:



a)

$$V_T = \frac{3}{2+3} 10 = 6 \quad V$$

$$R_T = 2 // 3 = \frac{6}{5} K = 1200 \; \Omega$$



$t < 0 \Rightarrow i(0^-) = 0. \quad , \quad v_c(0^-) = 0.$

$t > 0 \Rightarrow v_c(0^+) = v_c(0^-) = 0. \Rightarrow i(0^+) = \frac{6}{1200+800} = \underline{\underline{3 \; mA}}$

b) $i(\infty) = 0.$

c) $\tau = (1200 + 800)(2 \times 10^{-6}) = 4 \times 10^{-3}$ ms.

d) $i(t) = 0 + (3 - 0) e^{-250t}$ mA. ; $t \geq 0^+$

e) $v(t) = 6 - 1200 \, i = 6 - (1.2)(3) e^{-250t}$