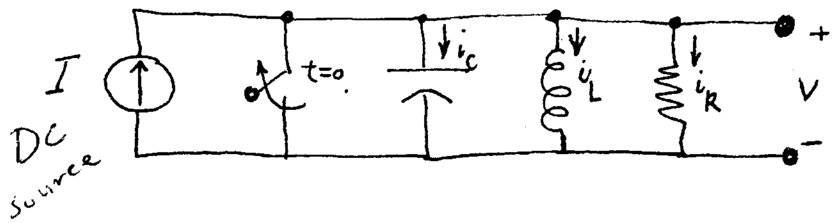
Step Response of a Parallel RLC:



$$i_L + i_R + i_C = I$$

$$i_L + \frac{v}{R} + C\frac{dv}{dt} = I \qquad (*)$$

$$v = L\frac{di_L}{dt} \Rightarrow \frac{dv}{dt} = L\frac{d^2 i_L}{dt^2}$$

$$\therefore \quad i_L + \frac{L}{R}\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} = I$$

non homogeneous DE

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{1}{LC} i_L = \frac{1}{LC} I.$$

Similar DE can be obtained in terms of v as follows:

$$(*) \Rightarrow \underbrace{\frac{1}{L}\int_{0^+}^{t} v(\tau)\,d\tau + I_0}_{i_L} + \frac{v}{R} + C\frac{dv}{dt} = I \quad , \quad t \geq 0^+$$

or

$$\frac{1}{L} v(t) + \frac{dv}{dt}\frac{1}{R} + C\frac{d^2 v}{dt^2} = 0. \quad ; \quad t \geq 0^+$$

$$\therefore \quad \frac{d^2 v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC} v(t) = 0 \quad ; \quad t \geq 0^+$$

$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad (1)$$

$$v = e^{-\alpha t}\left[B_1 \cos\omega_d t + B_2 \sin\omega_d t \right] \qquad (2)$$

$$v = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}. \qquad (3)$$

$$* \Longrightarrow \quad i_L = I - \frac{v}{R} - C\frac{dv}{dt}$$

$$(1) \Longrightarrow \quad i_L = I + \underbrace{A_1' e^{s_1 t} + A_2' e^{s_2 t}}$$

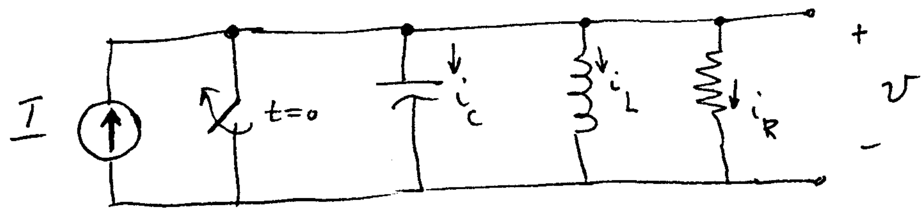
$$(2) \Longrightarrow \quad i_L = I + \underbrace{e^{-\alpha t}\left[B_1' \cos\omega_d t + B_2' \sin\omega_d t \right]}$$

$$(3) \Longrightarrow \quad i_L = I + \underbrace{D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}}$$

i_f

same form as the natural response.

$\#\frac{8.11}{270}$:



$I = 24\ mA$

$R = 400\ \Omega$

$L = 25\ mH$

$C = 25\ nF$

$i_L(0^-) = i_L(0^+) = 0.$

$v_c(0^-) = v_c(0^+) = 0$

a) $v(t) = ?$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0.$$

$$s^2 + \frac{1}{(400)(25\times10^{-9})}s + \frac{10^{12}}{(25)(25)} = 0.$$

$$s^2 + 10^5 s + 16\times10^8 = 0.$$

$$s_{1,2} = -5\times10^4 \pm \sqrt{25\times10^8 - 16\times10^8} = -5\times10^4 \pm 3\times10^4$$

$$s_1 = -8\times10^4$$

$$s_2 = -2\times10^4$$

$$\begin{cases} i_L(t) = A_1' e^{-8\times10 t} + A_2' e^{-2\times10 t} + I \\ i_L(0) = 0 \quad \leftarrow \text{given} \\ i_L'(0) = 0 \quad \leftarrow v_c(0) = \frac{di_L(0)}{dt} L \Rightarrow \frac{di_L(0)}{dt} = 0 \end{cases}$$

$$i_L(0) = A_1' + A_2' + \cancel{I}^{24} = 0 \Rightarrow \begin{cases} A_1' + A_2' = -24 \\ 4A_1' + A_2' = 0 \end{cases}$$

$$i_L'(0) = -8\times10^4 A_1' - 2\times10^{-4} A_2' = 0$$

$$A_1' = 8$$

$$A_2' = -32$$

$$\therefore \quad i_L(t) = 8e^{-8\times10^4 t} - 32e^{-2\times10^4 t} + 24 \quad mA$$

$$v(t) = L\frac{di_L(t)}{dt} = (25\times10^{-3})\left[-64\times10^4 e^{-8\times10^4 t} + 10^4 \cdot 64 e^{-2\times10^4 t}\right]$$

$$= 16000\left(-e^{-8\times10^4 t} + e^{-2\times10^4 t}\right) \quad mv$$

$$= \underline{\underline{16e^{-2\times10^4 t} - 16e^{-8\times10^4 t} \quad v.}} \begin{cases} 1)\ \lim_{t\to 0} v(t) = 0 \\ 2)\ \lim_{t\to\infty} v(t) = 0 \end{cases}$$

* notice that we could not evaluate $v(t)$ directly because $v'(0)$ is not known.

$$i_c(t)\Big|_{t=0} = c\frac{dv}{dt}\Big|_{t=0} = ?$$

$$i_R(t) = \frac{v}{R} = \frac{\overset{40}{\cancel{16000}}}{\cancel{400}}\left(e^{-2\times10^4 t} - e^{-8\times10^4 t}\right) \quad mA.$$

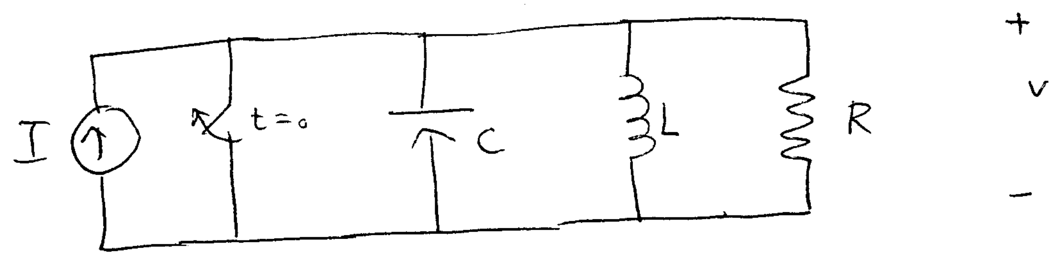
c) $i_c(t) = ?$

$$i_C(t) = C\frac{dv}{dt} = I - i_L(t) - i_R(t)$$

$$= \cancel{24} - \left[8e^{-8\times10^4 t} - 32e^{-2\times10^4 t} + \cancel{24}\right] - \left[40e^{-2\times10^4 t} - \cancel{40e^{-8\times10^4 t}}\right]$$

$$i_C(t) = 32\times e^{-8\times10^4 t} - 8\times e^{-2\times10^4 t} \quad mA.$$

$\frac{8.13}{270}$: (Exa $\frac{8.6}{258}$)



$I = 24$ mA

$L = 25$ mH

$C = 25$ nF

$R = 625\ \Omega$

$i_L(0^-) = i_L(0^+) = 0$

$v_c(0^-) = v_c(0^+) = 0 \implies \frac{di_L}{dt} = 0$

a) $v(t) = ?$

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s_{1,2} = -3.2\times10^4 \pm j\,2.4\times10^4 \quad s^{-1}$$

$$\begin{cases} i_L(t) = I + e^{-\alpha t}\left[B_1' \cos\omega_d t + B_2' \sin\omega_d t\right] \\ i_L(0) = 0 \\ i_L'(0) = 0 \end{cases} \implies \begin{matrix} B_1' = -24 \text{ mA} \\ B_2' = -32 \text{ mA} \end{matrix}$$

$$i_L(t) = 24 - e^{-32000t}\left[24 \cos 24000t + 32 \sin 24000t\right] \text{ mA}; \quad t \geq$$

$$v(t) = L\frac{di_L}{dt} = 40e^{-32000t}\sin 24000t\ ; \quad v,\ t \geq 0^+$$

b) $$i_C(t) = I - \cancel{i_R}(t) - i_L(t)$$

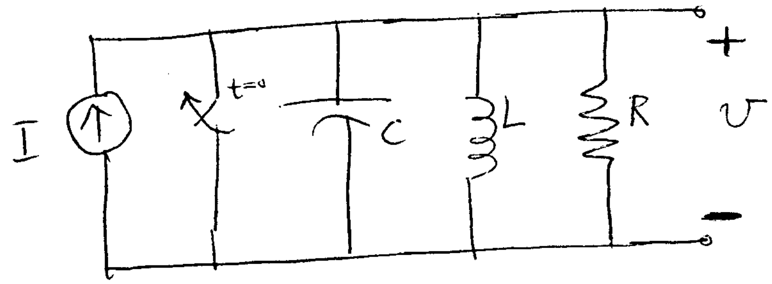
$$\frac{v}{R}$$

$$= 24e^{-32000t}\cos 24000t - 32e^{-32000t}\sin 24000t$$

mA.

$t \geq 0^+$

8.15 :
270



$W_L(0) = 0$

$W_C(0) = 153.6\ \mu J$

$R = 5k$, $L = 120\ H$, $C = \frac{10}{3}\ \mu F$

$I = 4\ mA.$

a) $i_L(t) = ?$

$W_L(0) = \frac{1}{2} L\, i_L^2(0) = 0 \Rightarrow i_L(0) = 0$

$W_C(0) = \frac{1}{2} \cancel{C}\, v_C^2(0) = 153.6 \quad \Rightarrow v_C(0) = 9.6\,V$
$\frac{10}{3}$

$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0 \Rightarrow s^2 + 60s + 2500 = 0$

$s_{1,2} = -30 \pm \sqrt{\underbrace{(30)^2}_{900} - 2500} = \underbrace{-30}_{\alpha} \pm j\underbrace{40}_{\omega_d} \quad \frac{1}{s}$

$$\begin{cases} i_L(t) = \overset{4}{\cancel{I}} + e^{-30t}\left[B_1' \cos 40t + B_2' \sin 40t \right]_{mA} ; \quad t \geq 0 \\ i_L(0) = 0 \\ i_L'(0) = ? \end{cases}$$

$$v_C(t)\Big|_{t=0} = L \left.\frac{di_L(t)}{dt}\right|_{t=0} = 9.6 \text{ V}$$

$$\frac{di_L(0)}{dt} = \frac{9.6}{120} = 80$$

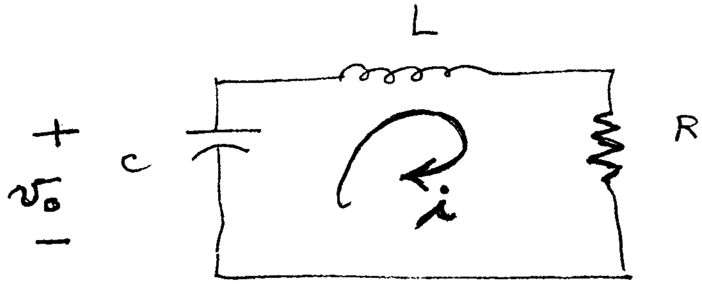
$$\therefore \quad B_1' = -4 \text{ mA}$$

$$B_2' = -1 \text{ mA.}$$

$$\therefore \quad i_L(t) = 4 - e^{-30t}\left[4\cos 40t + \sin 40t\right]; \quad t \geq 0$$
mA.

RLC series Circuit:

natural response:



$$v_c + v_L + v_R = 0.$$

$$R\,i(t) + \left(\frac{1}{c}\int^t i(\tau)\,d\tau + V_o\right) + L\,\frac{di(t)}{dt} = 0.$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{Lc}\,i = 0.$$

$$s^2 + \frac{R}{L}s + \frac{1}{Lc} = 0$$

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{Lc}} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$\alpha = \dfrac{R}{2L}$: Neper freq. ($\alpha = \dfrac{1}{2Rc}$ for parallel case)

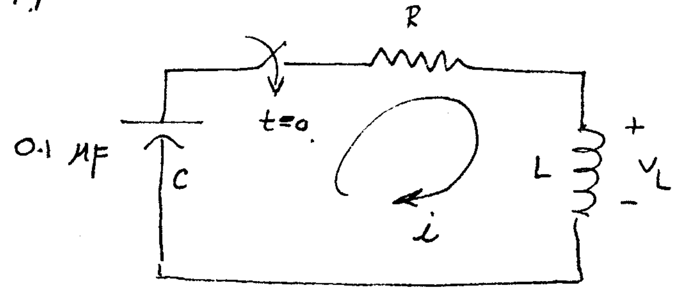
$$\omega_o^2 = \frac{1}{Lc}$$

(a) overdamped $\alpha^2 > \omega_o^2$

(b) underdamped $\alpha^2 < \omega_o^2$

(c) critically damped $\alpha^2 = \omega_o^2$

$\# \frac{8.19}{271}$:



$W_C(0) = 45\ \mu J$

$W_L(0) = 0.$

$s_1 = -2000, \quad s_2 = -8000 \Rightarrow (s+2000)(s+8000) = 0.$

$$s^2 + \underbrace{10000}_{\frac{R}{L}} s + \underbrace{16 \times 10^6}_{\frac{1}{LC}} = 0$$

(a) $R, L = ?$

$\frac{R}{L} = 10000$

$\frac{1}{LC} = \frac{1}{(L)(0.1)(\cancel{10^{-6}})} = 16 \times \cancel{10^6} \Rightarrow \underline{\underline{L = 1.6^{-1}\ H}} = .625\ H$

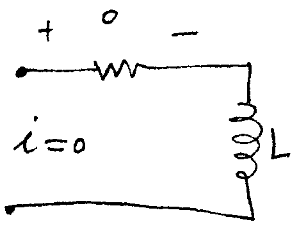
$R = 6.25K$

(b) $$W_L(0) = \frac{1}{2} L\, i_L^2(0) = 0. \Longrightarrow i(0) = 0.$$

$$W_C(0) = \frac{1}{2} C\, v_C^2(0) = \left(\frac{1}{2}\right)(.1)(\cancel{10^{-6}})\left(v_C^2(0)\right) = 45 \times \cancel{10^{-6}}$$

$$\therefore \quad v_C(0^-) = v_C(0^+) = 30 \text{ v}$$

For $t = \bar{0} \Longrightarrow$



$$\overset{+}{\underset{-}{}} \; v_C(0^+) = L \frac{di(0^+)}{dt} = 30 \text{ V.}$$

$$\frac{di(0^+)}{dt} = \frac{30}{.625} = 48. \ \frac{A}{Sec.}$$

(c) $$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$i(t) = A_1 e^{-2000t} + A_2 e^{-8000t}$$

$$\begin{cases} i(0) = A_1 + A_2 = 0. \\ i'(0) = -2000 A_1 - 8000 A_2 = 48. \end{cases} \Longrightarrow \begin{matrix} A_1 = 8 \text{ mA} \\ A_2 = -8 \text{ mA.} \end{matrix}$$

$$i(t) = 8 e^{-2000t} - 8 e^{-8000t}. \quad \text{mA} \quad (t \geq 0)$$

(d) $\frac{di(t)}{dt} = 0 \Rightarrow t_{max} = ?$

$$-\cancel{16000}\, e^{-2000t} + \overset{4}{\cancel{64000}}\, e^{-8000t} = 0$$

$$e^{-2000t} = 4e^{-8000t}$$

$$-2000t = \ln 4 - 8000t$$

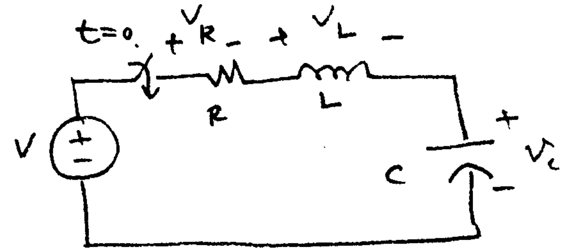
$$6000t = \ln 4 \Rightarrow t_{max} = \left(\frac{\ln 4}{6\cancel{000}}\right) 1\cancel{000} \text{ ms.}$$

$$= .231 \text{ ms.}$$

(e) $i(t)\Big|_{t_{max}} = i_{max} = i(.231) = 3.78 \text{ mA}$

(f) $v_L(t) = L\frac{di(t)}{dt} = -10e^{-2000t} + 40e^{-8000t}, \quad t \geq 0^+$ ✓

Step response:



$$V = Ri + L\frac{di}{dt} + v_c$$

$$i = C\frac{dv_c}{dt} \quad , \quad \frac{di}{dt} = C\frac{d^2v_c}{dt^2}$$

$$RC\frac{dv_c}{dt} + LC\frac{d^2v_c}{dt^2} + v_c = V$$

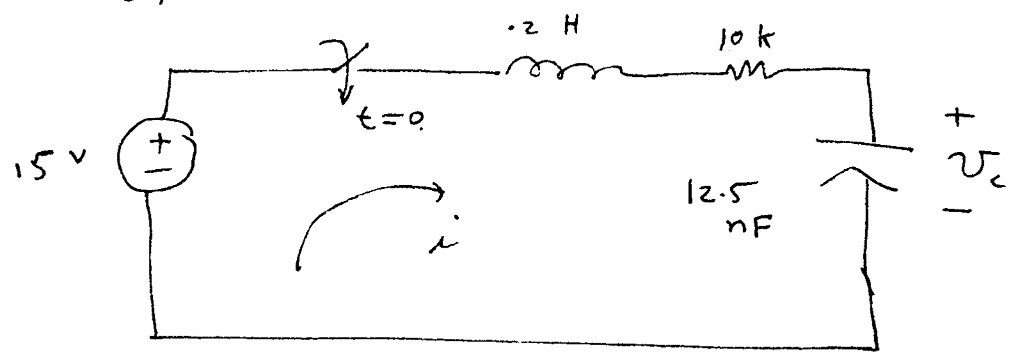
$$\frac{d^2v_c}{dt} + \frac{R}{L}\frac{dv_c}{dt} + \frac{1}{LC}v_c = \frac{V}{LC}$$

$$v_c = V + A_1' e^{s_1 t} + A_2' e^{s_2 t}$$

$$v_c = V + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t$$

$$v_c = V + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$$

#8.22 / 272 :



(a) $$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}\, i =$$

$$\frac{d^2 v_c}{dt^2} + \frac{R}{L}\frac{d v_c}{dt} + \frac{1}{LC}\, v_c = \frac{V}{LC}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + \underbrace{\frac{10\times10^3}{.2\ H}}_{50000} s + \underbrace{\frac{1}{(.2)(12.5\times10^{-9})}}_{4\times10^8} = 0$$

$$s_{1,2} = -25000 \pm \sqrt{(25^2\times10^6) - (400)(10^6)}$$

$$s_1 = -40000$$

$$s_2 = -10000$$

(b) $v_c(t) = V + A_1' e^{s_1 t} + A_2' e^{s_2 t}$

$$v_c(t) = 15 + A_1' e^{-40000t} + A_2' e^{-10000t}$$

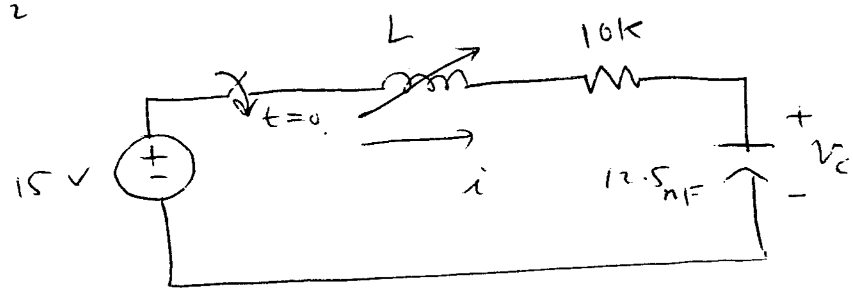
$v_c(0) = 0$ $\Longrightarrow$ $A_1' = 5\ V.$

$v_c'(0) = 0.$ $\quad A_2' = -20\ V$

$i_L(0) = C \dfrac{d v_c(0)}{dt} = 0$

(c) $i_L = i_c = C \dfrac{d v_c}{dt} = 205 \left[e^{-10000t} - e^{-40000t} \right]$ mA.

$\frac{8.24}{272}$:



a) $L = ?$

$$\sqrt{\alpha^2-\omega_0^2}=0 \Rightarrow \alpha^2=\omega_0^2 \Rightarrow \left(\frac{R}{2L}\right)^2=\frac{1}{LC}$$

$$\therefore \frac{R^2}{4L^{\cancel{2}}}=\frac{1}{\cancel{L}C} \Rightarrow L=\frac{R^2C}{4}=\frac{(10\times10^3)^2(12.5\times10^{-9})}{4}$$

$$= .3125 \text{ H}$$

b)
$$\begin{cases} i_L(t)=D_1' t e^{-\alpha t}+D_2' e^{-\alpha t} \\ i_L(0^+)=0 \\ v_L(0^+)=L\dfrac{di_L(0^+)}{dt}=15 \end{cases}$$

$\left(\text{at } t=0^+: \quad 15 = v_L(0^+) + \cancel{v_R(0^+)}^{\,0} + \cancel{v_C(0^+)}\right)$

$$\frac{di_L(0^+)}{dt}=\frac{15}{L}=48\ \frac{A}{s}.$$

$$D_2'=0, \quad D_1'=48\ \frac{A}{s}$$

$$\therefore \quad i_L(t)=48te^{-16000t} \ ; A \qquad t \geq 0^+.$$

c) $\frac{di_L}{dt} = 0. \Rightarrow i_L\left(\frac{1}{\alpha}\right) = \underline{1.104 \text{ mA}}$

$t = \frac{1}{\alpha}$

d) $t = \underline{62.5} \ \mu s$.

Chapter 9

Sinusoidal Steady-State Analysis

Thus far we have focused on circuits with DC sources; we are now ready to consider circuits energized by time-varying sources. In particular, we are interested in sinusoidal sources. The reasons are:

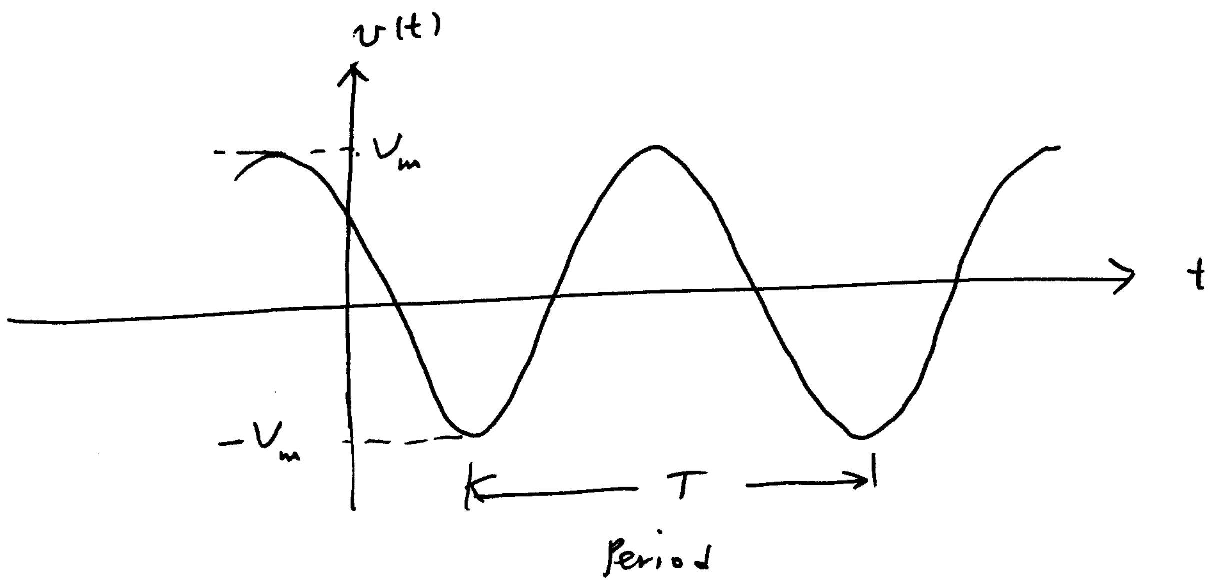
1) Generation, transmission, distribution, and consumption of electric energy occur under sin steady-state condition.

2) An understanding of sin behavior makes it possible to predict the behavior of circuits with nonsinusoidal sources.

3) steady-state sin behavior often simplifies the design of electrical systems. Thus a designer can spell out specifications in terms of a desired steady-state sin response.

<u>the sinusoidal source</u>

$$v(t) = V_m \cos(\omega t + \underbrace{\phi}_{\text{phase}}) \quad ; \quad \underbrace{\omega}_{\text{rad/sec}} = \frac{2\pi}{T} = 2\pi \underbrace{f}_{\text{cycle/sec (Hz)}}$$



Remarks

① ωt and ϕ must carry the same units.

$$\begin{cases} \omega t \to \text{rad} \\ \phi \to \text{rad} \end{cases}$$

$$\begin{cases} \omega t \to \text{degrees} \\ \phi \to \text{degrees} \end{cases}$$

$$\frac{180 \text{ Degrees}}{x \text{ Degrees}} = \frac{\pi \text{ radian}}{y \text{ radian}} \Rightarrow x^\circ = \frac{180^\circ}{\pi}(y \text{ rad}).$$

② The rms value. $\leftarrow$ important for power calculations

$f(t)$ periodic with T $\Rightarrow$

$$f_{rms} \triangleq \left(\frac{1}{T} \int_{t_0}^{t_0+T} f^2(t)\, dt \right)^{1/2}$$

when we have a sin voltage:

$$v(t) = V_m \cos(\omega t + \phi)$$

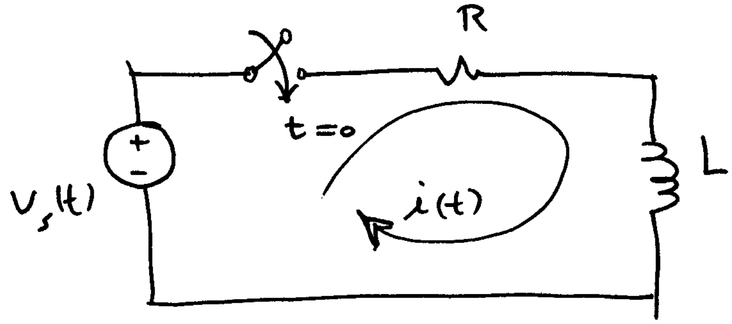
$$\therefore \quad V_{rms} = \left(\frac{1}{T} \int_T V_m^2 \cos^2(\omega t + \phi)\, dt \right)^{1/2}.$$

$$\cos^2\alpha = \frac{1 + \cos 2\alpha}{2}$$

$$\therefore \quad V_{rms} = \left(\frac{V_m^2}{2T} \int_T \left[1 + \cos 2(\omega t + \phi) \right] dt \right)^{1/2}$$

$$= \left(\frac{V_m^2}{2\cancel{T}} (\cancel{T} + 0) \right)^{1/2} = \frac{V_m}{\sqrt{2}} \,//.$$

the sinusoidal Response



$$v_s(t) = V_m \cos(\omega t + \phi).$$

$t \geq 0$:

$$-v_s(t) + Ri + L\frac{di}{dt} = 0$$

$$\therefore V_m \cos(\omega t + \phi) = Ri + L\frac{di}{dt}$$

$$\therefore i(t) = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta)\, e^{-(R/L)t}$$

Transient response

steady-state response

$$+ \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta),$$

this chapter focuses on the calculation of the steady-state response.

where

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right).$$

$$i_{ss}(t) = \frac{V_m}{\sqrt{R^2+\omega^2 L^2}} \cos(\omega t + \varphi - \theta)$$

Remarks

1. the ss-response is a sin function.

2. the freq of the ss-response is identical to the source freq (ω).

3. the maximum amplitude of the ss-response, in general, differs from V_m.

4. the phase angle of the response signal, in general, differs from that of the source θ.