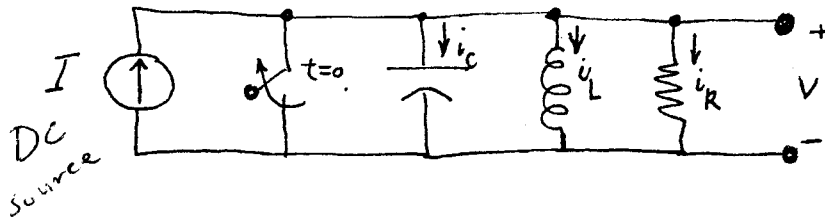


Step Response of a parallel RLC:



$$i_L + i_R + i_C = I$$

$$i_L + \frac{v}{R} + C \frac{dv}{dt} = I \quad (*)$$

$$v = L \frac{di_L}{dt} \Rightarrow \frac{dv}{dt} = L \frac{d^2 i_L}{dt^2}$$

$$\therefore i_L + \frac{L}{R} \frac{di_L}{dt} + LC \frac{d^2 i_L}{dt^2} = I$$

non homogeneous DE

$$\frac{d^2 i_L}{dt^2} + \frac{1}{RC} \frac{di_L}{dt} + \frac{1}{LC} i_L = \frac{1}{LC} I$$

Similar DE can be obtained in terms of v as follows:

$$(*) \Rightarrow \frac{1}{L} \int_{0^+}^t v(\tau) d\tau + I_0 + \frac{v}{R} + C \frac{dv}{dt} = I, \quad t \geq 0^+$$

or

$$\frac{1}{L} v(t) + \frac{dv}{dt} \frac{1}{R} + C \frac{d^2 v}{dt^2} = 0; \quad t \geq 0^+$$

$$\therefore \frac{d^2 v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{1}{LC} v(t) = 0; \quad t \geq 0^+$$

$$v = A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad (1)$$

$$v = e^{-\alpha t} [B_1 \cos \omega_d t + B_2 \sin \omega_d t] \quad (2)$$

$$v = D_1 \tau e^{-\alpha t} + D_2 e^{-\alpha t} \quad (3)$$

$$* \Rightarrow i_L = I - \frac{v}{R} - C \frac{dv}{dt}$$

$$(1) \Rightarrow i_L = I + \underbrace{A_1' e^{s_1 t} + A_2' e^{s_2 t}}$$

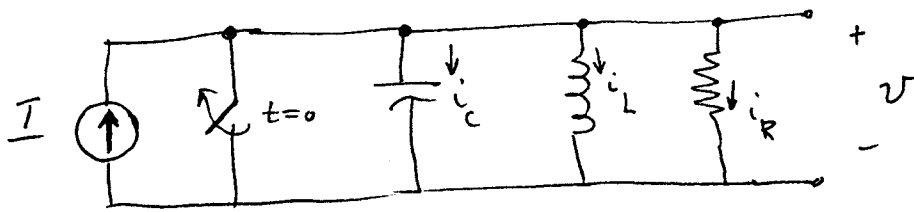
$$(2) \Rightarrow i_L = I + \underbrace{e^{-\alpha t} [B_1' \cos \omega_d t + B_2' \sin \omega_d t]}$$

$$(3) \Rightarrow i_L = I + \underbrace{D_1' \tau e^{-\alpha t} + D_2' e^{-\alpha t}}$$

if

same form
as the natural
response.

8.11 :
270



$$I = 24 \text{ mA}$$

$$R = 400 \text{ } \Omega$$

$$L = 25 \text{ mH}$$

$$C = 25 \text{ nF}$$

a) $v(t) = ?$

$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0.$$

$$s^2 + \frac{1}{(400)(25 \times 10^{-9})} s + \frac{10^{12}}{(25)(25)} = 0.$$

$$s^2 + 10^5 s + 16 \times 10^8 = 0.$$

$$s_{1,2} = -5 \times 10^4 \pm \sqrt{25 \times 10^8 - 16 \times 10^8} = -5 \times 10^4 \pm 3 \times 10^4$$

$$s_1 = -8 \times 10^4$$

$$s_2 = -2 \times 10^4$$

$$\begin{cases} i_L(t) = A_1' e^{-8 \times 10^4 t} + A_2' e^{-2 \times 10^4 t} + I \\ i_L(0) = 0 \leftarrow \text{given} \\ i_L'(0) = 0 \leftarrow v_L(0) = \frac{di_L(0)}{dt} L \Rightarrow \frac{di_L(0)}{dt} = 0 \end{cases}$$

$$\begin{cases} i_L(0) = A_1' + A_2' + \frac{24}{1} = 0 \Rightarrow \begin{cases} A_1' + A_2' = -24 \\ 4A_1' + A_2' = 0 \end{cases} \\ i_L'(0) = -8 \times 10^4 A_1' - 2 \times 10^4 A_2' = 0 \Rightarrow \end{cases}$$

$$A_1' = 8$$

$$A_2' = -32$$

$$\therefore i_L(t) = 8 e^{-8 \times 10^4 t} - 32 e^{-2 \times 10^4 t} + 24 \text{ mA}$$

$$v(t) = L \frac{di_L(t)}{dt} = (25 \times 10^{-3}) \left[-64 \times 10^4 e^{-8 \times 10^4 t} + 10^6 \times 4 e^{-2 \times 10^4 t} \right]$$

$$= 16000 \left(-e^{-8 \times 10^4 t} + e^{-2 \times 10^4 t} \right) \text{ mV}$$

$$= 16 e^{-2 \times 10^4 t} - 16 e^{-8 \times 10^4 t}$$

$$v. \begin{cases} 1) \lim_{t \rightarrow 0} v(t) = 0 \\ 2) \lim_{t \rightarrow \infty} v(t) = 0 \end{cases}$$

* notice that we could not evaluate $v(t)$ directly because $v'(0)$ is not known.

$$i_c(t) \Big|_{t=0} = c \frac{dv}{dt} \Big|_{t=0} = ?$$

$$i_R(t) = \frac{v}{R} = \frac{16000}{400} (e^{-2 \times 10^4 t} - e^{-8 \times 10^4 t}) \quad \text{mA.}$$

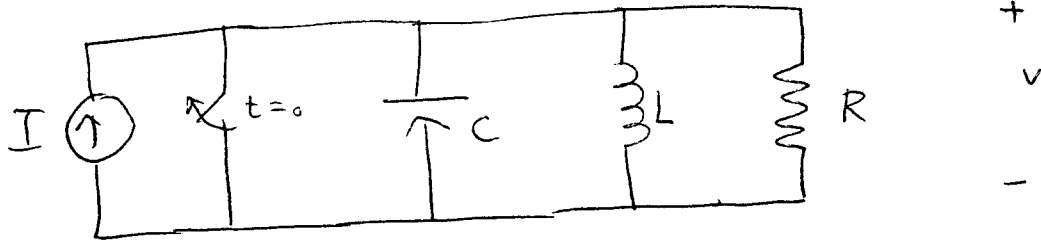
c) $i_C(t) = ?$

$$i_C(t) = C \frac{dv}{dt} = I - i_L(t) - i_R(t)$$

$$= 24 - \left[8e^{-8 \times 10^4 t} - 32e^{-2 \times 10^4 t} + 24 \right] - \left[40e^{-2 \times 10^4 t} - 40e^{-8 \times 10^4 t} \right]$$

$$i_C(t) = 32xe^{-8 \times 10^4 t} - 8xe^{-2 \times 10^4 t} \quad \text{mA.}$$

$$\frac{8.13}{270} : \quad \left(\text{EXA } \frac{8.6}{258} \right)$$



$$I = 24 \text{ mA}$$

$$L = 25 \text{ mH}$$

$$C = 25 \text{ nF}$$

$$R = 625 \text{ } \Omega$$

$$i_L(0^-) = i_L(0^+) = 0$$

$$v_C(0^-) = v_C(0^+) = 0 \quad \Rightarrow \quad \frac{di_L}{dt} = 0$$

a) $v(t) = ?$

$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$$

$$s_{1,2} = -3.2 \times 10^4 \pm j 2.4 \times 10^4 \text{ s}^{-1}$$

$$\left\{ \begin{array}{l} i_L(t) = I + e^{-\alpha t} \left[B_1' \cos \omega_d t + B_2' \sin \omega_d t \right] \\ i_L(0) = 0 \\ i_L'(0) = 0 \end{array} \right. \Rightarrow \begin{array}{l} B_1' = -24 \text{ mA} \\ B_2' = -32 \text{ mA} \end{array}$$

$$i_L(t) = 24 - e^{-32000t} \left[24 \cos 24000t + 32 \sin 24000t \right]; t \geq 0 \text{ mA}$$

$$v(t) = L \frac{di_L}{dt} = 40 e^{-32000t} \sin 24000t; \quad v, t \geq 0^+$$

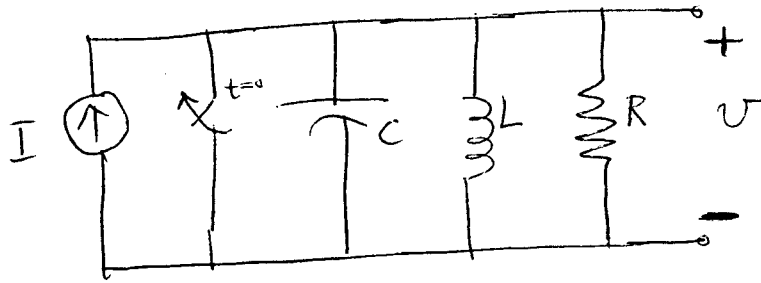
$$b) \quad i_c(t) = I - \cancel{i_k(t)} - i_L(t)$$
$$\quad \quad \quad \frac{v}{R}$$

$$= 24 e^{-32000t} \cos 24000t - 32 e^{-32000t} \sin 24000t$$

mA.

$t \geq 0^+$

8.15 :
270



$$W_L(0) = 0$$

$$W_C(0) = 153.6 \mu\text{J}$$

$$R = 5 \text{ k} \quad , \quad L = 120 \text{ H} \quad , \quad C = \frac{10}{3} \mu\text{F}$$

$$I = 4 \text{ mA}$$

$$a) \quad i_L'(t) = ?$$

$$W_L(0) = \frac{1}{2} L i_L^2(0) = 0 \Rightarrow i_L(0) = 0$$

$$W_C(0) = \frac{1}{2} \left(\frac{10}{3} \right) v_C^2(0) = 153.6 \Rightarrow v_C(0) = 9.6 \text{ V}$$

$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0 \Rightarrow s^2 + 60s + 2500 = 0$$

$$s_{1,2} = -30 \pm \sqrt{\frac{(30)^2}{900} - 2500} = -30 \pm j \frac{40}{5}$$

$$\left\{ \begin{array}{l} i_L(t) = \frac{4}{3} + e^{-30t} \left[B_1' \cos 40t + B_2' \sin 40t \right] ; \text{ mA} \\ i_L(0) = 0 \\ i_L'(0) = ? \end{array} \right. ; t \geq 0$$

$$v_L(t) \Big|_{t=0} = L \frac{di_L(t)}{dt} \Big|_{t=0} = 9.6 \text{ V}$$

$$\frac{di_L(0)}{dt} = \frac{9.6}{120} = 80$$

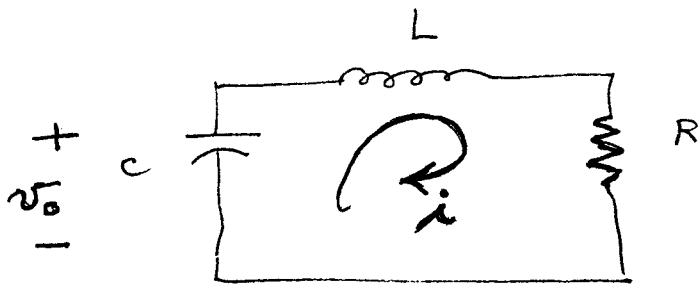
$$\therefore B_1' = -4 \text{ mA}$$

$$B_2' = -1 \text{ mA}$$

$$i_L(t) = 4 - e^{-30t} [4 \cos 40t + \sin 40t] \text{ mA}; \quad t \geq 0$$

RLC series circuit :

natural response :



$$v_c + v_L + v_R = 0.$$

$$Ri(t) + \left(\frac{1}{C}\right) \int^t i(\tau) d\tau + v_0 + L \frac{di(t)}{dt} = 0.$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{LC} i = 0.$$

$$s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$$

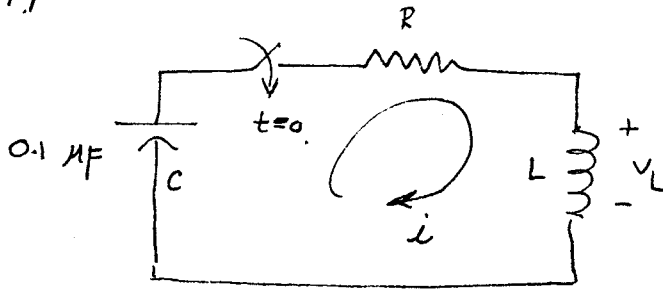
$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -d \pm \sqrt{d^2 - \omega_0^2}$$

$$d = \frac{R}{2L} \quad : \quad \text{Neper freq.} \quad \left(d = \frac{1}{2RC} \text{ for parallel case}\right)$$

$$\omega_0^2 = \frac{1}{LC}$$

- (a) overdamped $d^2 > \omega_0^2$
- (b) underdamped $d^2 < \omega_0^2$
- (c) critically damped $d^2 = \omega_0^2$

8.19 :
271



$$W_C(0) = 45 \text{ mJ}$$

$$W_L(0) = 0.$$

$$s_1 = -2000, \quad s_2 = -8000 \Rightarrow (s + 2000)(s + 8000) = 0.$$

$$s^2 + \underbrace{10000}_{\frac{R}{L}} s + \underbrace{16 \times 10^6}_{\frac{1}{LC}} = 0.$$

(a) $R, L = ?$

$$\frac{R}{L} = 10000$$

$$\frac{1}{LC} = \frac{1}{(L)(0.1 \times 10^{-6})} = 16 \times 10^8 \Rightarrow \underline{\underline{L = 1.6^{-1} \text{ H} = .625 \text{ H}}}$$

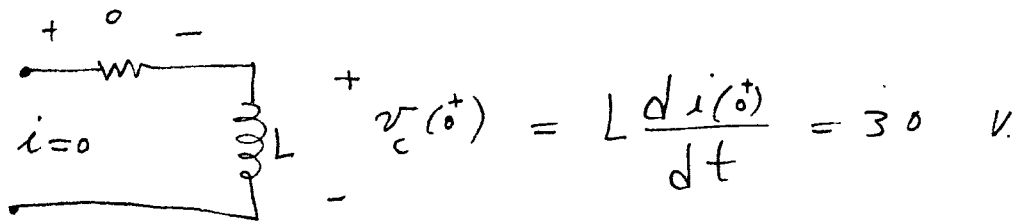
$$R = 625 \text{ k}$$

$$\textcircled{b} \quad W_L(0) = \frac{1}{2} L i_L^2(0) = 0 \Rightarrow i(0) = 0.$$

$$W_C(0) = \frac{1}{2} C v_C^2(0) = \left(\frac{1}{2}\right)(.1)(10^{-6}) (v_C^2(0)) = 45 \times 10^{-6}$$

$$\therefore v_C(0^-) = v_C(0^+) = 30 \text{ V}$$

For $t = 0 \Rightarrow$



$$\frac{di(0^+)}{dt} = \frac{30}{.625} = 48 \frac{\text{A}}{\text{Sec.}}$$

$$\textcircled{c} \quad i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$i(t) = A_1 e^{-2000t} + A_2 e^{-8000t}$$

$$\begin{cases} i(0) = A_1 + A_2 = 0. & \Rightarrow A_1 = 8 \text{ mA} \\ i'(0) = -2000A_1 - 8000A_2 = 48. & A_2 = -8 \text{ mA}. \end{cases}$$

$$i(t) = 8 e^{-2000t} - 8 e^{-8000t} \text{ mA} \quad (t \geq 0)$$

(d)

$$\frac{di(t)}{dt} = 0 \Rightarrow t_{\max} = ?$$

$$-16000 e^{-2000t} + 64000 e^{-8000t} = 0$$

$$e^{-2000t} = 4 e^{-8000t}$$

$$-2000t = \ln 4 - 8000t$$

$$6000t = \ln 4 \Rightarrow t_{\max} = \left(\frac{\ln 4}{6000} \right) 1000 \text{ ms.}$$

$$= .231 \text{ ms.}$$

(e)

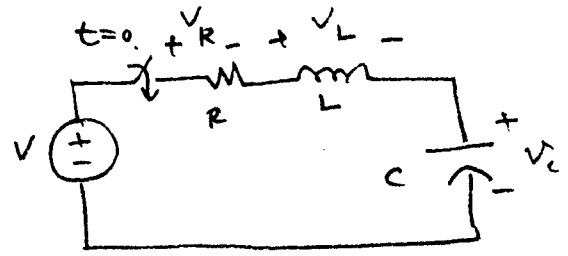
$$i(t) \Big|_{t_{\max}} = i_{\max} = i(.231) = 3.78 \text{ mA}$$

(f)

$$v_L(t) = L \frac{di(t)}{dt} = -10 e^{-2000t} + 40 e^{-8000t}, t \geq 0^+$$

Step Response :

$$V = Ri + L \frac{di}{dt} + v_c$$



$$i = C \frac{dv_c}{dt}, \quad \frac{di}{dt} = C \frac{d^2 v_c}{dt^2}$$

$$RC \frac{dv_c}{dt} + LC \frac{d^2 v_c}{dt^2} + v_c = V$$

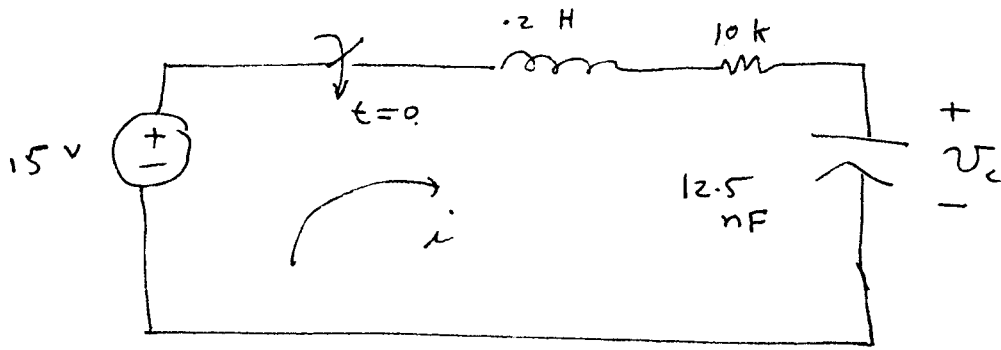
$$\frac{d^2 v_c}{dt^2} + \frac{R}{L} \frac{dv_c}{dt} + \frac{1}{LC} v_c = \frac{V}{LC}$$

$$v_c = V + A_1' e^{s_1 t} + A_2' e^{s_2 t}$$

$$v_c = V + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$$

$$v_c = V + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$$

8.22 :
272.



a)
$$\frac{d^2 i}{dt^2} + \frac{R}{L} \frac{di}{dt} + \frac{1}{Lc} i =$$

$$\frac{d^2 v_c}{dt^2} + \frac{R}{L} \frac{dv_c}{dt} + \frac{1}{Lc} v_c = \frac{V}{Lc}$$

$$s^2 + \frac{R}{L} s + \frac{1}{Lc} = 0.$$

$$s^2 + \frac{10 \times 10^3}{0.2 \text{ H}} s + \frac{1}{(0.2)(12.5 \times 10^{-9})} = 0$$

$50000 \qquad 4 \times 10^8$

$$s_{1,2} = -25000 \pm \sqrt{(25 \times 10^6) - (400)(10^6)}$$

$$s_1 = -40000$$

$$s_2 = -10000$$

$$(b) v_c(t) = V + A_1' e^{s_1 t} + A_2' e^{s_2 t}$$

$$v_c(t) = 15 + A_1' e^{-40000 t} + A_2' e^{-10000 t}$$

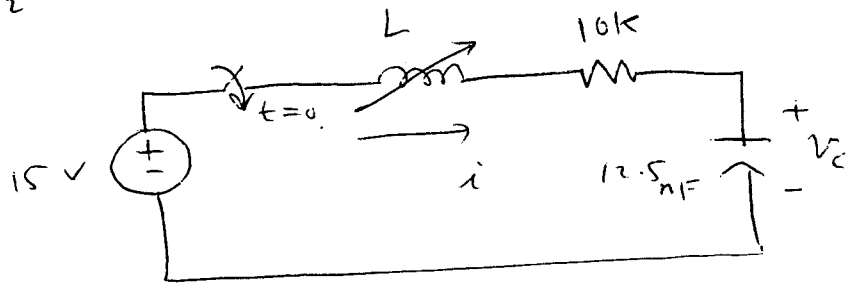
$$v_c(0) = 0 \Rightarrow A_1' = 5 \text{ V.}$$

$$v_c'(0) = 0 \Rightarrow A_2' = -20 \text{ V}$$

$$\uparrow i_L(0) = C \frac{dv_c(0)}{dt} = 0$$

$$(c) i_L = i_c = C \frac{dv_c}{dt} = 205 \left[e^{-10000 t} - e^{-40000 t} \right] \text{ mA.}$$

$$\frac{8.24}{272}$$



a) $L = ?$

$$\sqrt{\alpha^2 - \omega_0^2} = 0 \Rightarrow \alpha^2 = \omega_0^2 \Rightarrow \left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$

$$\therefore \frac{R^2}{4L^2} = \frac{1}{LC} \Rightarrow L = \frac{R^2 C}{4} = \frac{(10 \times 10^3)^2 (12.5 \times 10^{-9})}{4} = 0.3125 \text{ H}$$

b)
$$i_L(t) = D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$$

$$i_L(0^+) = 0$$

$$V_L(0^+) = L \frac{d i_L(0^+)}{dt} = 15 \quad (\text{at } t=0^+ : 15 = V_L(0^+) + V_R(0^+) + V_C(0^+))$$

$$\frac{d i_L(0^+)}{dt} = \frac{15}{L} = 48 \frac{\text{A}}{\text{s}}$$

$$D_2' = 0, \quad D_1' = 48 \frac{\text{A}}{\text{s}}$$

$$\therefore i_L(t) = 48 t e^{-16000 t} ; \text{ A } \quad t \geq 0^+$$

$$c) \quad \frac{di_L}{dt} = 0 \quad \Rightarrow \quad \dot{i}_L \left(\frac{1}{\alpha} \right) = \underline{1.104 \text{ mA}}$$

$$t = \frac{1}{\alpha}$$

$$d) \quad t = \underline{67.5 \text{ } \mu\text{s}}$$

Chapter 9

Sinusoidal Steady-State Analysis

Thus far we have focused on circuits with DC sources; we are now ready to consider circuits energized by time-varying sources. In particular, we are interested in sinusoidal sources. The

reasons are:

1) Generation, transmission, distribution, and consumption of electric energy occur under sinusoidal steady-state condition.

2) An understanding of sinusoidal behavior makes it possible to predict the behavior of circuits with non-sinusoidal sources.

3) steady-state sin behavior often simplifies the design of electrical systems. Thus a designer can spell out specifications in terms of a desired steady-state sin response.

the sinusoidal source

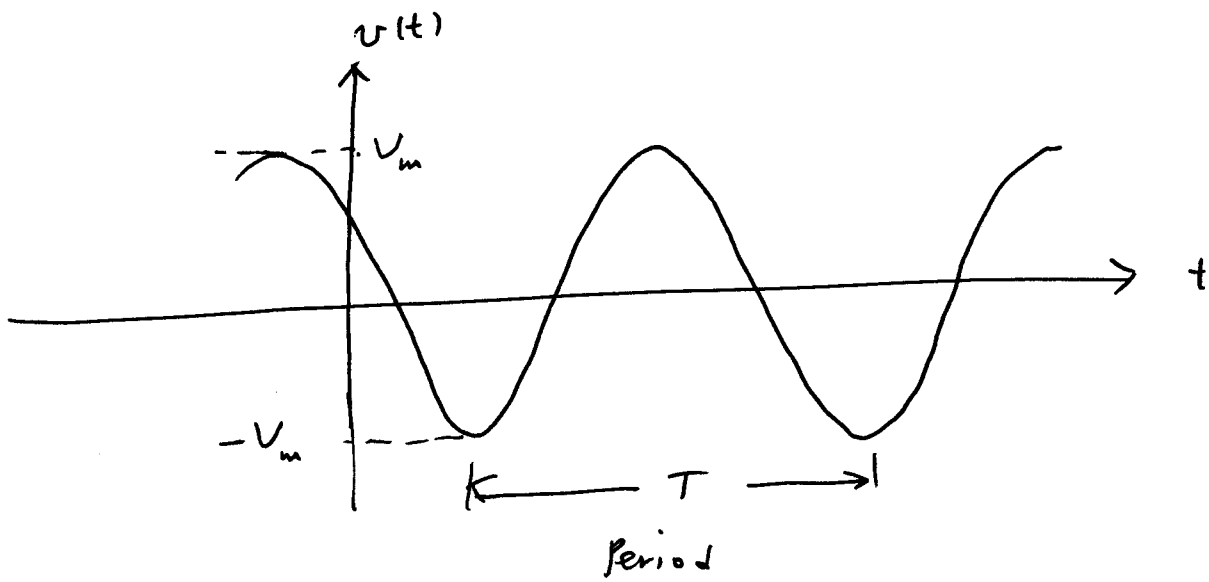
$$v(t) = V_m \cos(\omega t + \phi)$$

phase
↓
 ϕ

rad/sec

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Cycle/sec (Hz)



Remarks

① ωt and ϕ must carry the same units.

$$\left\{ \begin{array}{l} \omega t \rightarrow \text{rad} \\ \phi \rightarrow \text{rad} \end{array} \right.$$

$$\left\{ \begin{array}{l} \omega t \rightarrow \text{degrees} \\ \phi \rightarrow \text{degrees} \end{array} \right.$$

$$\frac{180 \text{ Degrees}}{x \text{ Degrees}} = \frac{\pi \text{ radian}}{y \text{ radian}} \Rightarrow x^\circ = \frac{180^\circ}{\pi} (y \text{ rad}).$$

② The rms value. ← important for power calculations

$f(t)$ periodic with $T = D$

$$f_{rms} \triangleq \left(\frac{1}{T} \int_{t_0}^{t_0+T} f^2(t) dt \right)^{1/2}$$

When we have a sin voltage:

$$v(t) = V_m \cos(\omega t + \phi)$$

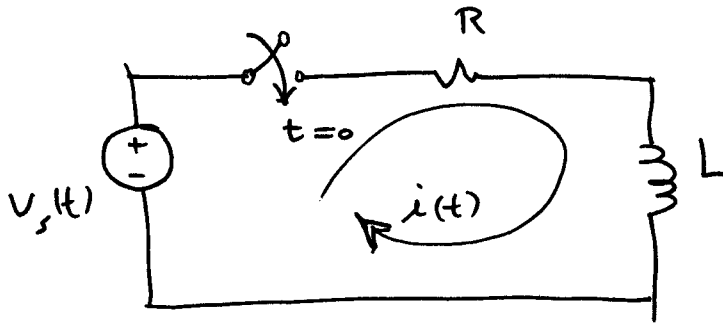
$$\therefore V_{rms} = \left(\frac{1}{T} \int_T V_m^2 \cos^2(\omega t + \phi) dt \right)^{1/2}$$

$$\cos^2 = \frac{1 + \cos 2\omega t}{2}$$

$$\therefore V_{rms} = \left(\frac{V_m^2}{2T} \int_T [1 + \cos 2(\omega t + \phi)] dt \right)^{1/2}$$

$$= \left(\frac{V_m^2}{2T} (T + 0) \right)^{1/2} = \frac{V_m}{\sqrt{2}} //$$

the sinusoidal Response



$$v_s(t) = V_m \cos(\omega t + \phi).$$

$t \geq 0$:

$$-v_s(t) + Ri + L \frac{di}{dt} = 0$$

$$\therefore V_m \cos(\omega t + \phi) = Ri + L \frac{di}{dt}$$

$$\therefore i(t) = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \theta) e^{-(R/L)t}$$

$$+ \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

where

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right).$$

Transient
Response

steady-state
Response

this chapter focuses
on the calculation of the
steady-state response.

Remarks

$$i_{ss}(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \varphi - \theta)$$

1. the ss-response is a sin function.
2. the freq of the ss-response is identical to the source freq (ω).
3. the maximum amplitude of the ss-response, in general, differs from V_m .
4. the phase angle of the response signal, in general, differs from that of the source \odot .